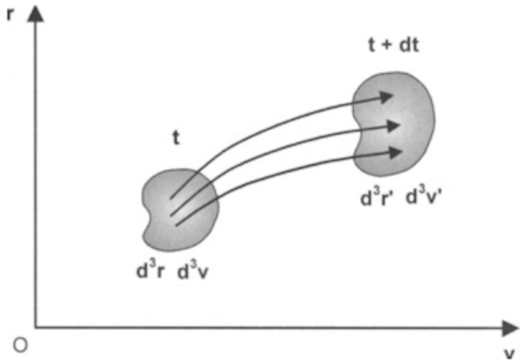

Fig. 4 *In the absence of collisions the particles within the volume element $d^3r\ d^3v$ about $(\mathbf{r},\mathbf{v})$, at an instant t, will occupy after a time interval dt a new volume element $d^3r'\ d^3v'$, about $(\mathbf{r}',\mathbf{v}')$.*

$$\mathbf{r}'(t+dt) = \mathbf{r}(t) + \mathbf{v}\ dt$$

$$\mathbf{v}'(t+dt) = \mathbf{v}(t) + \mathbf{a}\ dt$$

$$d^3r'\ d^3v' = |J|\ d^3r\ d^3v$$

Show that: $|J| = 1$

5.6 Show that the Vlasov equation for a homogeneous plasma under the influence of a uniform external magnetostatic field $\mathbf{B}_0$, in the equilibrium state, is satisfied by any homogeneous distribution function, $f(v_\parallel, v_\perp)$, which is cylindrically symmetric with respect to the magnetostatic field.

5.7 The entropy of a system can be expressed, in terms of the distribution function, as

$$S = -k \int_r \int_v f\ \ln(f)\ d^3r\ d^3v$$

Show that, for a system that obeys the collisionless Boltzmann equation, the total time derivative of the entropy vanishes.

5.2 Consider the following *two-dimensional* Maxwellian distribution function:

$$f(v_x, v_y) = n_0 \left(\frac{m}{2\pi kT}\right)\ exp\ \left[-\frac{m(v_x^2 + v_y^2)}{2kT}\right]$$

(a) Verify that n_0 represents correctly the particle number density, that is, the number of particles per unit area.
(b) Sketch, in a three-dimensionsal perspective view, the surface for this distribution function, plotting $f(v_x, v_y)$ in terms of v_x and v_y. Draw, on this surface, curves of constant v_x, curves of constant v_y, and curves of constant f.

7.9 Consider the particles in the Earth's atmosphere under equilibrium conditions in the presence of the Earth's gravitationl field. Assume a horizontally stratified (x, y plane) atmosphere with constant temperature T and consider a constant value $\mathbf{g} = -g\hat{\mathbf{z}}$ for the acceleration due to gravity. Derive an expression for the number density $n_\alpha(z)$ as a function of height z, for the type α species, in terms of the number density $n_\alpha(z_0)$ at a base level z_0 and of the *scale height* $H_\alpha = kT/m_\alpha g$. How is the expression for $n_\alpha(z)$ modified, when T and g vary with height?