6.5 A plasma is made up of a mixture of various particle species, the type α species having mass m_α, number density n_α, average macroscopic velocity $\mathbf{u}_\alpha$, random velocity $\mathbf{c}_\alpha = \mathbf{v} - \mathbf{u}_\alpha$, scalar pressure $p_\alpha = n_\alpha k T_\alpha$, temperature $T_\alpha = (m_\alpha/3k) < c_\alpha^2 >$, pressure dyad $\mathcal{P}_\alpha = n_\alpha m_\alpha < \mathbf{c}_\alpha \mathbf{c}_\alpha >$, and heat flow vector $q_\alpha = (n_\alpha m_\alpha/2) < c_\alpha^2 \mathbf{c}_\alpha >$. Similar quantities can be defined for the plasma *as a whole*, for example, we can define the *total* number density by

$$n_0 = \sum_\alpha n_\alpha$$

the *average* mass by

$$m_0 = \frac{1}{n_0} \sum_\alpha n_\alpha m_\alpha$$

and the *average* flow velocity by

$$\mathbf{u}_0 = \frac{1}{n_0 m_0} \sum_\alpha n_\alpha m_\alpha \mathbf{u}_\alpha$$

We can also define an *alternative* random velocity for the type α species, with reference to $\mathbf{u}_0$, as $\mathbf{c}_{\alpha 0} = \mathbf{v} - \mathbf{u}_0$, as well as an *alternative* absolute temperature by

$$T_{\alpha 0} = \frac{m_\alpha < c_{\alpha 0}^2 >}{3k}$$

a corresponding pressure dyad by

$$\mathcal{P}_{\alpha 0} = n_\alpha m_\alpha < \mathbf{c}_{\alpha 0} \mathbf{c}_{\alpha 0} >$$

and heat flow vector by

$$\mathbf{q}_{\alpha 0} = \tfrac{1}{2} n_\alpha m_\alpha < c_{\alpha 0}^2 \mathbf{c}_{\alpha 0} >$$

(a) Show that, for the plasma as a whole, the *total* pressure dyad is given by

$$\mathcal{P}_0 = \sum_\alpha (\mathcal{P}_\alpha + n_\alpha m_\alpha \mathbf{w}_\alpha \mathbf{w}_\alpha)$$

and the *total* scalar pressure by

$$p_0 = \sum_\alpha (p_\alpha + \tfrac{1}{3} n_\alpha m_\alpha w_\alpha^2)$$

where $\mathbf{w}_\alpha = \mathbf{u}_\alpha - \mathbf{u}_0$ is the macroscopic *diffusion* velocity.
(b) If $\mathbf{c}_\alpha$ is isotropic, that is, $< c_{\alpha i}^2 >= (1/3) < c_\alpha^2 >$, for $i = x, y, z$, show that the *total* heat flow vector is given by

$$\mathbf{q}_0 = \sum_\alpha \left(\mathbf{q}_\alpha + \tfrac{5}{2} p_\alpha \mathbf{w}_\alpha + \tfrac{1}{2} n_\alpha m_\alpha w_\alpha^2 \mathbf{w}_\alpha\right)$$

(c) If an *average* temperature T_0, for the plasma as a whole, is defined by requiring that $p_0 = n_0 k T_0$, show that

$$T_0 = \frac{1}{n_0} \sum_\alpha n_\alpha \left(T_\alpha + \frac{m_\alpha w_\alpha^2}{3k}\right)$$

(d) Verify that

$$\tfrac{1}{2} \textstyle\sum_\alpha n_\alpha m_\alpha < c_{\alpha 0}^2 >= \tfrac{3}{2} n_0 k T_0$$

so that there is an average thermal energy of $kT_0/2$ per degree of freedom.

6.1 Consider a system of particles characterized by the distribution function given in problem 5.1 (in Chapter 5).
(a) Show that the absolute temperature of the system is given by

$$T = \frac{m v_0^2}{3k}$$

where m is the mass of each particle and k is Boltzmann's constant.
(b) Obtain the following expression for the pressure dyad

$$\mathcal{P} = \tfrac{1}{3} \rho_m v_0^2 \,\mathbf{1}$$

where $\rho_m = nm$ and $\mathbf{1}$ is the unit dyad.
(c) Verify that the heat flow vector $\mathbf{q} = 0$.

From 5.1: $f(v) = K_0 \quad for \quad |v_i| \le v_0 \quad (i = x, y, z)$

$f(v) = 0 \quad otherwise,$

where K_0 is a nonzero positive constant. Determine the value of K_0