D

6.1.

1. $y' = \sqrt{3x} = \sqrt{3}\, x^{\frac{1}{2}}$

$\frac{dy}{dx} = \sqrt{3}\, x^{\frac{1}{2}}$

$\int 1\, dy = \sqrt{3} \int x^{\frac{1}{2}} dx$

$y = \sqrt{3}\, \frac{2}{3} x^{\frac{3}{2}} + C$

$y = \frac{2}{3} \sqrt{3x^3} + C$

$x \geq 0,\ C \in \mathbb{R}$

2. $y' = -2xy$

$\frac{dy}{dx} \cdot \frac{1}{y} = -2x$

$\int \frac{1}{y}\, dy = -2 \int x\, dx$

$\ln|y| = -x^2 + C$

$|y| = e^{-x^2+C} = e^{-x^2} \cdot e^{C}$

$y = K \cdot e^{-x^2}$

$x \in \mathbb{R},\ K \in \mathbb{R}$

3. $y^2 y' = \cos x$

$y^2 \frac{dy}{dx} = \cos x$

$\int y^2 dy = \int \cos x\, dx$

$\frac{y^3}{3} = \sin x + C$

$y^3 = 3 \sin x + C$

$y = \sqrt[3]{3 \sin x + C}$

$x \in \mathbb{R},\ C \in \mathbb{R}$

4. $e^{-y}(1+y') = 1$

$e^{-y} + e^{-y} y' = 1$

$e^{-y} + e^{-y} \frac{dy}{dx} = 1$

$e^{-y} \frac{dy}{dx} = 1 - e^{-y}$

$e^{-y} dy = (1 - e^{-y})\, dx$

$\int \frac{e^{-y}}{1-e^{-y}}\, dy = \int 1\, dx$ →

$\left| \begin{array}{l} t = 1 - e^{-y} \\ dt = e^{-y} dy \end{array} \right|$

$\int \frac{1}{t}\, dt = \int 1\, dx$

$\ln|t| = x + C$

$\ln|1 - e^{-y}| = x + C$

$1 - e^{-y} = e^{x+C} = e^{x} \cdot e^{C}$ →

$1 - \frac{1}{e^{y}} = \frac{e^{y} - 1}{e^{y}} = e^{x} \cdot e^{C} = K e^{x}$

$e^{y} - 1 = K e^{x} e^{y}$

$-1 = K e^{x} e^{y} - \cancel{e^{y}} e^{y} = e^{y}(K e^{x} - 1)$

$1 = e^{y}(1 - K e^{x}) = e^{y}(1 + K e^{x})$

$e^{y} = 1/(1 + K e^{x})$

$y = \ln\left(1/(1 + K e^{x})\right) = -\ln(1 + K e^{x})$

$x \in \mathbb{R},\ K \in \mathbb{R}$

5. $\frac{y'}{y-1} = x + 1 \to y = 1\ ?$

$\frac{1}{y-1} \cdot \frac{dy}{dx} = x + 1$

$\int \frac{dy}{y-1} = \int x + 1\, dx$

$\ln|y-1| = \frac{x^2}{2} + x + C$

$y - 1 = e^{\left(\frac{x^2}{2} + x + C\right)} = K e^{\frac{x^2}{2} + x}$

$y = 1 + K e^{\frac{x^2}{2} + x}$

$x \in \mathbb{R},\ K \in \mathbb{R} \setminus \{0\}$

6. $y' = 6x^2 + 10x - 6$

$\frac{dy}{dx} = 6x^2 + 10x - 6$

$\int 1\, dy = \int 6x^2 + 10x - 6\, dx$

$y = 2x^3 + 5x^2 - 6x + C$

$x \in \mathbb{R},\ C \in \mathbb{R}$

7. $\frac{1}{y} y' = -2 \to y = 0\ ?$

$\frac{1}{y} \frac{dy}{dx} = -2$

$\int \frac{1}{y}\, dy = \int -2\, dx$

$\ln|y| = -2x + C$

$y = e^{-2x+C} = e^{-2x} \cdot K$

$x \in \mathbb{R},\ K \in \mathbb{R} \setminus \{0\}$

6.2.1. $y' = \frac{1}{x+3}$, $y(-2)=4$

$$\frac{dy}{dx} = \frac{1}{x+3}$$

$$\int 1\,dy = \int \frac{1}{x+3}\,dx = \int (x+3)^{-1}\,dx = \left| \begin{matrix} t = x+3 \\ dt = dx \end{matrix} \right| = \int t^{-1}\,dt = \ln|t| = \ln|x+3|$$

$y = \ln|x+3| + C$ obecné řešení

part. řešení: $y(-2)=4 \rightarrow 4 = \ln|-2+3| + C = \ln 1 + C = C$

$\rightarrow C=4$

$$y = \ln|x+3| + 4$$

2. $y' \sin x \sin y = \cos x \cos y$, $y(\pi) = 2\pi$

$$\frac{dy}{dx}\,\frac{\sin y}{\cos y} = \frac{\cos x}{\sin x}$$

$$\int \frac{\sin y}{\cos y}\,dy = \int \frac{\cos x}{\sin x}\,dx = \left| \begin{matrix} t = \cos y \\ dt = -\sin y\,dy \end{matrix} \right| \left| \begin{matrix} w = \sin x \\ dw = \cos x\,dx \end{matrix} \right| = \int \frac{-1}{t}\,dt = \int \frac{1}{w}\,dw$$

$\rightarrow -\ln|t| = \ln|w| \rightarrow -\ln|\cos y| = \ln|\sin x| + C$

$$\ln|\cos y| = -\ln|\sin x| + C = \ln\frac{1}{|\sin x|} + C$$

$|\cos y| = \frac{1}{|\sin x|} + C \rightarrow \sin x \neq 0$; $C \in \mathbb{R}$

$x \neq 0 + k\pi,\ k \in \mathbb{R}$

obecné řešení

part. řešení: $y(\pi) = 2\pi \longrightarrow$ nemá riešenie