$$\sum_{n=1}^{\infty} \frac{(-1)^n (x+2)^n}{n+\sqrt{n}}, \quad x_0 = -2$$

$$\frac{(-1)^n}{n+\sqrt{n}} = a_n$$

$$\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{\frac{\cancel{(-1)^{n+1}}}{n+1+\sqrt{n+1}}}{\frac{\cancel{(-1)^{n}}}{n+\sqrt{n}}} \right| =$$

$$= \lim_{n \to \infty} \frac{n+\sqrt{n}}{n+1+\sqrt{n+1}} = \underline{\underline{1}} = \frac{1}{R} \Rightarrow R = 1$$

$X_0 = -2,\ R = 1 \Rightarrow (-3, -1)$

$\cancel{X = -3}$ $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \cdot (-3+2)^n}{n+\sqrt{n}} = \sum_{n=1}^{\infty} \frac{(-1)^n \cdot (-1)^n}{n+\sqrt{n}} = \sum_{1}^{\infty} \frac{1}{n+\sqrt{n}} \geq$

$\displaystyle\geq \sum_{1}^{\infty} \frac{1}{n+n} = \sum_{1}^{\infty} \frac{1}{2n} = \frac{1}{2} \cdot \sum_{1}^{\infty} \frac{1}{n} = \underline{\underline{\infty}}$

$\underline{\underline{X = -1}}$ $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \cdot (-1+2)^n}{n+\sqrt{n}} = \sum_{n=1}^{\infty} \frac{(-1)^n \cdot 1}{n+\sqrt{n}}$ ALT. Ř.

& $\displaystyle\frac{1}{n+\sqrt{n}} \searrow 0$

$\Rightarrow$ LEIB. KR. TO $\underline{\underline{\text{KONV.}}}$

KONV. PRO $x \in (-3, -1]$

ABS. K. PRO $x \in (-3; -1)$

$$\sum_{1}^{\infty} \frac{2^n}{n^2} \cdot x^n \quad , \quad x_0 = 0 \quad , \quad a_n = \frac{2^n}{n^2}$$

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left(\frac{2^{\cancel{n}+1}}{(n+1)^2} \cdot \frac{n^2}{\cancel{2^n}} \right) = \lim_{n\to\infty} \frac{2 \cdot n^2}{n^2 + 2n + 1} = 2$$

$$\Rightarrow R = \frac{1}{2}$$

? $\left(-\frac{1}{2}, \frac{1}{2}\right)$?

$$x = -\frac{1}{2} \Rightarrow \sum_{n=1}^{\infty} \frac{2^n}{n^2} \cdot \left(-\frac{1}{2}\right)^n = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$ ABS. KONV.

$$x = \frac{1}{2} \Rightarrow \sum_{n=1}^{\infty} \frac{2^n}{n^2} \cdot \left(\frac{1}{2}\right)^n = \sum_{n=1}^{\infty} \frac{1}{n^2}$$ ABS. KONV.

ABS. KONV. $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

$$\sum_{n=1}^{\infty} \alpha^{n^2} \cdot x^n \quad , \quad 0 < \alpha < 1 \quad , \quad x_0 = 0, \; a_n = \alpha^{n^2}$$

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{\alpha^{n^2}} = \lim_{n\to\infty} \alpha^{\frac{n^2}{n}} =$$

$$= \lim_{n\to\infty} \alpha^n = 0 \Rightarrow R = \infty \Rightarrow$$

konv. $x \in \mathbb{R}$

$$\sum_{n=3}^{\infty} \frac{x^n}{n \cdot 5^n}, \quad x_0 = 0, \quad \frac{1}{R} = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{n \cdot \cancel{5^n}}{(n+1)\cdot 5^{\cancel{n+1}}} = \frac{1}{5}$$

$$a_n = \frac{1}{n \cdot 5^n}$$

$$\Rightarrow R = 5 \qquad (-5, 5)$$

$\underline{x = -5}$

$$\sum_{n=3}^{\infty} \frac{(-5)^n}{n \cdot 5^n} = \sum_{n=3}^{\infty} \frac{(-1)^n}{n} \Rightarrow K$$

$\underline{x = 5}$

$$\sum_{n=3}^{\infty} \frac{5^n}{n \cdot 5^n} = \sum_{n=3}^{\infty} \frac{1}{n} \Rightarrow D$$

$[-5, 5)$

$$\left(\sum_{n=3}^{\infty} \frac{x^n}{n\cdot 5^n}\right)' = \sum_{n=3}^{\infty} \frac{\cancel{n}\cdot x^{n-1}}{\cancel{n}\cdot 5^n} = \frac{1}{5}\cdot \sum_{n=3}^{\infty}\left(\frac{x}{5}\right)^{n-1} = \left| \; q=\frac{x}{5} \; \right| =$$

$$= \frac{1}{5}\cdot\frac{\left(\frac{x}{5}\right)^2}{1-\frac{x}{5}} = \frac{1}{\cancel{5}}\cdot\frac{\frac{x^2}{25}}{\frac{5-x}{\cancel{5}}} = \underline{\underline{\frac{x^2}{25\cdot(5-x)}}}$$

$$\sum_{n=3}^{\infty} \frac{x^n}{n\cdot 5^n} = \frac{1}{25}\cdot\int \frac{x^2}{5-x}\,dx = \frac{1}{25}\cdot\int -x-5+\frac{25}{5-x}\,dx =$$

$$= \frac{1}{25}\cdot\left(-\frac{x^2}{2}-5x-25\cdot\ln|5-x|\right)+C$$

$$x=0 \Rightarrow \quad 0 = \frac{1}{25}\cdot\left(-0-0-25\cdot\ln 5\right)+C$$

$$\sum \frac{0^n}{n\cdot 5^n} \quad \Big| \quad 0 = -\ln 5 + C \Rightarrow C = \ln 5$$

$$\sum_{n=3}^{\infty} \frac{x^n}{n \cdot 5^n} = -\frac{x^2}{50} - \frac{x}{5} - \ln|5-x| + \ln 5 = \dots$$

$$x \in [-5, 5)$$

$$x = -5 \Rightarrow \sum_{n=3}^{\infty} \frac{(-5)^n}{n \cdot 5^n} = \sum_{n=3}^{\infty} \frac{(-1)^n}{n} = -\frac{25}{50} + \frac{5}{5} - \ln 10 + \ln 5 =$$

$$= -\frac{1}{2} + 1 + \ln \frac{5}{10} = \frac{1}{2} + \ln \frac{1}{2}$$

$$= \underline{\frac{1}{2} - \ln 2}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} = \frac{1}{2} - \ln 2 + (-1) + \frac{1}{2} = -\ln 2$$

$$\sum_{n=1}^{\infty} \frac{(n-1)\cdot(e-1)^n}{n\cdot e^n + e^n} = \sum_{n=1}^{\infty} \frac{n-1}{n+1}\cdot\left(\frac{e-1}{e}\right)^n \Rightarrow \boxed{\sum_{n=1}^{\infty} \frac{n-1}{n+1}\cdot x^n}$$

$$x_0 = 0,\ a_n = \frac{n-1}{n+1}$$

$$\boxed{x = \frac{e-1}{e}}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left(\frac{n}{n+2}\cdot\frac{n+1}{n-1}\right) = \underline{\underline{1}} = \frac{1}{R}$$

$$\Rightarrow R = 1 \Rightarrow x \in (-1,1) \ni \frac{e-1}{e}$$

✓

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad \Big| \int dx$$

$$\underbrace{\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1}} = -\int \frac{-1}{1-x}\,dx = \underbrace{-\ln|1-x| + C}$$

$$x=0 \Rightarrow \quad 0 = -\ln|1| + C \quad \Rightarrow \quad C = 0$$

$$\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1} = -\ln|1-x| \quad \Big| \frac{1}{x^2}$$

$$\sum_{n=0}^{\infty} \frac{x^{n-1}}{n+1} = \frac{-\ln(1-x)}{x^2} \quad \Big/ \frac{d}{dx}$$

$$\sum_{n=0}^{\infty} \frac{1}{n+1}\cdot(n-1)\cdot x^{n-2} = \frac{1}{x^2\cdot(1-x)} + \frac{2\cdot\ln(1-x)}{x^3} \quad \Big/ \cdot x^2$$

$$\sum_{n=0}^{\infty} \frac{n-1}{n+1}\cdot x^{n} = \frac{1}{1-x} + \frac{2\cdot\ln(1-x)}{x}$$

$$\sum_{n=1}^{\infty} \frac{n-1}{n+1}\, x^{n} = \boxed{\frac{1}{1-x} + \frac{2\cdot\ln(1-x)}{x} - (-1)}$$

$$\sum_{n=1}^{\infty} \frac{n-1}{n+1} \cdot \left(\frac{e-1}{e}\right)^n = \; \dots = \underline{\underline{e - \frac{2e}{e-1} + 1}}$$

$$(x-\pi)\cdot(x+\pi)\cdot(x-2\pi)\cdot(x+2\pi)\cdot\ \ldots$$

$$ax^2+bx+c = a\cdot(x-x_1)\cdot(x-x_2)$$