10.1 Consider a solid-state plasma with the same number of electrons (e) and holes (h). Using the linearized Langevin equation (with $\alpha = e,\ h$)

$$m_\alpha \frac{\partial \mathbf{u}_\alpha}{\partial t} = q_\alpha(\mathbf{E} + \mathbf{u}_\alpha \times \mathbf{B}_0) - \nu_{c\alpha} m_\alpha \mathbf{u}_\alpha$$

taking $m_e = m_h$, $\nu_{ce} = \nu_{ch}$, assuming a time dependence for both $\mathbf{E}$ and $\mathbf{u}_\alpha$ of the form $\exp(-i\omega t)$, and choosing a Cartesian coordinate system with the z axis pointing along the constant and uniform magnetic field $\mathbf{B}_0$, show that the conductivity dyad is given by

$$\mathcal{S} = 2 \begin{pmatrix} \sigma_\perp & 0 & 0 \\ 0 & \sigma_\perp & 0 \\ 0 & 0 & \sigma_0 \end{pmatrix}$$

with $\sigma_\perp$ and σ_0 given by (5.3) and (5.5), respectively. Explain, in physical terms, why we have $\sigma_H = 0$ in this case.

$$\sigma_\perp = \frac{(\nu_c - i\omega)^2}{(\nu_c - i\omega)^2 + \Omega_{ce}^2}\sigma_0 \tag{5.3}$$

$$\sigma_H = \frac{(\nu_c - i\omega)\Omega_{ce}}{(\nu_c - i\omega)^2 + \Omega_{ce}^2}\sigma_0 \tag{5.4}$$

$$\sigma_0 = \frac{n_e e^2}{m_e(\nu_c - i\omega)} = \frac{n_e e^2(\nu_c + i\omega)}{m_e(\nu_c^2 + \omega^2)} \tag{5.5}$$

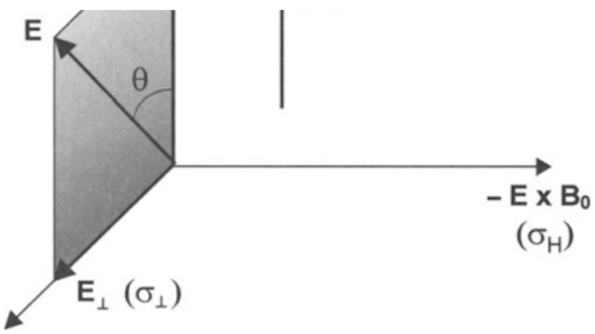
10.3 Consider the equation $\mathbf{J} = \mathcal{S} \cdot \mathbf{E}$, with $\mathcal{S}$ as given in (4.23). If we choose a Cartesian coordinate system such that $E_x = E_\perp$, $E_y = 0$, $E_z = E_\parallel$, and $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$ (refer to Fig. 2), verify that in this coordinate system we have

$$J_x = \sigma_\perp E_\perp$$

$$J_y = \sigma_H E_\perp$$

$$J_z = \sigma_\parallel E_\parallel$$

Interpret physically this result with reference to Fig. 2.



$$\mathcal{S} = \begin{pmatrix} \sigma_\perp & -\sigma_H & 0 \\ \sigma_H & \sigma_\perp & 0 \\ 0 & 0 & \sigma_\parallel \end{pmatrix} \tag{4.23}$$

Fig. 2 *Relative orientation of the vector fields $\mathbf{E}_\parallel$, $\mathbf{E}_\perp$, and $-\mathbf{E} \times \mathbf{B}_0$. The conductivities $\sigma_\parallel$, $\sigma_\perp$, and σ_H govern the magnitude of the electric currents flowing along these directions, respectively.*