SKUPINA C

5.1 Neurčité integrály

1. $\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}\,dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2}{3}\sqrt{x^3} + c$

2. $\int \frac{3}{4}\,dx = \frac{3}{4}\int 1\,dx = \frac{3}{4}x + c$

3. $\int 4x^{-3}\,dx = 4\int x^{-3}\,dx = 4\frac{x^{-2}}{-2} + c = -2x^{-2} + c$

4. $\int 3\sqrt{x}\,dx = 3\int x^{\frac{1}{2}}\,dx = 3\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c = 3\cdot\frac{2}{3}x^{\frac{3}{2}} + c = 2x^{\frac{3}{2}} + c$

5. $\int e^x\left(1 + \frac{e^x}{3}\right)dx = \int e^x + \frac{e^{2x}}{3}\,dx = \int e^x dx + \int \frac{e^{2x}}{3}\,dx = \int e^x dx + \frac{1}{3}\int e^{2x}\,dx = e^x + \frac{1}{3}\cdot\frac{1}{2}e^{2x} + c =$

$= e^x + \frac{e^{2x}}{6} + c$

6. $\int\left(\frac{(2\sqrt{x}+1)^2}{x^2} + \cos^{-2}x\right)dx = \int\frac{(2\sqrt{x}+1)^2}{x^2}\,dx + \int\frac{1}{\cos^2 x}\,dx = \int\frac{4x+4\sqrt{x}+1}{x^2}\,dx + \int\frac{1}{\cos^2 x}\,dx =$

$= \int\frac{4}{x}\,dx + \int\frac{4\sqrt{x}}{x^2}\,dx + \int\frac{1}{x^2}\,dx + \int\frac{1}{\cos^2 x}\,dx = 4\int\frac{1}{x}\,dx + 4\int \underbrace{x^{\frac{1}{2}}\cdot x^{-2}}_{x^{-\frac{3}{2}}}\,dx + \int x^{-2}\,dx + \int\frac{1}{\cos^2 x}\,dx =$

$= 4\ln|x| + 4\cdot\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + \frac{x^{-1}}{-1} + \operatorname{tg} x + c = 4\ln|x| - 8\frac{1}{\sqrt{x}} - \frac{1}{x} + \operatorname{tg} x + c$

7. $\int(\sqrt{x}+1)(x-\sqrt{x}+1)\,dx = \int x\sqrt{x} \cancel{-x} + \cancel{\sqrt{x}} + \cancel{x} - \cancel{\sqrt{x}} + 1\,dx = \int x\sqrt{x} + 1\,dx =$

$= \int x\cdot x^{\frac{1}{2}}\,dx + \int 1\,dx = \int x^{\frac{3}{2}}\,dx + \int 1\,dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + x + c = \frac{2}{5}x^{\frac{5}{2}} + x + c$

8. $\int(4x^5 + x^3 - 5)\,dx = 4\int x^5\,dx + \int x^3\,dx - 5\int 1\,dx = 4\frac{x^6}{6} + \frac{x^4}{4} - 5x + c =$

$= \frac{2}{3}x^6 + \frac{1}{4}x^4 - 5x + c$

9. $\int\frac{x^4 - 10x^2 + 5}{x^2}\,dx = \int x^2\,dx - 10\int 1\,dx + 5\int\frac{1}{x^2}\,dx = \int x^2\,dx - 10\int 1\,dx + 5\int x^{-2}\,dx =$

$= \frac{x^3}{3} - 10x + 5\frac{x^{-1}}{-1} + c = \frac{x^3}{3} - 10x - \frac{5}{x} + c$

10. $\int\frac{\sqrt{x}}{x^2}\,dx = \int x^{\frac{1}{2}}\cdot x^{-2}\,dx = \int x^{-\frac{3}{2}}\,dx = \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + c = -2\frac{1}{\sqrt{x}} + c$

11. $\int\frac{5}{x^{\frac{2}{7}}}\,dx = 5\int x^{-\frac{2}{7}}\,dx = 5\frac{x^{\frac{5}{7}}}{\frac{5}{7}} + c = 7x^{\frac{5}{7}} + c$

12. $\int\frac{x^3 - 2x + 1}{x^3}\,dx = \int 1\,dx - 2\int x^{-2}\,dx + \int x^{-3}\,dx = x - 2\frac{x^{-1}}{-1} + \frac{x^{-2}}{-2} + c = x + \frac{2}{x} - \frac{1}{2x^2} + c$

13. $\int\left(\frac{3}{x^4} + \frac{1}{\sqrt{x}}\right)dx = 3\int x^{-4}\,dx + \int x^{-\frac{1}{2}}\,dx = 3\frac{x^{-3}}{-3} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c = -\frac{1}{x^3} + 2\sqrt{x} + c$

5.2 Substituční metoda

1. $$\int \sin(2x-5)\,dx = \left| \begin{aligned} t &= 2x-5 \\ dt &= 2\,dx \\ \tfrac{dt}{2} &= dx \end{aligned} \right| = \int \sin t \,\frac{dt}{2} = \frac{1}{2}\int \sin t\,dt = -\frac{1}{2}\cos t = \underline{-\frac{1}{2}\cos(2x-5)+c}$$

2. $$\int \frac{3\ln^2 x}{x}\,dx = \left| \begin{aligned} t &= \ln x \\ dt &= \tfrac{1}{x}\,dx \\ x\,dt &= dx \end{aligned} \right| = \int \frac{3t^2}{\cancel{x}}\,\cancel{x}\,dt = 3\int t^2\,dt = 3\,\frac{t^3}{3} = \underline{\ln^3 x + c}$$

3. $$\int \frac{1}{\sqrt{5-4x}}\,dx = \left| \begin{aligned} t &= 5-4x \\ dt &= -4\,dx \\ -\tfrac{dt}{4} &= dx \end{aligned} \right| = \int \frac{1}{\sqrt{t}}\left(-\frac{1}{4}\right)dt = -\frac{1}{4}\int t^{-\frac{1}{2}}\,dt = -\frac{1}{4}\,\frac{t^{\frac{1}{2}}}{\frac{1}{2}} = -\frac{1}{2}\sqrt{t} = \underline{-\frac{1}{2}\sqrt{5-4x}+c}$$

4. $$\int \frac{e^{2x}-1}{e^x}\,dx = \left| \begin{aligned} t &= e^x \\ dt &= e^x\,dx \\ \tfrac{dt}{e^x} &= dx \end{aligned} \right| = \int \frac{t^2-1}{t}\,\frac{1}{t}\,dt = \int \frac{t^2-1}{t^2}\,dt = \int 1\,dt - \int t^{-2}\,dt = t - \frac{t^{-1}}{-1} =$$

$$= \underline{e^x + \frac{1}{e^x} + c}$$

5. $$\int x e^{-x^2}\,dx = \left| \begin{aligned} t &= -x^2 \\ dt &= -2x\,dx \\ -\tfrac{dt}{2x} &= dx \end{aligned} \right| = \int x e^t\left(-\frac{1}{2x}\right)dt = \int e^t\cdot\left(-\frac{1}{2}\right)dt = -\frac{1}{2}\int e^t\,dt = -\frac{1}{2}e^t = \underline{-\frac{1}{2}e^{-x^2}+c}$$

6. $$\int \frac{1}{6}\left(1-\frac{x}{6}\right)^{-2}dx = \left| \begin{aligned} t &= 1-\tfrac{x}{6} \\ dt &= -\tfrac{1}{6}\,dx \\ -dt &= \tfrac{1}{6}\,dx \end{aligned} \right| = -\int t^{-2}\,dt = -\frac{t^{-1}}{-1} = t^{-1} = \frac{1}{1-\frac{x}{6}}+c = \frac{1}{\frac{6-x}{6}}+c = \underline{\frac{6}{6-x}+c}$$

7. $$\int \frac{1}{\cos^2(1-x)}\,dx = \left| \begin{aligned} t &= 1-x \\ dt &= -dx \\ -dt &= dx \end{aligned} \right| = -\int \frac{1}{\cos^2 t}\,dt = -\mathrm{tg}\,t = \underline{-\mathrm{tg}\,(1-x)+c}$$

8. $$\int 6x^2 e^{-2x^3}\,dx = \left| \begin{aligned} t &= -2x^3 \\ dt &= -6x^2\,dx \\ -dt &= 6x^2\,dx \end{aligned} \right| = -\int e^t\,dt = -e^t = \underline{-e^{-2x^3}+c}$$

9. $$\int \frac{\sin x}{2\sqrt{\cos^3 x}}\,dx = \left| \begin{aligned} t &= \cos x \\ dt &= -\sin x\,dx \\ -dt &= \sin x\,dx \end{aligned} \right| = -\int \frac{1}{2\sqrt{t^3}} = -\frac{1}{2}\int t^{-\frac{3}{2}} = -\frac{1}{2}\,\frac{t^{-\frac{1}{2}}}{-\frac{1}{2}} = t^{-\frac{1}{2}} = \underline{\frac{1}{\sqrt{\cos x}}+c}$$

10. $$\int \frac{4\cos x}{\sqrt[3]{1+2\sin x}}\,dx = \left| \begin{aligned} t &= 1+2\sin x \\ dt &= 2\cos x\,dx \\ \tfrac{dt}{2} &= \cos x\,dx \end{aligned} \right| = \int \frac{4}{\sqrt[3]{t}}\,\frac{1}{2}\,dt = 2\int t^{-\frac{1}{3}}\,dt = 2\,\frac{t^{\frac{2}{3}}}{\frac{2}{3}} = 3t^{\frac{2}{3}} =$$

$$= \underline{3(1+2\sin x)^{\frac{2}{3}} + c}$$

11. $$\int \sqrt{1+2x}\,dx = \left| \begin{aligned} t &= 1+2x \\ dt &= 2\,dx \\ \tfrac{1}{2}\,dt &= dx \end{aligned} \right| = \int \sqrt{t}\cdot\frac{1}{2}\,dt = \frac{1}{2}\int t^{\frac{1}{2}}\,dt = \frac{1}{2}\,\frac{t^{\frac{3}{2}}}{\frac{3}{2}} = \frac{1}{3}t^{\frac{3}{2}} =$$

$$= \underline{\frac{(1+2x)^{\frac{3}{2}}}{3} + c}$$

5.3.

(1.) $\int_1^4 3\sqrt{x}\,dx = 3\int_1^4 \sqrt{x}\,dx = 3\int_1^4 x^{\frac{1}{2}}dx = 3\left[\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_1^4 =$

$= 3\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4 = 3\left[\frac{4^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1^{\frac{3}{2}}}{\frac{3}{2}}\right] = 3\left[\frac{8}{\frac{3}{2}} - \frac{1}{\frac{3}{2}}\right] = 3\left[\frac{7}{\frac{3}{2}}\right] = \underline{\underline{14}}$

(2.) $\int_2^5 \frac{4}{x}\,dx = 4\int_2^5 \frac{1}{x}\,dx = 4\left[\ln|x|\right]_2^5 = 4(\ln 5 - \ln 2) =$

$= \underline{\underline{4\ln\frac{5}{2}}}$

(3.) $\int_0^{\pi} 5\sin 4x\,dx = 5\int_0^{\pi} \sin 4x\,dx = \left|\begin{matrix} s = 4x \\ ds = 4\end{matrix}\right| = 5\int_{4\cdot 0}^{4\cdot\pi} \sin s\,ds =$ $\to \sin\pi = 0$

$= 5\left[\sin 4x - \sin 4x\right]_0^0 = 5\left[\sin 4\cdot 0 - \sin 4\cdot 0\right] = 5\left[\sin 0 - \sin 0\right] =$

$= 5[0 - 0] = \underline{\underline{0}}$

(4.) $\int_0^{\pi/2} \sin x\,dx = \left[\sin \pi/2 - \sin 0\right] = [1 - 0] = \underline{\underline{1}}$

(5.) $\int_1^2 \frac{2(1+\ln x)}{x}dx = 2\int_1^2 \frac{(1+\ln x)}{x}dx = 2\left[\frac{\ln^2(x)}{2} + x\right]_1^2 =$

$= 2\left[\frac{\ln^2(2)}{2} + 2 - \frac{\ln^2(1)}{2} + 1\right] = 2\left[\frac{\ln^2 2 + 4}{2} - \frac{\ln^2 1 + 2}{2}\right] =$

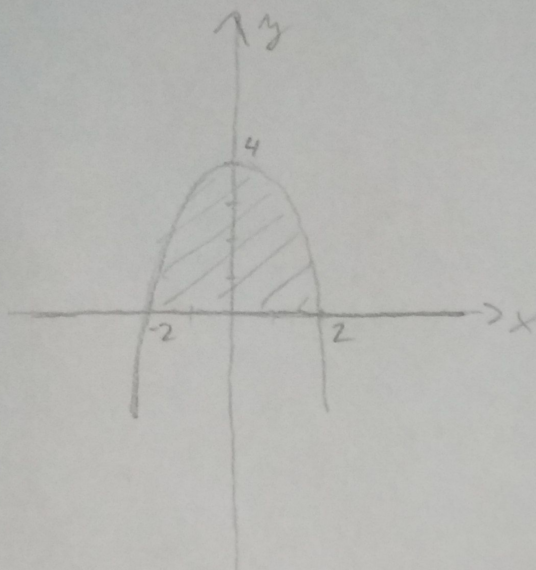
$= 2\left[\frac{\ln^2 2 + 4 - (\ln^2 1 + 2)}{2}\right] = \cancel{2}\left[\frac{\ln^2 2 + 4 - \underbrace{\ln^2 1}_{0} - 2}{\cancel{2}}\right] = \underline{\underline{\ln^2 2 + 2}}$

5 4

1. $y = 4 - x^2$, $y = 0$

↳ $[-2;0]$, $[2,0]$

$[0,4]$



$$\int_{-2}^{2} 4 - x^2 - 0\,dx = 4\int_{-2}^{2} 1\,dx - \int_{-2}^{2} x^2\,dx =$$

$$= [4x]_{-2}^{2} - \left[\frac{x^3}{3}\right]_{-2}^{2} =$$

$$= 8-(-8) - \left(\frac{8}{3} - \left(-\frac{8}{3}\right)\right) = 16 - \frac{16}{3} =$$

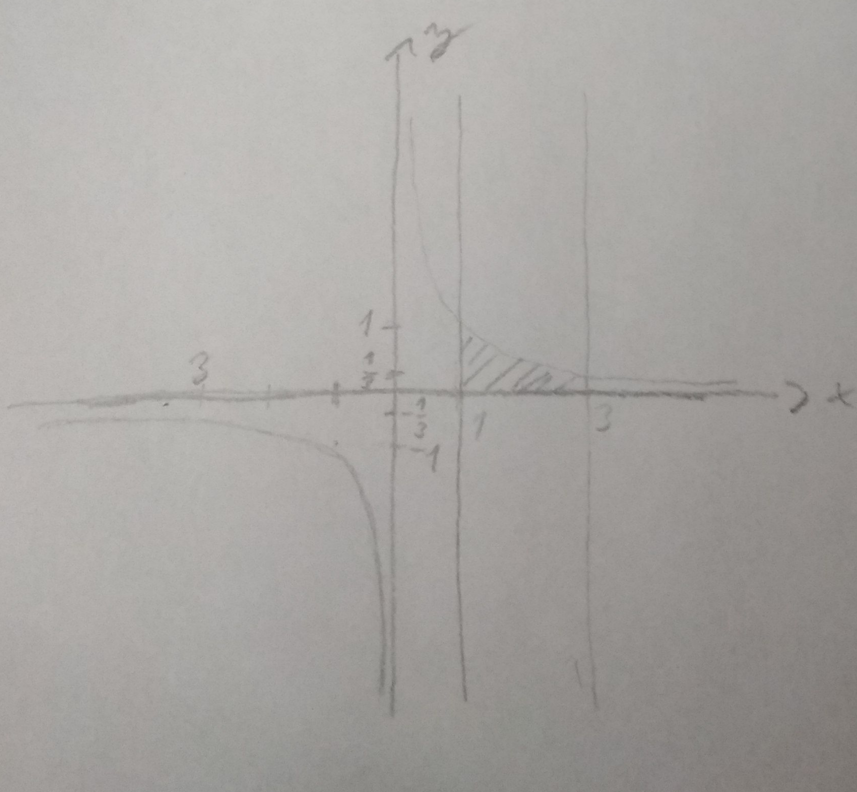
$$= \frac{48}{3} - \frac{16}{3} = \underline{\frac{32}{3}}$$

2. $yx = 1$, $x = 1$, $x = 3$, $y = 0$

↳ $y = \frac{1}{x}$

$[1;1]$, $[3;\frac{1}{3}]$

$[-1;-1]$, $[-3;-\frac{1}{3}]$



$$\int_{1}^{3} \frac{1}{x} - 0\,dx = \int_{1}^{3} \frac{1}{x}\,dx =$$

$$= [\ln x]_{1}^{3} = \ln 3 - \ln 1 =$$

$$= \underline{\ln 3}$$