Examination test from Discrete mathematics 4th term, 30/1/2017

Name and surname	1	2	3	4	5	Sum

16 points/task, 100 min.

1. Let $X = \{0, 1, 2\}$ be a set and $\mathcal{A} \subseteq \mathcal{P}(X)$ a system of its subsets. For each of the formulas find a system which satisfies the formula and a system which violates it. The later one denote as $\mathcal{B}$. If there is no such $\mathcal{A}$ or $\mathcal{B}$ prove that.

a) $(\exists x \in X)(\forall Y \in \mathcal{A})(x \in Y)$.

b) $(\forall Y, Z \in \mathcal{A})(Y - Z \in \mathcal{A})$.

c) $(\forall x, y \in X)(\forall Y \in \mathcal{A})((x \neq y \ \wedge x \notin Y) \ \rightarrow \ y \in Y)$.

d) $(\forall x \in X)(\forall Y \in \mathcal{A})(\{x\} \cup Y \in \mathcal{A})$.

2. Construct some isotonne mapping $f : (\mathbb{N}, \leq) \rightarrow (\mathbb{N}, \leq)$ (or prove that it does not exist) which does not have a fix-point (i.e. $(\forall x) f(x) \neq x$) and is

a) injective (one-to-one),

b) surjective (onto).

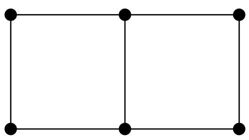
3. Let ρ, σ be relations on set X. Prove that:

a) $(\rho \cup \sigma)^{-1} = \rho^{-1} \cup \sigma^{-1}$,

b) $(\rho \circ \sigma)^{-1} = \sigma^{-1} \circ \rho^{-1}$.

4. Find Hasse diagrams of all mutually non-isomorphic preordered sets with four elements which do not have a greatest nor a smallest element.

5. a) Find some weight $w : E \to \{1, 2\}$ for the depicted graph $G = (V, E)$ such that there is exactly one minimal spanning tree.



b) Calculate a number of all such weights.