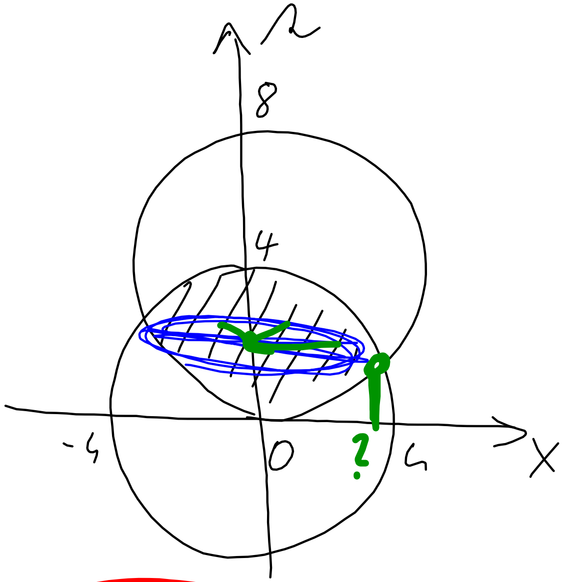
$V = ?$, $x^2+y^2+z^2 \le 16$, $x^2+y^2+z^2 \le 8z$

$x^2+y^2+(z-4)^2 \le 16$



① VÁLC.

$x = \rho \cdot \cos\varphi$

$y = \rho \cdot \sin\varphi$

$z = z$, $|J| = \rho$

$\varphi \in [0, 2\pi]$

$\rho \in [0, ?]$

$16 - 8z = 0$

$8z = 16$

$z = 2 \Rightarrow x^2+y^2+2^2 = 16$

$x^2+y^2 = 12$

$\rho^2 \cdot (\cos^2\varphi + \sin^2\varphi) = 12 \Rightarrow \rho = \sqrt{12}$

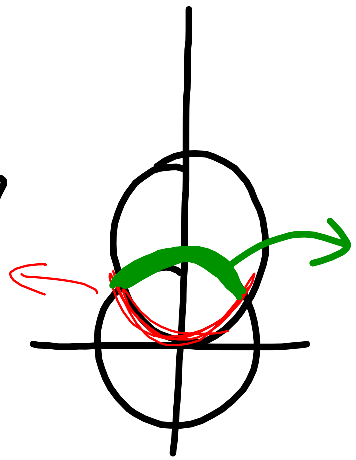
$$z \in \left[4-\sqrt{16-\rho^2}, \sqrt{16-\rho^2}\right]$$

$$x^2+y^2+z^2=8z$$

$$(z-4)^2=16-x^2-y^2$$

$$z=4\pm\sqrt{16-x^2-y^2}$$

$$z=4-\sqrt{16-\rho^2}$$



$$x^2+y^2+z^2=16$$

$$z^2=16\underbrace{-x^2-y^2}_{-\rho^2}$$

$$z=\pm\sqrt{16-\rho^2}$$

$$V = \int_0^{\sqrt{12}} \int_0^{2\pi} \int_{4-\sqrt{16-\rho^2}}^{\sqrt{16-\rho^2}} 1 \cdot \rho \; dz \, d\varphi \, d\rho =$$

$$= (2\pi - 0) \cdot \int_0^{\sqrt{12}} \rho \cdot \left(\sqrt{16-\rho^2} - 4 + \sqrt{16-\rho^2}\right) d\rho = \left| \; t = 16-\rho^2 \; \right|$$

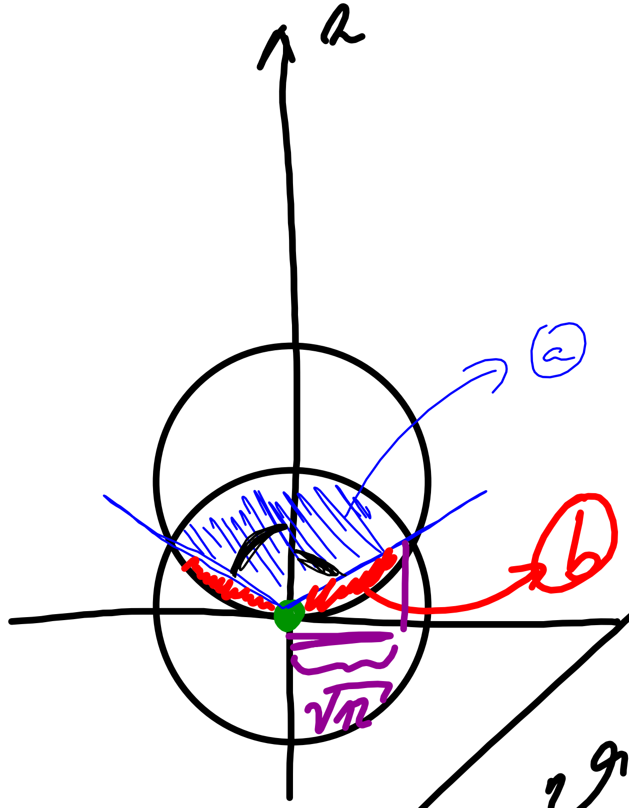
$$= 2\pi \cdot \int_0^{\sqrt{12}} \underline{2 \cdot \rho \cdot \sqrt{16-\rho^2}} - 4 \cdot \rho \; d\rho = \left| \begin{array}{l} t = 16-\rho^2 \\ dt = -2\rho \, d\rho \\ 2\rho \, d\rho = -dt \end{array} \right| \left| \begin{array}{l} \rho = \sqrt{12} \\ \Rightarrow t = 4 \\ \rho = 0 \\ \Rightarrow t = 16 \end{array} \right| =$$

$$= 2\pi \cdot \int_{16}^{4} \sqrt{t} \cdot (-1) \, dt + 2\pi \cdot (-4) \cdot \int_0^{\sqrt{12}} \rho \; d\rho =$$

$$= -2\pi \cdot \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right]_{16}^{4} - 8\pi \cdot \left[\frac{\rho^2}{2} \right]_0^{\sqrt{12}} =$$

$$= -2\pi \cdot \frac{2}{3} \cdot \left(4^{3/2} - 16^{3/2}\right) - 8\pi \cdot \frac{1}{2} \cdot (12 - 0) =$$

$$= \frac{4}{3}\pi (64 - 8) - 4\pi \cdot 12 = \frac{56 \cdot 4}{3}\pi - 48\pi = \frac{80}{3}\pi$$



$$x = \rho \cdot \cos\varphi \cdot \sin\vartheta$$

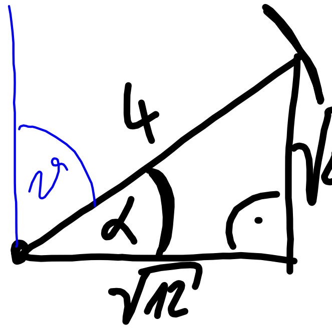
$$y = \rho \cdot \sin\varphi \cdot \sin\vartheta$$

$$z = \rho \cdot \cos\vartheta$$

$$|J| = \rho^2 \cdot \sin\vartheta$$

a) $\varphi \in [0, 2\pi]$, $\rho \in [0, 4]$

$$\vartheta \in [0, ?] = \left[0, \frac{\pi}{3}\right]$$



$$\sqrt{4^2 - (\sqrt{12})^2} = 2 \Rightarrow \sin\alpha = \frac{2}{4} = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{6}$$

$$\vartheta = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$$

$$V_a = \int_0^{2\pi} \int_0^{\frac{\pi}{3}} \int_0^{4} \left(\rho^2 \cdot \sin\vartheta\right) \, d\rho \, d\vartheta \, d\varphi = \ldots =$$

$$= \underline{\underline{\frac{64}{3}\pi}}$$

$V_b = ?$

$$\boxed{\varphi \in [0, 2\pi], \quad \vartheta \in \left[\frac{\pi}{3}, \frac{\pi}{2}\right] \\ \rho \in [0, 8\cdot\cos\vartheta]}$$

$x^2 + y^2 + z^2 \leq 8z$

$\rho^2 \cdot \cos^2\varphi \cdot \sin^2\vartheta + \rho^2 \cdot \sin^2\varphi \cdot \sin^2\vartheta + \rho^2 \cdot \cos^2\vartheta = 8 \cdot \rho \cdot \cos\vartheta$

$= \rho^2 \sin^2\vartheta \cdot (\underbrace{\cos^2\varphi + \sin^2\varphi}_{=1}) + \rho^2 \cdot \cos^2\vartheta =$

$= \rho^2 (\underbrace{\sin^2\vartheta + \cos^2\vartheta}_{=1}) = \rho^2$

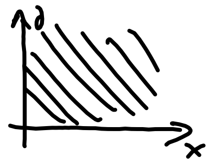
$\rho \leq 8 \cdot \cos\vartheta$

$$V_6 = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \int_{0}^{2\pi} \int_{0}^{8\cdot\cos\vartheta} \left(\varrho^2 \cdot \sin\vartheta\right) d\varrho \; d\varphi \; d\vartheta = \ldots$$

$$= \ldots = \frac{16}{3}\pi$$

$$\int_0^\infty e^{-x^2}dx \cdot \int_0^\infty e^{-y^2}dy = \int_0^\infty e^{-x^2}\cdot \int_0^\infty e^{-y^2}dy\, dx =$$

$$= \int_0^\infty \int_0^\infty e^{-x^2-y^2}\,dy\,dx = \left| \begin{array}{l} x=\rho\cdot\cos\varphi \\ y=\rho\cdot\sin\varphi \\ |J|=\rho \end{array} \right| \begin{array}{l} x^2+y^2=\rho^2 \end{array}$$



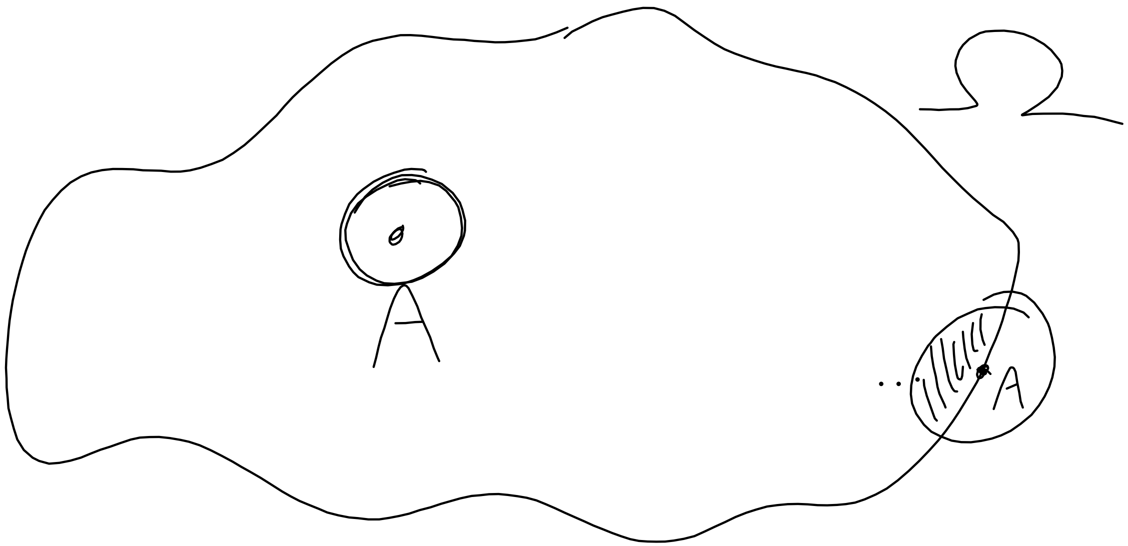
$$\varphi\in\left[0,\tfrac{\pi}{2}\right] \qquad \rho\in(0,\infty)$$

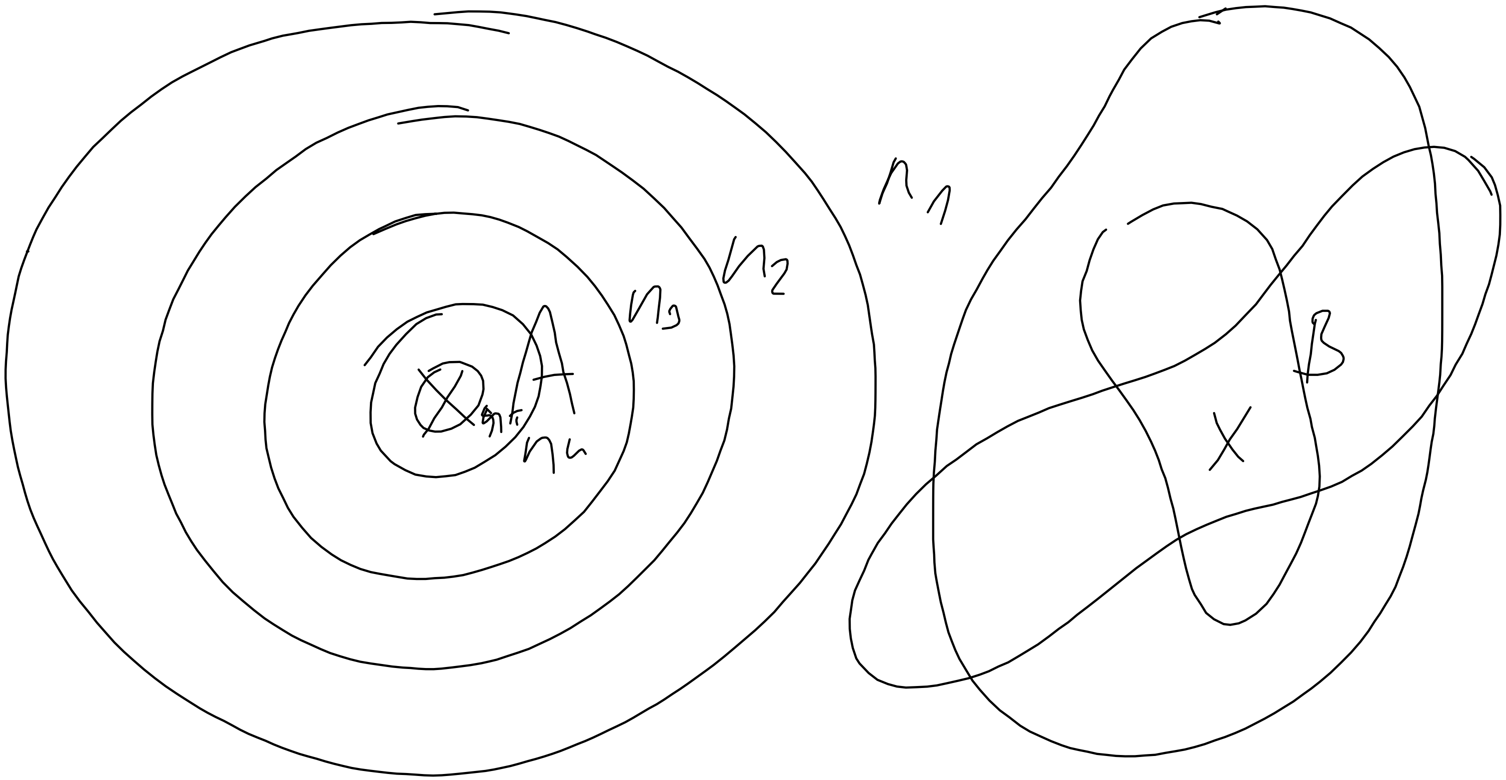
$$= \int_0^{\frac{\pi}{2}} \left(\int_0^\infty e^{-\rho^2}\cdot\rho\, d\rho \right) d\varphi =$$

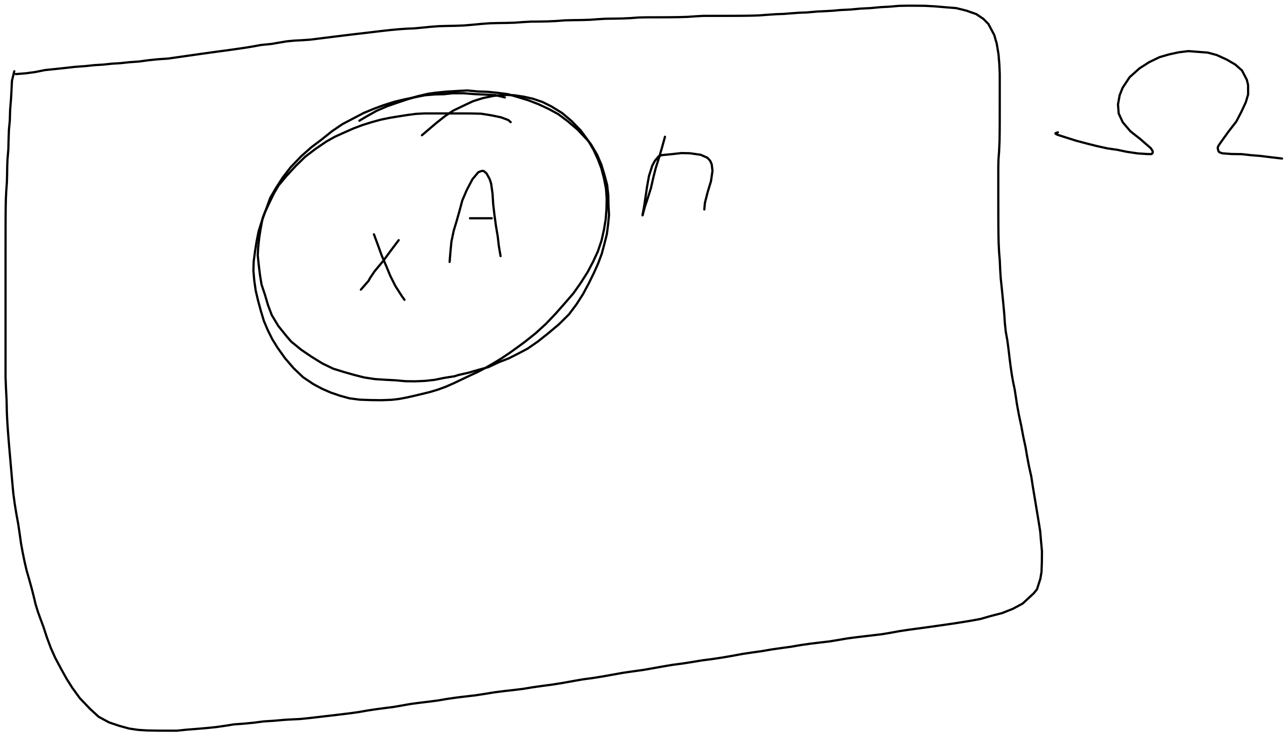
$$= \left(\int_0^\infty e^{-\rho^2}\cdot\rho\, d\rho \right)\cdot \underbrace{\int_0^{\frac{\pi}{2}} 1\, d\varphi}_{=\frac{\pi}{2}} = \left| \begin{array}{l} t=-\rho^2 \\ dt=-2\rho\, d\rho \\ \rho\, d\rho=-\frac{1}{2}\,dt \end{array} \right| \begin{array}{l} \rho=\infty \\ \Rightarrow t=-\infty \\ \rho=0 \\ \Rightarrow t=0 \end{array} \Bigg| =$$

$$= \frac{\pi}{2}\cdot\int_0^{-\infty} e^t\cdot\left(-\frac{1}{2}\right)dt = \frac{\pi}{4}\cdot\left[e^t\right]_{-\infty}^{0} = \frac{\pi}{4} - \underset{\substack{\uparrow \\ \text{LIMITA}}}{0} = \frac{\pi}{4}$$

$$\Rightarrow \int_0^\infty e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$$


A
A

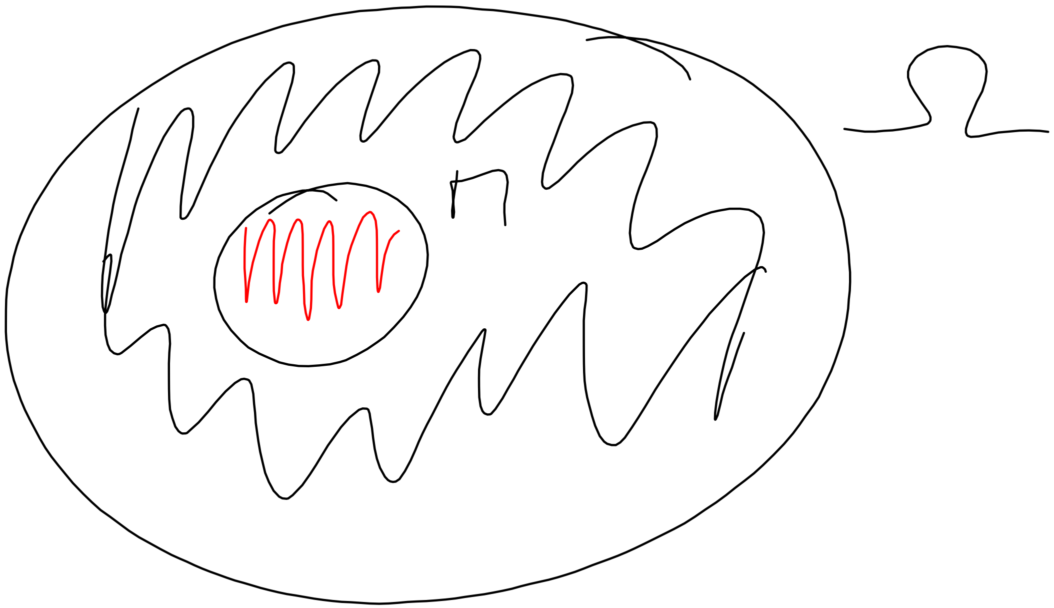

A
n_1
n_2
n_3
n_4
x
B

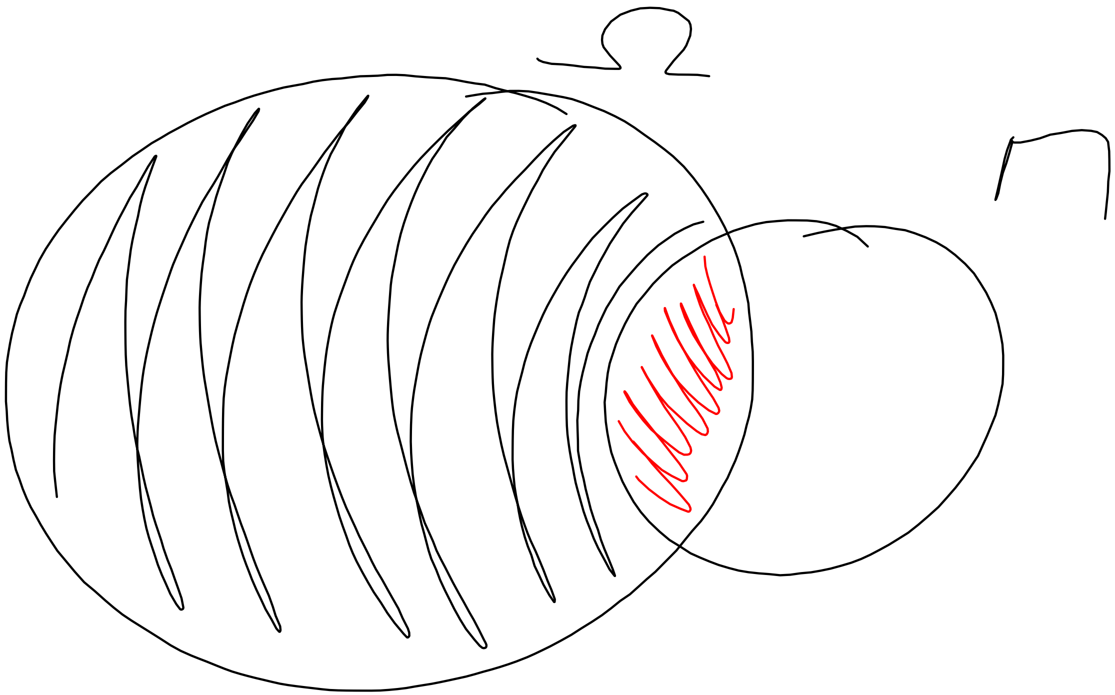


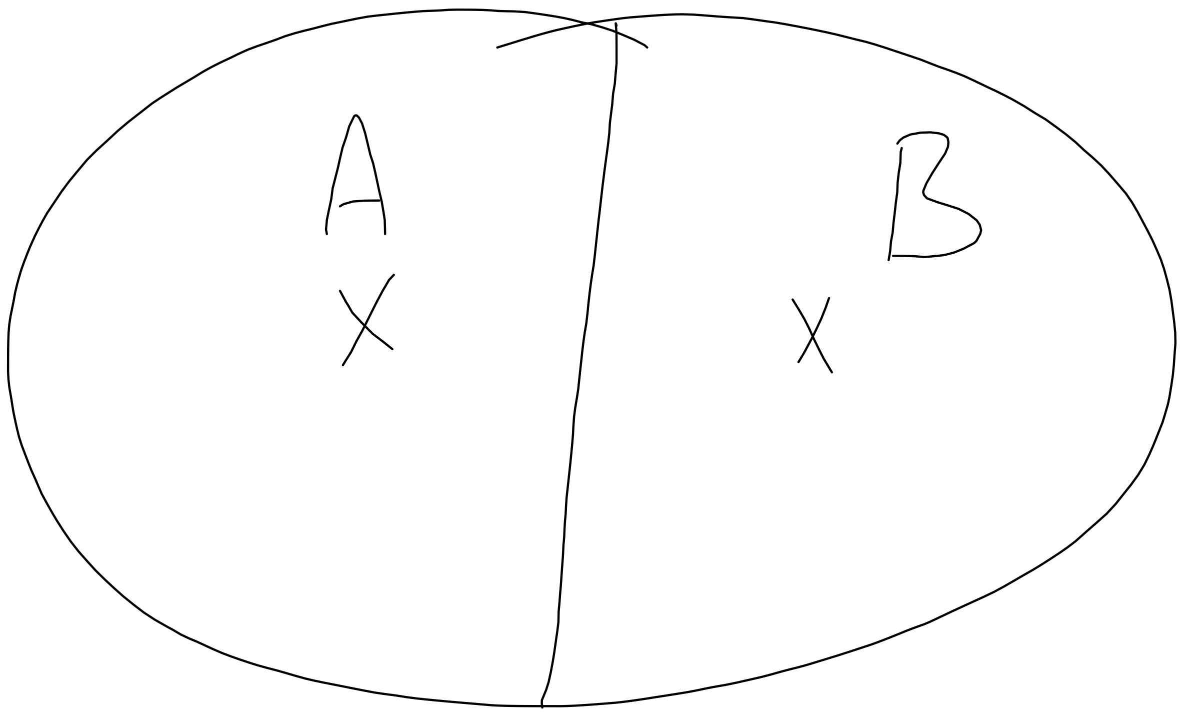
x
$\bar{A}$
A
Ω

$$\int_0^1 \frac{1}{x}\,dx = \lim_{t\to 0^+} \int_t^1 \frac{1}{x}\,dx$$

$$\int_a^b f(x)\,dx = \left| \text{S.B.} = c \in (a,b) \right| = \int_c^b f(x)\,dx + \int_a^c f(x)\,dx$$






A
x
B
x

$$2 \cdot \frac{1}{x} - 1\frac{2}{x}$$

$$\int_0^1 \frac{1}{x^\alpha}\,dx \qquad \alpha \geq 0$$

$$\int_0^1 x^5\,dx$$