SKUPINA B

5.1.

1. $\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = \frac{x^{-1}}{-1} = -\frac{1}{x} + C$

2. $\int x^{12}\,dx = \frac{x^{13}}{13} + c$

3. $\int 4x^3\,dx = \frac{4x^4}{4} = x^4 + c$

4. $\int \frac{3x}{4}\,dx = 4\cdot\int 3x\,dx = \frac{3x^2}{8} + c$

5. $\int \frac{e^{2x}-1}{e^x-1}\,dx = \int e^x - 1 = ex + x + c$

6. $\int \frac{\cos^3 x - 0{,}8}{\cos^2 x}\,dx = \int \frac{\cos^3 x}{\cos^2 x} \int - \frac{0{,}8}{\cos^2 x} = \int \cos x - \int \frac{0{,}8}{\cos^2 x} = \sin x - 0{,}8\,tg\,x + c$

7. $\int \frac{4x - 2\sqrt{x}}{x}\,dx = \int \frac{\sqrt{x}\cdot(4\sqrt{x}-2)}{x}\,dx = \int \frac{4\sqrt{x}-2}{\sqrt{x}}\,dx = \int \frac{4\sqrt{x}}{\sqrt{x}}\,dx - \int \frac{2}{x^{\frac{1}{2}}}\,dx = \int 4\,dx - \int \frac{2}{x^{\frac{1}{2}}}\,dx =$

$= 4x - 4\sqrt{x} + c = 4(x - \sqrt{x}) + c$

8. $\int (8\cos x - 3\sin x)\,dx = 8\sin x + 3\cos x + c$

9. $\int \left(\frac{4x}{\sqrt{3x}} + (3-2x)^2\right) dx = \int \left(\frac{4x}{\sqrt{3x}} + 9 - 12x + 4x^2\right) dx =$

10. $\int x\cdot(2x-5)\,dx = \int 2x^2 - 5x\,dx = \frac{2x^3}{3} - \frac{5x^2}{2} + c$

11. $\int \frac{50}{(5x)^3}\,dx = \int \frac{\cancel{50}^{\,2}}{\cancel{125}_{\,5}\cdot x^3}\,dx = \int \frac{2}{5x^3}\,dx = \frac{2}{5}\cdot\int x^{-3}\,dx = \frac{2}{5}\cdot\frac{x^{-2}}{-2} = -\frac{2}{5x^2} + c$

12. $\int \left(x^3 - \frac{1}{x} + \frac{\sqrt[4]{x}}{2}\right) dx = \int \left(x^3 - \frac{1}{x} + \frac{x^{\frac{1}{4}}}{2}\right) dx = \frac{x^4}{4} - \ln|x| + \frac{2x\sqrt[4]{x}}{5} + c$

13. $\int \frac{e^{2x}-1}{ex}\,dx = ex + e^{-x} + c$

[5.2]

1. $\int 3e^{-3x+1}\,dx = \left| \begin{matrix} t = -3x+1 \\ dt = -3dx \\ -dt = 3dx \end{matrix} \right| = \int e^{t} \cdot (-dt) = -\int e^{t}\,dt = -e^{-3x+1} + c$

2. $\int \cos x \sqrt{\sin x}\,dx = \left| \begin{matrix} t = \sin x \\ dt = \cos x \end{matrix} \right| = \int \cos x \sqrt{t}\,dt =$

$= \int t^{\frac{1}{2}}\,dt = \frac{t^{\frac{3}{2}}}{\frac{3}{2}}\,dt = \frac{2t^{\frac{3}{2}}}{3} = \frac{2\sin^{\frac{3}{2}} x}{3} + c$

3. $\int \frac{1}{\sqrt{3-2x}}\,dx = \left| \begin{matrix} t = 3-2x \\ dt = -2 \\ -\frac{dt}{2} = 1 \end{matrix} \right| = \int -\frac{1}{2\sqrt{t}}\,dt = -\frac{1}{2}\int \frac{1}{\sqrt{t}} = -\frac{1}{2}\cdot 2\sqrt{t} = -\sqrt{t} =$

$= -\sqrt{3-2x} + c$

4. $\int \sqrt[3]{5-6x}\,dx = \int \left| \begin{matrix} t = 5x-6x \\ dt = -6 \\ -\frac{1}{6}\cdot dt = 1 \end{matrix} \right| = \int \sqrt[3]{t}\,dt \cdot -\frac{1}{6}dt = -\frac{1}{6}\int t^{\frac{1}{3}}\,dt = -\frac{1}{6}\cdot\frac{t^{\frac{4}{3}}}{\frac{4}{3}} =$

$-\frac{1}{6}\cdot\frac{3t^{\frac{4}{3}}}{4} = -\frac{2}{12}\cdot\frac{9t^{\frac{4}{3}}}{12} = -\frac{18t^{\frac{4}{3}}}{144} = -\frac{t^{\frac{4}{3}}}{8} = -\frac{(5-6x)^{\frac{4}{3}}}{8} + c$

5. $\int \sin x \cos^{5} x\,dx = \left| \begin{matrix} t = \cos x \\ dt = -\sin x \\ -dt = \sin x \end{matrix} \right| = \int t^{5} \cdot (-dt) = -\int t^{5}\,dt = -\frac{t^{6}}{6} = -\frac{\cos^{6} x}{6} + c$

6. $\int (2x+1)^{3}\,dx = \left| \begin{matrix} t = 2x+1 \\ dt = 2 \\ \frac{1}{2}dt = 1 \end{matrix} \right| = \int t^{3}\,\frac{1}{2}dt = \frac{1}{2}\int t^{3}\,dt = \frac{1}{2}\cdot\frac{t^{4}}{4} = \frac{t^{4}}{8} =$

$= \frac{(2x+1)^{4}}{8} + c$

~~7.~~ $\cancel{\int \frac{\cos x}{3\sin^{\frac{2}{3}} x}\,dx = \left| \begin{matrix} t = \cos x \\ dt = -\sin x \\ -dt = \sin x \end{matrix} \right| = \int \frac{t}{3\sin^{\frac{2}{3}} x}\,dt = -3\int \frac{t}{\sin^{\frac{2}{3}} x}}$

7. $\int \frac{\cos x}{3\sin^{\frac{2}{3}} x} = \frac{1}{3}\int \frac{\cos x}{\sin^{\frac{2}{3}} x} = \left| \begin{matrix} t = \sin x \\ dt = \cos x \end{matrix} \right| = \frac{1}{3}\int \frac{dt}{t^{\frac{2}{3}}} = \frac{1}{3}\int dt \cdot t^{-\frac{2}{3}} =$

$\frac{1}{3}\cdot\frac{t^{\frac{1}{3}}}{\frac{1}{3}} = t^{\frac{1}{3}} = \sin^{\frac{1}{3}} x + c$

8. $\int 6x \sin 3x^{2}\,dx = \left| \begin{matrix} t = 3x^{2} \\ dt = 6x \end{matrix} \right| = \int \sin t \cdot dt = -\cos t = -\cos 3x^{2} + c$

9. $\int \frac{8x^{2}}{\sqrt[3]{(8x^{3}+27)^{2}}}\,dx = \left| \begin{matrix} t = 8x^{3}+27 \\ dt = 24x^{2} \\ \frac{dt}{3} = 8x^{2} \end{matrix} \right| = \frac{1}{3}\int \frac{dt}{\sqrt[3]{(t)^{2}}} = \frac{1}{3}\cdot t^{-\frac{2}{3}} = \frac{1}{3}\cdot\frac{t^{\frac{1}{3}}}{\frac{1}{3}} = t^{\frac{1}{3}} =$

$(8x^{3}+27)^{\frac{1}{3}} + c$

~~10.~~ $\cancel{\int 6\tan 3x\,dx = 6\int \tan 3x\,dx}$

~~18. $\int 6\tan 3x\,dt = 2\int \tan t\,dx = | t = 2\tan x$~~

10. $\int 6\tan 3x\,dx = 6\cdot\int \tan 3x = \begin{vmatrix} t=3x \\ dt=3\end{vmatrix} = 6\int \frac{\tan(t)}{3}\,dt = 6\cdot\frac{1}{3}\int \tan(t)\,dt =$

$= 2\cdot\int\frac{\sin(t)}{\cos(t)} = \Big| \Big| -2\cdot\int\frac{1}{\cos(t)} = -2\ln|\cos(t)| = -2\ln|\cos 3x|$

11. $\int\frac{3x}{(x^2+1)^2}\,dx = \begin{vmatrix} t = x^2+1\\ dt = 2x \\ \frac{1}{2}dt = x\end{vmatrix} = \int\frac{3}{2t^2}\,dt = \frac32\int\frac{1}{t^2} = \frac32\int -\frac{1}{(2-1)t^{2-1}} =$

$\frac32\cdot -\frac1t = -\frac{3}{2t} = -\frac{3}{2(x^2+1)} + c$

[5.3]

1. $\int_0^4\sqrt{x}\,dx = \int_0^4 x^{\frac12}\,dx = \left[\frac{x^{\frac32}}{\frac32}\right]_0^4 = \left[\frac{2\sqrt{x^3}}{3}\right]_0^4 = \frac{16}{3} - 0 = \frac{16}{3}$

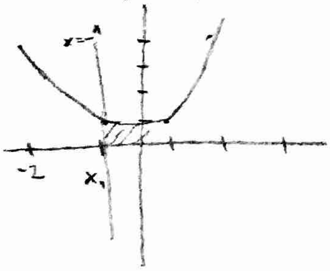
2. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\,dx = [\sin x]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 1-(-1) = \underline{\underline{2}}$

3. $\int_0^3 e^{\frac{x}{3}}\,dx = \begin{vmatrix} t = \frac{x}{3} \\ dt = \frac13 dx \\ 3dt = dx\end{vmatrix} = \int_0^1 e^t\cdot 3dt = 3\int_0^1 e^t dt = 3\cdot[e^t]_0^1 = 3\cdot(e-1)$

4. $\int_0^\pi \frac{2\sin x}{5+4\cos x}\,dx = \frac15\cdot\int_0^\pi\frac{2\sin x}{4\cos x}\,dx = \frac15\int_0^\pi\frac{\sin x}{2\cos x}$

5. $\int_{-1}^1 2x^3\,dx = \left[\frac{2x^4}{4}\right]_{-1}^1 = \left[\frac{x^4}{2}\right]_{-1}^1 = \left(\frac{1^4}{2} - \left(-\frac{1^4}{2}\right)\right) = 1$

[5.4] $y=0$, $x=-1$, $y=x^2$



$y=x^2$

$x=1$	$y=1$	$x=-2$	$y=4$
$x=2$	$y=4$		
$x=3$	$y=9$		
$x=-1$	$y=1$		

$\int_{-1}^0 x^2 - 0\,dx = \int_{-1}^0 x^2\,dx = \left[\frac{x^3}{3}\right]_{-1}^0 = 0-\left(-\frac13\right) = \underline{\frac13}$