Integrály 1

A

$$\int x^3 \, dx = \frac{x^4}{4} + c$$

Vzorec: $\int x^n \, dx = \frac{x^{n+1}}{n+1}$

$$\int e^x \, dx = e^x + c$$

Vzorec: $\int e^x \, dx = e^x$

$$\int \frac{x+1}{x^2+2x+9} \, dx = \frac{1}{2} \int \frac{2x+2}{x^2+2x+9} \, dx = \frac{1}{2} \ln|x^2+2x+9| + c$$

$(x^2+2x+9)' = 2x+2$

Vzorec: $\int \frac{f'(x)}{f(x)} \, dx = \ln|f(x)|$

$$\int x^4 - x^2 \sqrt[5]{x^3} \, dx = \int x^4 \, dx - \int x^2 \cdot x^{\frac{3}{5}} \, dx = \int x^4 \, dx - \int x^{\frac{13}{5}} \, dx = \frac{x^5}{5} - \frac{5x^{\frac{18}{5}}}{18} + c$$

$\frac{18}{5} \leftarrow \left(\frac{13}{5} + \frac{5}{5}\right)$

$\int a - b \, dx = \int a \, dx - \int b \, dx$

$\int x^n \, dx = \frac{x^{n+1}}{n+1}$

$$\int \frac{1}{x^4} \, dx = \int x^{-4} \, dx = \frac{x^{-4+1}}{-3} + c = -\frac{1}{x^3} + c$$

B

$$\int \frac{1}{x} \, dx = \ln|x| + c$$

$\int \frac{1}{x} \, dx = \ln|x|$

$$\int \frac{x+1}{x^2+2x+9} \, dx = \frac{1}{2} \ln|x^2+2x+9| + c \text{ viz } A$$

$$\int \frac{1}{x^2} \, dx = \int x^{-2} \, dx = -\frac{1}{x} + c$$

$\int x^n \, dx = \frac{x^{n+1}}{n+1}$

$$\int 3x^5 = \frac{x^6}{2} + c$$

$\int x^n \, dx = \frac{x^{n+1}}{n+1}$

$$\int a + 3\cos x \, dx = \int a \, dx + 3\int \cos x \, dx = ax + 3\sin x + c$$

$\int a + b \, dx = \int a \, dx + \int b \, dx$

$\int \cos x \, dx = \sin x$

Integraly 1

C

$\int 1 dx = \int x^0 dx = x + c$

$\int x^n dx = \frac{x^{n+1}}{n+1}$

$\int \sqrt{x} + \frac{1}{x-1} + 2 dx = \int x^{\frac{1}{2}} dx + \int \frac{1}{x-1} dx + \int 2 dx$

$\int a + b dx = \int a dx + \int b dx$

$= \frac{2x^{\frac{3}{2}}}{3} + \ln|x-1| + 2x + c$

$\int \frac{f'(x)}{f(x)} = \ln|f(x)|$

$\int tg^2 x dx = \int \frac{\sin^2 x}{\cos^2 x} dx = \int \frac{1-\cos^2 x}{\cos^2 x} dx$

$\int \frac{a-b}{c} dx = \int \frac{a}{c} - \int \frac{b}{c}$

$= \int \frac{1}{\cos^2 x} dx - \int \frac{\cancel{\cos^2 x}}{\cancel{\cos^2 x}} dx = tg\, x - x + c$

$\int 7x^6 - 5x^4 + 2x - 1 dx = x^7 - x^5 + x^2 - x + c$

$\int x^n dx = \frac{x^{n+1}}{n+1}$

$\int cotg^2 x\, dx = \int \frac{\cos^2 x}{\sin^2 x} dx = \int \frac{1-\sin^2 x}{\sin^2 x} dx = \int \frac{1}{\sin^2 x} dx - \int dx$

$= -cotg\, x - x + c$

D

$\int x^n dx = \frac{x^{n+1}}{n+1} + c \qquad n \neq -1$

$\int \frac{1}{2x-5} dx = \frac{1}{2} \int \frac{2}{2x-5} dx = \frac{\ln|2x-5|}{2} + c$

$\int \frac{f'(x)}{f(x)} = \ln|f(x)|$

$(2x-5)' = 2$

$\int tg\, x\, dx = \int \frac{\sin x}{\cos x} = -\int \frac{-\sin x}{\cos x} dx = -\ln|\cos x| + c \quad (\cos x)' = -\sin x$

$\int 7e^x - \frac{5}{x} dx = \int 7e^x dx - \int \frac{5}{x} dx$

$\int a - b dx = \int a dx - \int b dx$

$\int e^x = e^x$

$= 7\int e^x dx - 5\int \frac{1}{x} dx$

$\int \frac{1}{x} = \ln|x|$

$= 7e^x - 5\ln|x| + c$

$\int \frac{x^2 \sqrt{x}}{x^5} dx = \int \frac{x^2 \cdot x^{\frac{1}{2}}}{x^5} dx = \int x^{2+\frac{1}{2}-5} dx$

$\frac{x^a \cdot x^b}{x^c} = x^{a+b-c}$

$\int x^{-\frac{5}{2}} dx = -\frac{2x^{-\frac{3}{2}}}{3} + c = -\frac{2}{3x^{\frac{3}{2}}} + c$

Integrály 2

A

$$\int \frac{\ln^2 x}{x}\,dx = \left| \begin{matrix} t & \ln x \\ dt & \frac{1}{x}dx \\ x\,dt = dx & \end{matrix} \right| = \int \frac{t^2}{\cancel{x}} \cancel{x}\,dx = \int t^2\,dt = \frac{1}{3}t^3 + c$$

$$= \frac{1}{3}\ln^3 x + c$$

$$\int \frac{3x}{(x^2+4)^3}\,dx = \left| \begin{matrix} t & x^2+4 \\ dt & 2x\,dx \\ \frac{1}{2x}dt = dx & \end{matrix} \right| = \int \frac{3\cancel{x}}{t^3 2\cancel{x}}\,dt = \frac{3}{2}\int t^{-3}\,dt =$$

$$= \frac{3}{2}\cdot -\frac{1}{2t^2} + C = -\frac{3}{4t^2} + C$$

$$= -\frac{3}{4(x^2+4)^2} + c$$

$$\int \frac{e^{2x}}{\sqrt{e^x-1}}\,dx = \left| \begin{matrix} t & e^x - 1 \\ dt & e^x\,dx \\ \frac{dt}{e^x} = dx & \end{matrix} \right| = \int \frac{t+1}{\sqrt{t}}\,dt = \int \frac{t}{\sqrt{t}}\,dt + \int \frac{1}{\sqrt{t}}\,dt$$

$u = e^x - 1$

$e^x = u + 1$

$e^{2x} = e^x \cdot e^x = (u+1)^2$

$$= \int \sqrt{t}\,dt + \int \frac{1}{\sqrt{t}}\,dt =$$

$$= \frac{2t^{\frac{3}{2}}}{3} + 2\sqrt{t}$$

$$= \frac{2(e^x-1)^{\frac{3}{2}}}{3} + 2\sqrt{e^x-1} + c$$

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Integraly 2

3

$$\int 10x(x^2+13)^{12}dx = \left| \begin{array}{ll} t & x^2+13 \\ dt & 2x\,dx \\ dx = \frac{dt}{2x} & \end{array} \right| = \int \cancel{10}^{5}\cancel{x} \cdot t^{12} \frac{dt}{\cancel{2x}} =$$

$$= 5\int t^{12}dt = \frac{5}{13}t^{13} + C$$

$$= \frac{5(x^2+13)^{13}}{13} + C$$

$$\int 8x^2(x^3+2)^5 dx = \left| \begin{array}{ll} t & x^3+2 \\ dt & 3x^2dx \\ dx = \frac{dt}{3x^2} & \end{array} \right| = \int 8\cancel{x^2} \cdot t^5 \frac{dt}{3\cancel{x^2}}$$

$$= \frac{8}{3}\int t^5 dt = \frac{\cancel{8}^{4}\, t^6}{3 \cdot \cancel{6}_{3}} + C$$

$$= \frac{4(x^3+2)^6}{9} + C$$

$$\int 5xe^{x^2}dx = \left| \begin{array}{ll} t & x^2 \\ dt & 2x\,dx \\ dx = \frac{dt}{2x} & \end{array} \right| = \int 5\cancel{x}\, e^t \frac{dt}{2\cancel{x}} =$$

$$= \frac{5}{2}\int e^t dt = \frac{5}{2}e^t + C$$

$$= \cancel{\frac{5e^{x^2}}{2}} = \frac{5e^{x^2}}{2} + C$$

Integraly 2

C

$$\int \frac{x}{x^2+1}\,dx = \left|\begin{array}{ll} t & x^2+1 \\ dt & 2x\,dx \\ \frac{dt}{2x} = dx & \end{array}\right| = \frac{1}{2}\int \frac{1}{t}\,dt = \frac{1}{2}\ln|t| + c$$

$$= \frac{1}{2}\ln|x^2+1| + c$$

$$\int x(x^2-1)^{10}\,dx = \left|\begin{array}{ll} t & x^2-1 \\ dt & 2x\,dx \\ \frac{dt}{2x} = dx & \end{array}\right| = \int \cancel{x}\, t^{10}\,\frac{dt}{2\cancel{x}} = \frac{1}{2}\int t^{10}\,dt$$

$$= \frac{1}{2}\cdot\frac{t^{11}}{11} + c$$

$$= \frac{(x^2-1)^{11}}{22} + c$$

$$\int \frac{7\ln^4 x}{x}\,dx = \left|\begin{array}{ll} t & \ln x \\ dt & \frac{1}{x}\,dx \\ x\,dt = dx & \end{array}\right| \int \frac{7t^4\,\cancel{x}\,dt}{\cancel{x}} = 7\int t^4\,dt$$

$$= 7\,\frac{t^5}{5} + c$$

$$= 7\,\frac{\ln^5 x}{5} + c$$

Integrály 2

①

$$\int (3x-4)^7\,dx = \left|\begin{matrix} t & 3x-4 \\ dt & 3\,dx \\ \frac{dt}{3} = dx & \end{matrix}\right| = \int t^7 \frac{dt}{3} = \frac{1}{3}\int t^7\,dt$$

$$= \frac{1}{3}\,\frac{t^8}{8} + c = \frac{(3x-4)^8}{24} + c$$

$$\int 3x\sqrt[4]{x^2+5}\,dx = \left|\begin{matrix} t & x^2+5 \\ dt & 2x\,dx \\ \frac{dt}{2x} = dx & \end{matrix}\right| = \int \frac{3\cancel{x}\,t^{\frac{1}{4}}\,dt}{2\cancel{x}} =$$

$$= \frac{3}{2}\int t^{\frac{1}{4}}\,dt = \frac{3\cdot\cancel{4}^{\,2}\,t^{\frac{5}{4}}}{\cancel{2}\cdot 5} + c$$

$$= \frac{6(x^2+5)^{\frac{5}{4}}}{5} + c$$

$$\int e^{\cos 2x}\sin x\cos x\,dx \left|\begin{matrix} t & \cos x \\ dt & -\sin x \\ dx = -\frac{1}{\sin x}\,dt & \end{matrix}\right|$$

$\cos 2x = \cos^2 x - \sin^2 x$

$\sin^2 x = 1 - \cos^2 x$

$$= \int -\left(-e^{2\cos^2 x - 1}\cos x\right)\sin x\,dx$$

$$= -e^{-1}\int t\,e^{2t^2}\,dt = \left|\begin{matrix} u & 2t^2 \\ du & 4t\,dt \\ \frac{du}{4t} = dt & \end{matrix}\right|$$

$$= -e^{-1}\cdot\frac{1}{4}\int e^{u}\,du =$$

$$= -e^{-1}\,\frac{e^{2t^2}}{4}$$

$$= \frac{-e^{2u^2-1}}{4} = -\frac{e^{2\cos^2 x - 1}}{4} + c$$

$$= -\frac{e^{\cos 2x}}{4} + c$$

Integrály 3

A

$$\int_1^3 \frac{1}{x+1}\,dx = \Big[\ln|x+1|\Big]_1^3 = \ln|1+3| - \ln|1+1| = \ln|4| - \ln|2| = \ln\left|\frac{\cancel{4}\,2}{\cancel{2}}\right| = \ln|2|$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin 2x}{\cos x}\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{2\sin x \cancel{\cos x}}{\cancel{\cos x}}\,dx = 2\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin x\,dx =$$

$$= -2\Big[\cos x\Big]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = -2\left(\cos\frac{\pi}{3} - \cos\frac{\pi}{6}\right) =$$

$$= -2\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right) = \sqrt{3} - 1$$

$$\int_0^1 \frac{1}{\sqrt{x^2-1}}\,dx = \text{nelze}$$

$x \neq \pm 1 \vee x \neq 0$

to nemá nelze

$$\int_6^{12} \frac{x^2 - 13x + 19}{x^2}\,dx = \int_6^{12} \frac{\cancel{x^2}\,1}{\cancel{x^2}\,1}\,dx - \int_6^{12} \frac{13\cancel{x}}{x^{\cancel{2}}}\,dx + \int_6^{12} \frac{19}{x^2}\,dx =$$

$$= \int_6^{12} 1\,dx - \int_6^{12} \frac{13}{x}\,dx + \int_6^{12} \frac{19}{x^2}\,dx =$$

$$= \Big[x\Big]_6^{12} - 13\Big[\ln|x|\Big]_6^{12} + 19\left[-\frac{1}{x}\right]_6^{12} =$$

$$= 12 - 6 - 13\ln|12| + 13\ln|6| + \frac{19}{6} - \frac{19}{12}$$

$$= -\frac{156\ln|12| - 156\ln|6| - 91}{12}$$

$$= -13\ln\left|\frac{12}{6}\right| + \frac{91}{12}$$

Integraly 3

B

$$\int_2^5 2x+3\,dx = \left[x^2+3x\right]_2^5 = 5^2+3\cdot5-2^2-6 = 40-10 = 30$$

15?

$$\int_1^4 \sqrt{x}\,dx = \int_1^4 x^{\frac{1}{2}}dx = \left[\frac{2x^{\frac{3}{2}}}{3}\right]_1^4 = \left[\frac{2}{3}\sqrt{x^3}\right]_1^4 = \sqrt{4^3}-\sqrt{1^3} =$$

$$= \frac{2}{3}(8-1) = \frac{2}{3}7 = \frac{14}{3}$$

$$\int_0^{\frac{\pi}{4}} \frac{1}{2\cos^2 x}\,dx = \frac{1}{2}\int \frac{1}{\cos^2 x}\,dx = \frac{1}{2}\left[\operatorname{tg} x\right]_0^{\frac{\pi}{4}} =$$

$$= \frac{1}{2}\left(\operatorname{tg}\frac{\pi}{4} - \operatorname{tg} 0\right) = \frac{1}{2}(1-0) = \frac{1}{2}$$

$$\int_1^e 1-e^x+\frac{100}{x}\,dx = \left[x-e^x+100\ln|x|\right]_1^e =$$

$$= e-1-e^e+e+\overset{100}{100\ln|e|}-\overset{0}{100\ln|1|}$$

$$= 2e-e^e+99$$

Integraly 3

ⓒ

$$\int_1^3 3x^2-2x+1\,dx = \left[x^3-x^2+x\right]_1^3 = 3^3-1-3^2+1+3-1$$
$$= 27-9+2$$
$$= \underline{\underline{20}}$$

$$\int_0^5 \frac{\cos^4 x-\sin^4 x}{\cos 2x}\,dx = \int_0^5 \frac{\cancel{(\cos^2 x-\sin^2 x)}(\cos^2 x+\sin^2 x)}{\cancel{\cos^2 x-\sin^2 x}}\,dx =$$
$$= \int_0^5 \underbrace{(\cos^2 x+\sin^2 x)}_{1}\,dx = \int_0^5 dx = \left[x\right]_0^5$$
$$= 5-0 = \underline{\underline{5}}$$

$$\int_0^4 \frac{2x^2-50}{x-5}\,dx = 2\int_0^4 \frac{x^2-25}{x-5}\,dx = 2\int_0^4 \frac{(\cancel{x-5})(x+5)}{\cancel{x-5}}\,dx$$
$$= 2\int_0^4 x+5\,dx = 2\left[\frac{x^2}{2}+5x\right]_0^4 =$$
$$= \left[x^2+10x\right]_0^4 = 4^2+10\cdot 4-0 = 16+40 = \underline{\underline{56}}$$

$$\int_{-1}^{1} \frac{x^4-2x^2+1}{x-1}\,dx = \int_{-1}^{1} \frac{(\cancel{x-1})^2(x+1)^2}{\cancel{x-1}}\,dx =$$

$$= \int_{-1}^{1} (x-1)(x+1)^2\,dx = \int_{-1}^{1} (x^3+x^2-x-1)\,dx$$
$$= \int_{-1}^{1} x^3\,dx + \int_{-1}^{1} x^2\,dx - \int_{-1}^{1} x\,dx - \int_{-1}^{1} 1\,dx$$
$$= \left[\frac{x^4}{4}+\frac{x^3}{3}-\frac{x^2}{2}-x\right]_{-1}^{1} = \underline{\underline{-\frac{4}{3}}}$$

$$3(1)^4+4(1)^3-6(1)^2-12(1)$$
$$-\left(3(-1)^4+4(-1)^3-6(-1)^2-12(-1)\right) =$$
$$= \cancel{3}+4-\cancel{6}-12-\cancel{3}+4+\cancel{6}-12$$
$$= -16$$
$$= -\frac{16}{12} = \underline{\underline{-\frac{4}{3}}}$$

$$= \left[\frac{3x^4+4x^3-6x^2-12x}{12}\right]_{-1}^{1} = \frac{\cancel{3+4-6-12-3+4+6+12}}{12}$$
$$=$$

Integrally 3

D

$$\int_0^{\frac{\pi}{2}} \cos x \, dx = \left[\sin x\right]_0^{\frac{\pi}{2}} = \sin\frac{\pi}{2} - \sin 0 = 1 - 0 = 1$$

$$\int_0^1 \frac{e^x}{1+e^x} dx = \left[\ln|1+e^x|\right]_0^1 = \ln|1+e^1| - \ln|1+e^0| =$$

$\ln a - \ln b = \ln \frac{a}{b}$

$$= \ln|e+1| - \ln|2| = \ln\left|\frac{1+e}{2}\right|$$

$$a^2 \int_0^{\frac{\pi}{2}} \cos^2 x \, dx = a^2\left[\frac{x}{2} + \frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{2}} = a^2\left(\frac{\pi}{4} + \frac{1}{4}\sin\pi - 0 + 0\right)$$

$$= a^2\left(\frac{\pi}{4} + 0 - 0 + 0\right) = \frac{1}{4}\pi a^2$$

$$\int_1^{10} \frac{1}{x^2} dx = \left[-\frac{1}{x}\right]_1^{10} = -\frac{1}{10} + \frac{10}{10} = \frac{9}{10}$$