Výpočet čísla π

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$$

$$\arctan 1 = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$$

(Gregory - Leibnitz)

$$\arctan 1 \approx \sum_{j=0}^{N-1} \frac{(-1)^j}{2j+1} \approx \sum_{j=1}^{N} \frac{-(-1)^j}{2j-1}$$

$$\frac{\pi}{2} - 2\sum_{j=1}^{N} \frac{-(-1)^j}{2j-1} \sim \sum_{m=0}^{\infty} \frac{E_{2m}}{(2N)^{2m+1}}$$

```
import numpy as np

N=1000

sgn=1
x=0.0
for j in range(N):
    x=x+sgn/(2*j+1)
    sgn=-sgn

sgn=-(-1)**N
y=0.0
for j in range(N,0,-1):
    y=y+sgn/(2*j-1)
    sgn=-sgn

print(np.pi)
print(4*x)
print(4*y)
```

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$$

Machinova formula

$$\frac{\pi}{4} = 4\arctan\frac{1}{5} - \arctan\frac{1}{239}$$

$$\arg(5+i) = \arctan\frac{1}{5}$$

$$\arg(239+i) = \arctan\frac{1}{239}$$

$$= 4\arg(5+i) - \arg(239+i)$$

$$= \arg(5+i)^4 + \arg(239-i) = \arg(5+i)^4(239-i)$$

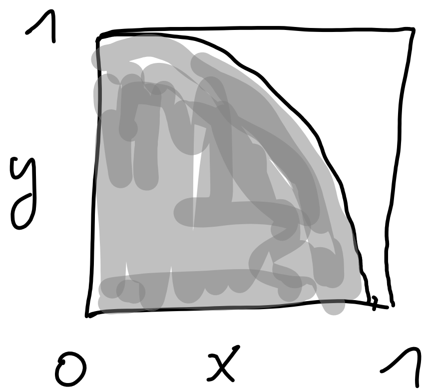
$$= \arg[114244(1+i)] = \arg(1+i) = \frac{\pi}{4}$$

$$\frac{\pi}{4} \approx 4\sum_{j=0}^{N-1}\frac{(-1)^j}{2j+1}\frac{1}{5^{2j+1}} - \sum_{j=0}^{N-1}\frac{(-1)^j}{2j+1}\frac{1}{239^{2j+1}}$$

$$\frac{\pi}{4} = \arctan\frac{1}{2} + \arctan\frac{1}{5} + \arctan\frac{1}{8}$$

```
import numpy as np

N=15

x1=np.zeros(N)
x2=np.zeros(N)

sgn=1
for j in range(N):
    k=2*j+1
    x1[j]=sgn*(1/5.0)**k/k
    x2[j]=sgn*(1/239.0)**k/k
    sgn=-sgn

for j in range(1,N+1):
    print("%2d %.15f" % ( j, 4*

print("   %.15f"%np.pi)
```

Monte Carlo výpočet čísla π



$= \frac{\pi}{4}$

náhodná čísla $(x, y) \in [0,1]^2$

zkontrolujeme $x^2 + y^2 \leq 1$

$$\frac{\#\text{ body s } x^2+y^2 \leq 1}{\#\text{ všechny body}} \to \frac{\pi}{4}$$

Viètův vzorec (16. stol.)

$$\frac{2}{\pi} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \cdot \ldots$$

Kapradina

(x_n, y_n)

1) $x_{n+1} = 0.85 x_n + 0.04 y_n$ 85%

$y_{n+1} = -0.04 x_n + 0.85 y_n + 1.6$

2) $x_{n+1} = 0.2 x_n - 0.26 y_n$ 7%

$y_{n+1} = 0.23 x_n + 0.22 y_n + 1.6$

3) $x_{n+1} = -0.15 x_n + 0.28 y_n$ 7%

$y_{n+1} = 0.26 x_n + 0.24 y_n + 0.44$

4) $x_{n+1} = 0$ 1%

$y_{n+1} = 0.16 y_n$