5.1

1. $\int \frac{1}{x^2}dx = \ln x^2 + c$

2. $\int x^{12}dx = \frac{x^{13}}{13} + c$

3. $\int 4x^3 dx = \cancel{4}\frac{x^4}{\cancel{4}} = x^4 + c$

4. $\int \frac{3x}{4}dx = \frac{3\frac{x^2}{2}}{4} = \frac{3x^2}{8} + c$

5. $\int \frac{e^{2x}-1}{e^x-1}dx = \frac{(\cancel{e^x-1})\cdot(e^x+1)}{\cancel{e^x-1}} = \int e^x + 1 = e^x + x + c$

6. $\int \frac{\cos^3 x - 0,8}{\cos^2 x}dx = \int\frac{\cos^3 x}{\cos^2 x} - \int\frac{0,8}{\cos^2 x} = \int \cos x - \int\frac{0,8}{\cos^2 x} = \sin x - 0,8\, tg x + c$

7. $\int \frac{4x - 2\sqrt{x}}{x} = \int\frac{4x}{x} - \int\frac{2\sqrt{x}}{x} = 4\int\frac{x}{x} - 2\int\frac{\sqrt{x}}{x} = 4\int 1 - 2\int\frac{\sqrt{x}}{x} =$

$4x - 2\frac{\frac{x^{\frac{1}{2}}}{1}}{\frac{x^2}{2}} = 4x - 2\frac{2x^{\frac{1}{2}}}{x^2} = 4x - \frac{4\sqrt{x}}{x^2}$

8. $\int (8\cos x - 3\sin x)dx = 8\sin x + 3\cos x + c$

9. $\int\left(\frac{4x}{\sqrt{3x}} + (3-2x)^2\right)dx = \int\left(\frac{4x}{\sqrt{3x}} + 9 - 12x + 4x^2\right)dx = \frac{\frac{4x^2}{2}}{\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}} + 9x - \frac{12x^2}{2} +$

$\frac{4x^3}{3} = \frac{4x^2\cdot\frac{3}{2}}{2\cdot 3x^{\frac{3}{2}}} + 9x - 6x^2 + \frac{4x^3}{3} = \frac{\frac{12}{2}x^2}{2\sqrt{3x^3}} + 9x - 6x^2 + \frac{4x^3}{3} =$

$\frac{3x^2}{\sqrt{3x^3}} + 9x - 6x^2 + \frac{4x^3}{3} + c$

10. $\int x(2x-5)dx = \int 2x^2 - 5x = \frac{2x^3}{3} - \frac{5x^2}{2} = \frac{2}{3}x^3 - \frac{5}{2}x^2 + c$

11. $\int \frac{50}{(5x)^3}dx = \frac{50}{5^3}\int x^{-3}dx = \frac{2}{5}\left(-\frac{x^{-2}}{2}\right) = -\frac{1}{5x^2}$

12. $\int\left(x^3 - \frac{1}{x} + \sqrt[4]{\frac{x}{2}}\right)dx = \frac{x^4}{4} - \ln|x| + \int\frac{x^{\frac{1}{4}}}{2} = \frac{x^4}{4} - \ln|x| + \frac{\frac{x^{\frac{5}{4}}}{\frac{5}{4}}}{\frac{2x}{1}} =$

$\frac{x^4}{4} - \ln|x| + \frac{x^{\frac{5}{4}}}{\frac{10}{4}x} = \frac{x^4}{4} - \ln|x| + \frac{\sqrt[4]{x^5}}{\frac{5}{2}x} = \frac{x^4}{4} - \ln|x| + \frac{2}{5}x + \sqrt[4]{x^5} + c$

13. $\int \frac{e^{2x}-1}{e^x}dx$

5.2

1. $\int 3e^{-3x+1}dx = \left| \begin{matrix} -3x+1=t \\ -3dx=dt \\ 3dx=-dt \end{matrix} \right| \int e^{t} - dt = -e^{t} = e^{-3x+1}+C$

4. $\int \sqrt[3]{5-6x}\,dx = \left| \begin{matrix} 5-6x=t \\ -6dx=dt \\ dx=\frac{dt}{-6} \end{matrix} \right| = -\frac{1}{6}\int \sqrt[3]{t}\,dt = -\frac{1}{6}\int t^{\frac{1}{3}}dt = -\frac{1}{6}\cdot\frac{3}{4}t^{\frac{4}{3}}+C$

$= -\frac{3}{24}t^{\frac{4}{3}} = -\frac{1}{8}\sqrt[3]{t^4} = -\frac{1}{8}\sqrt[3]{(5-6x)^4}$

6. $\int (2x+1)^3dx = \left| \begin{matrix} 2x+1=t \\ 2dx=dt \\ dx=\frac{dt}{2} \end{matrix} \right| = \int (t)^3\frac{dt}{2} = \frac{t^4}{4}\cdot\frac{dt}{2} = \frac{t^4dt}{8} = \frac{(2x+1)^4}{8}$

11. $\int \frac{3x}{(x^2+1)^2}dx = \left| \begin{matrix} x^2+1=t \\ 3xdx=dt \\ dx=\frac{dt}{3x} \end{matrix} \right| = \int \frac{\cancel{3x}}{(t)^2}\cdot\frac{dt}{\cancel{3x}} = \frac{dt}{t^2} = \frac{\frac{t^2}{2}}{\frac{t^3}{3}} = \frac{3\cancel{t^2}}{2\cancel{t^3}} = \frac{3}{2t}$

$= \frac{3}{2(x^2+1)}$

9. $\int \frac{8x^2}{\sqrt[3]{8x^3+27}}dx - \left| \begin{matrix} 8x^3+27=t \\ 8x^2dx=dt \\ dx=\frac{dt}{8x^2} \end{matrix} \right| = \int \frac{\cancel{8x^2}}{\sqrt[3]{t^2}}\cdot\frac{dt}{\cancel{8x^2}} = \frac{dt}{t^{\frac{2}{3}}} = \frac{\frac{t^2}{2}}{\frac{t^{\frac{5}{3}}}{\frac{5}{3}}}$

$= \frac{\cancel{t^2}\cdot\frac{5}{3}}{2\cdot\cancel{t^{\frac{5}{3}}}} = \frac{t^{1/3}}{\frac{1}{3}} = 3t^{1/3}$

$\hookrightarrow 3(8x^3+27)^{1/3}$

5.3

1. $\int_0^4 \sqrt{x}\,dx = \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 = \frac{4^{\frac{3}{2}}}{\frac{3}{2}} - \frac{0}{\frac{3}{2}} = \frac{\sqrt{4^3}}{\frac{3}{2}} - 0 = \frac{2\sqrt{4^3}}{3} = \frac{2\sqrt{64}}{3} = \frac{16}{3}$

$x^{\frac{1}{2}}$

2. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\,dx = [\sin x]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right) = +1-(-1) = 2$

3. $\int_0^3 e^{\frac{x}{3}} = \left[e^{\frac{x}{3}}\right]_0^3 = e^{\frac{3}{3}} - e^{\frac{0}{3}} = e^1 - 1$

$\sqrt[3]{e^x}$

4. $\int_0^\pi \frac{2\sin x}{5+4\cos x} = \left[\frac{2-\cos x}{5x+4\sin x}\right]_0^\pi$

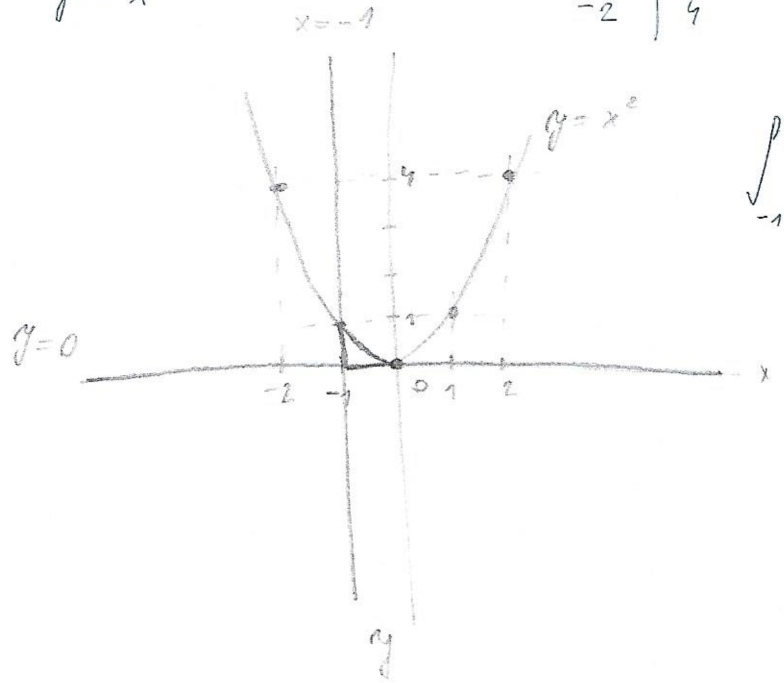
5. $\int_{-1}^1 2x^3\,dx = \left[\frac{2x^4}{4}\right]_{-1}^1 = \frac{2\cdot 1^4}{4} - \frac{2\cdot(-1)^4}{4} = \frac{2}{4} - \frac{2}{4} = 0$

(1) $y = 0$

$x = -1$

$y = x^2$

| x | y |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| -1 | 1 |
| -2 | 4 |




$$\int_{-1}^{0} x^2 = \left[\frac{x^3}{3}\right]_{-1}^{0}$$

$$\left(\frac{-1^3}{3}\right) - \left(\frac{0^3}{3}\right) = -\frac{1}{3} - \frac{0}{3}$$

$$= -\frac{1}{3}$$

(2) $y = x^2 + 4x$

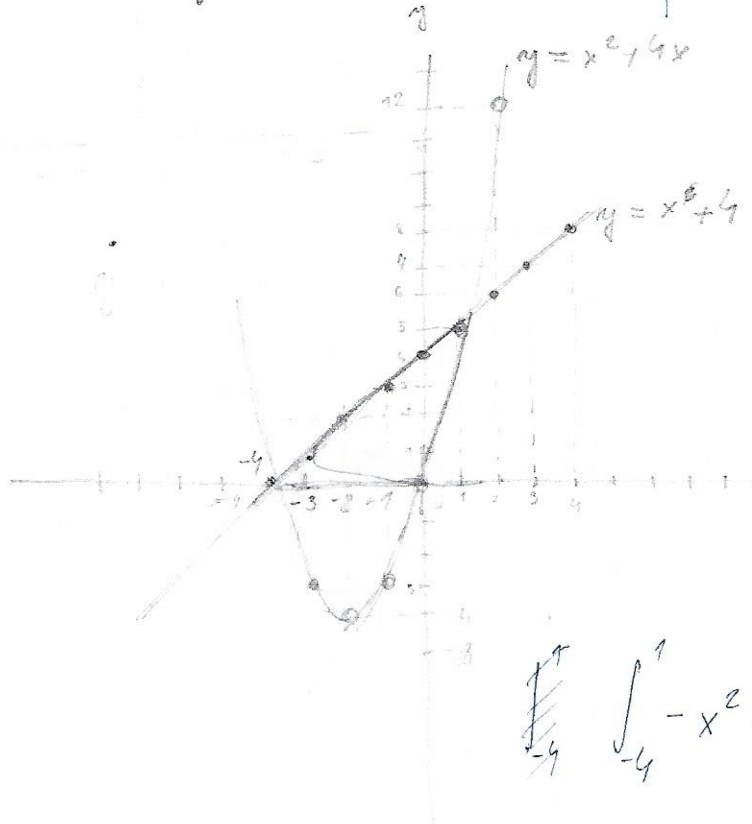
$y = x + 4$

$\hookrightarrow x = y - 4$

| x | y |
|---|---|
| 1 | 5 |
| 2 | 6 |
| 3 | 7 |
| 4 | 8 |
| −1 | ~~#~~ 3 |
| −2 | 2 |
| 0 | 4 |

| x | y |
|---|---|
| 1 | 5 |
| 2 | 12 |
| ~~3~~ | |
| −1 | −3 |
| −2 | −4 |
| ~~−3~~ | |
| 0 | 0 |
| −3 | |

$\overset{9}{(-3)^2} + \overset{-12}{4 \cdot (-3)} = -3$




x

$$\int_{-4}^{1} (x+4) - (x^2+4x)\,dx$$

$$\int_{-4}^{1} x + 4 - x^2 + 4x \quad dx =$$

$$\int_{-4}^{1} -x^2 + 5x + 4\,dx$$

$$\int_{-4}^{1} -x^2 + \int_{-4}^{1} 5x + \int_{-4}^{1} 4dx =$$

$$\hookrightarrow 4\int_{-4}^{1} dx$$

$$\left[-\frac{x^3}{3}\right]_{-4}^{1} + \left[\frac{5x^2}{2}\right]_{-4}^{1} + 4\left[x\right]_{-4}^{1}$$

$$\left[\left(-\frac{1^3}{3}\right) - \left(-\frac{\overset{-64}{-4^3}}{3}\right)\right] + \left[\frac{5\cdot(1)^2}{2} - \frac{5\cdot\overset{16}{(-4)^2}}{2}\right] + 4\left(1 \overset{+}{-} (-4)\right) =$$

$$\left[\left(-\frac{1}{3}\right) - + \left(-\frac{64}{3}\right)\right] + \left[\frac{5}{2} - \frac{80}{2}\right] + 20 =$$

$$\left(\frac{63}{3}\right) + - \left(-\frac{75}{2}\right) + 20 =$$

$$21 - \frac{75}{2} + 20 = \frac{42}{2} - \frac{75}{2} + \frac{40}{2} = \frac{7}{2}$$