$$
\begin{aligned}
x_1' &= -3x_1 + 4x_2 - 2x_3 \\
x_2' &= x_1 + x_3 \\
x_3' &= 6x_1 - 6x_2 + 5y_3
\end{aligned}
$$

$$A = \begin{pmatrix} -3 & 4 & -2 \\ 1 & 0 & 1 \\ 6 & -6 & 5 \end{pmatrix}, \quad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\Rightarrow \quad x' = Ax$$

$$|A - \lambda I| = \begin{vmatrix} -3-\lambda & 4 & -2 \\ 1 & -\lambda & 1 \\ 6 & -6 & 5-\lambda \end{vmatrix} = \ldots = \underbrace{(\lambda-2)\cdot(1-\lambda)\cdot(1+\lambda) = 0}$$

$$\Rightarrow \boxed{\lambda_1 = 1, \quad \lambda_2 = -1, \quad \lambda_3 = 2}$$

(ii) $\underline{\lambda_1 = 1}$

$x' = A \cdot x$

$$\underbrace{x = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot e^{1t}} \Rightarrow x' = \begin{pmatrix} a \\ b \\ c \end{pmatrix} e^{t}$$

$$I \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \cancel{e^{t}} = \begin{pmatrix} -3 & 4 & -2 \\ 1 & 0 & 1 \\ 6 & -6 & 5 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \cancel{e^{t}}$$

$$0 = \begin{pmatrix} -3-1 & 4 & -2 \\ 1 & 0-1 & 0 \\ 6 & -6 & 5-1 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

3 ROCE O 3 NEZN.

$$0 = A \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} - I \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = (A - I) \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} p \\ p \\ 0 \end{pmatrix}, \quad p \in \mathbb{R}$$

HOMOG. S KONST. KOEF. $\Rightarrow$

$$\underline{\begin{pmatrix} P_1(t) \\ \vdots \\ P_n(t) \end{pmatrix}} \cdot e^{\lambda_j t}$$

$\Delta P_q = \underbrace{k_j}_{} - 1$

NÁS. λ_j

$$p = 1 \Rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \cdot e^{t}$$

---

ii) $\lambda_2 = -1 \Rightarrow x = \begin{pmatrix} a \\ b \\ c \end{pmatrix} e^{-t} \Rightarrow x' = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \left(-e^{-t}\right)$

$x' = Ax$

$$-\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \cancel{e^{-t}} = A \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \cancel{e^{-t}}$$

$$0 = (A + I) \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} -3+1 & 3 & -2 & a \\ 1 & 0+1 & 1 & b \\ 6 & -6 & 5+1 & c \end{array}\right) \Rightarrow$$

$$x = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \cdot e^{-t}$$

(iii) $\lambda_3 = 2 \Rightarrow x = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot e^{2t} \Rightarrow x' = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot 2 \cdot e^{2t}$

$x' = A\cancel{x}$

$I \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot 2 \cdot \cancel{e^{2t}} = A \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \cancel{e^{2t}}$

$0 = (A - 2 \cdot I) \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

$\left(\begin{array}{ccc|c} -3-2 & 4 & -2 & a \\ 1 & 0-2 & 1 & b \\ 6 & -6 & 5-2 & c \end{array}\right) \Rightarrow$

$\Rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ p/2 \\ p \end{pmatrix}, \; p \in \mathbb{R}$

$p = 2 \Rightarrow \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \Rightarrow x = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \cdot e^{2t}$

OB. ŘEŠ.:

$$x = c_1 \cdot \underbrace{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \cdot e^{t}} + c_2 \cdot \underbrace{\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \cdot e^{-t}} + c_3 \cdot \underbrace{\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \cdot e^{2t}}$$

FUND. MATICE

$$\tilde{X}(t) = \begin{pmatrix} e^{t} & -e^{-t} & 0 \\ e^{t} & 0 & e^{2t} \\ 0 & e^{t} & 2e^{2t} \end{pmatrix}$$

$$X(t_0) = X_0$$

$$X(t) = \tilde{X}(t) \cdot C$$

$$\underbrace{X(t_0)}_{= X_0} = \tilde{X}(t_0) \cdot C \Rightarrow \boxed{C = \tilde{X}^{-1}(t_0) \cdot X(t_0)}$$

$$\Rightarrow \quad X(t) = \tilde{X}(t) \cdot \tilde{X}^{-1}(t_0) \cdot X(t_0)$$

$$\boxed{X(0) = I} \qquad X(t) = \tilde{X}(t) \cdot (\hat{X}(0))^{-1} \cdot I$$

$$\tilde{X}(t) = \begin{pmatrix} 0 & -e^{-t} & e^{t} \\ e^{2t} & 0 & e^{t} \\ 2e^{2t} & e^{-t} & 0 \end{pmatrix} \Rightarrow \hat{X}(0) = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}$$

$$(\hat{X}(0))^{-1} = \begin{pmatrix} 1 & -1 & 1 \\ -2 & 2 & -1 \\ -1 & 2 & -1 \end{pmatrix}$$

$$X(t) = \begin{pmatrix} 0 & -e^{-t} & e^{t} \\ e^{2t} & 0 & e^{t} \\ 2e^{2t} & e^{-t} & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 & 1 \\ -2 & 2 & -1 \\ -1 & 2 & -1 \end{pmatrix} = \ldots$$

$$x_1' = -x_1 + x_2 + x_3$$
$$x_2' = x_1 - x_2 + x_3$$
$$x_3' = x_1 + x_2 - x_3$$

$$A = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}, \quad |A - \lambda I| = (1-\lambda)\cdot(\lambda+2)^2$$

(i) $\lambda_1 = 1$ $\quad \left(\begin{array}{ccc|c} -2 & 1 & 1 & a \\ 1 & -2 & 1 & b \\ 1 & 1 & -2 & c \end{array}\right) \Rightarrow x = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot e^{1t}$

(iii) $\lambda_{2,3} = -2 \Rightarrow x = \begin{pmatrix} at+b \\ ct+d \\ et+f \end{pmatrix} \cdot e^{-2t}$

$x' = A \cdot x$

$$x' = \begin{pmatrix} a \\ c \\ e \end{pmatrix} \cdot \cancel{e^{-2t}} + \begin{pmatrix} at+b \\ ct+d \\ et+f \end{pmatrix} \cdot (-2) \cdot \cancel{e^{-2t}} = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} at+b \\ ct+d \\ et+f \end{pmatrix} \cancel{e^{-2t}}$$

PO ÚPRAVÁCH:

$$\begin{array}{cccccc} a & b & c & d & e & f \end{array}$$
$$\begin{pmatrix} 1 & -1 & 0 & -1 & 0 & -1 \\ 0 & -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & -3 & 0 \end{pmatrix} \Rightarrow \quad 1\cdot c + 2\cdot e = 0 \Rightarrow c = 0$$

$e = 0$

$$\begin{pmatrix} a - b - d - f = 0 \\ -b - d - f = 0 \end{pmatrix}$$

VOLBA $f = 1,\ d = 0 \Rightarrow b = -1$, $a = 0$

$$\Rightarrow \begin{pmatrix} at+b \\ \vdots \\ et+f \end{pmatrix} \cdot e^{-2t} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \cdot e^{-2t}$$

VOLBA $f = 0,\ d = 1 \Rightarrow b = -1,\ a = 0 \Rightarrow \quad x = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \cdot e^{-2t}$

OB. ŘEŠ.:

$$X = c_1 \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot e^{t} + c_2 \cdot \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \cdot e^{-2t} + c_3 \cdot \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \cdot e^{-2t}$$

$$y'' - 2y' + y = \frac{e^{x}}{x}$$

$$y'' - 2y' + y = 0 \quad \Rightarrow \quad y_h = c_1 \cdot e^{1x} + c_2 \cdot x \cdot e^{1x}$$

$$\lambda^2 - 2\lambda + 1 = 0$$

$$(\lambda - 1)^2 = 0$$

$$\lambda_{1,2} = 1$$

$$\boxed{y = c_1(x) \cdot e^{x} + c_2(x) \cdot x \cdot e^{x}}$$

$$y = c_1(x)\cdot \overset{y_1}{e^x} + c_2(x)\cdot \overset{y_2}{x\cdot e^x}$$

$$\begin{pmatrix} y_1 & y_2 \\ y_1' & y_2' \end{pmatrix} \cdot \begin{pmatrix} c_1' \\ c_2' \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{e^x}{x} \end{pmatrix}$$

$$\boxed{\begin{array}{ll} (1) & c_1'\cdot e^x + c_2'\cdot x\cdot e^x = 0 \\ (2) & c_1'\cdot e^x + c_2'\cdot(e^x + x\cdot e^x) = \frac{e^x}{x} \end{array}}$$

$$(2)-(1) \Rightarrow c_2'\cdot e^x = \frac{e^x}{x}$$

$$\boxed{c_2' = \frac{1}{x}}$$

$$(1) \Rightarrow c_1'\cdot e^x + e^x = 0$$

$$c_1' + 1 = 0$$

$$\boxed{c_1' = -1}$$

$$c_1 = \int -1\,dx = \underbrace{\boxed{-x + K_1}}_{c_1(x)}$$

$$c_2 = \int \frac{1}{x}\,dx = \underbrace{\boxed{\ln|x| + K_2}}_{c_2(x)}$$

$$y = (-x + k_1) \cdot e^x + (\ln|x| + k_2) \cdot x \cdot e^x$$

$$= \underbrace{-xe^x + \ln|x| \cdot x \cdot e^x} + \underbrace{k_1 e^x + k_2 x e^x}_{y_h}$$

$$y'' - 2y' + 5y = 5 \cdot e^{2x} \cdot \sin x = e^{2x} \cdot (5 \cdot \sin x + 0 \cdot \cos x)$$

$$\lambda^2 - 2\lambda + 5 = 0$$

$$\lambda_{1,2} = 1 \pm 2i$$

$$\boxed{\text{St} = 0}$$

$$y = x^{r} \cdot e^{2x} \cdot (P_n(x) \cdot \sin 1x + P_L(x) \cdot \cos x)$$

$$2 + 1i \Rightarrow \boxed{r = 0}$$

$$y = x^0 \cdot e^{2x} \cdot (A \cdot \sin x + B \cdot \cos x)$$

$$y = e^{2x}\cdot(A\cdot\sin x + B\cdot\cos x)$$

$$y' = \underbrace{2\cdot e^{2x}\cdot(A\cdot\sin x + B\cdot\cos x)} + \underbrace{e^{2x}\cdot(A\cdot\cos x - B\cdot\sin x)}$$

$$y'' = \underbrace{4e^{2x}\cdot(A\cdot\sin x + B\cdot\cos x) + 2\cdot e^{2x}\cdot(A\cdot\cos x - B\sin x)} +$$

$$+ \underbrace{2\cdot e^{2x}\cdot(A\cdot\cos x - B\cdot\sin x) + e^{2x}\cdot(-A\sin x - B\cos x)}$$

$$4e^{2x}\cdot(A\sin x + B\cdot\cos x) + 2\cdot e^{2x}\cdot(A\cdot\cos x - B\cdot\sin x) +$$

$$+ 2\cdot e^{2x}\cdot(A\cdot\cos x - B\cdot\sin x) + e^{2x}\cdot(-A\cdot\sin x - B\cdot\cos x)$$

$$-4\cdot e^{2x}\cdot(A\sin x + B\cdot\cos x) - 2\cdot e^{2x}\cdot(A\cos x - B\cdot\sin x)$$

$$+ 5\cdot e^{2x}\cdot(A\cdot\sin x + B\cdot\cos x) = 5\cdot e^{2x}\cdot\sin x$$

$\boxed{e^{2x}\cdot\sin x}$:

$$4A - 2B - 2B - A - 4A + 2B + 5A = 5$$

$$\boxed{4A - 2B = 5}$$

$\boxed{e^{2x} \cdot \cos x}$ $\quad 2A + 4B = 0$

$$\left.\begin{array}{l} 4A - 2B = 5 \\ \cancel{2A + 4B = 0} \\ A + 2B = 0 \end{array}\right\} \oplus \Rightarrow 5A = 5 \Rightarrow \boxed{\begin{array}{l} A = 1 \\ \\ B = -\frac{1}{2} \end{array}}$$

$$1 + 2B = 0 \Rightarrow B = -\frac{1}{2}$$

$$y_p = e^{2x} \cdot \left(1 \cdot \sin x - \frac{1}{2} \cos x\right)$$

$$y = y_h + y_p = c_1 \cdot \underline{e^x \cdot \cos 2x} + c_2 \cdot \underline{e^x \cdot \sin 2x} + e^x \cdot \left(\sin x - \frac{1}{2} \cos x\right)$$

$$\lambda_{1,2} = 1 \pm 2i \Rightarrow \tilde{y}_1 = e^{(1+2i)\cdot x}$$
$$\tilde{y}_2 = e^{(1-2i)\cdot x}$$

$$\Rightarrow \underbrace{y_1 = e^{1\cdot x} \cdot \cos 2x}, \quad \underbrace{y_2 = e^{1x} \cdot \sin 2x}$$

$$y_h = c_1 y_1 + c_2 y_2$$

$$y' - 3x^2 y = (x+2)\cdot e^{x^3}, \quad y(0) = 5$$

$$\mu(x) = e^{\int -3x^2\,dx} = e^{-3\frac{x^3}{3}} = e^{-x^3}$$

$$\underbrace{y'\cdot e^{-x^3} - 3x^2 y\cdot e^{-x^3}} = (x+2)\cdot \cancel{e^{x^3}\cdot e^{-x^3}}$$

$$\left(y\cdot e^{-x^3}\right)' = x+2 \quad \Big/ \int dx$$

$$y\cdot e^{-x^3} = \frac{x^2}{2}+2x+c, \quad c\in\mathbb{R} \quad \Big/ \cdot e^{x^3}$$

$$\boxed{y = e^{x^3}\cdot\left(\frac{x^2}{2}+2x+c\right), \quad c\in\mathbb{R}}$$

$$\boxed{y = e^{x^3}\cdot\left(\frac{x^2}{2}+2x+c\right),\ c\in\mathbb{R}}$$

$$y(0)=5$$

$$\boxed{x=0,\ y=5}$$

$$5 = 1\cdot(0+0+c) \Rightarrow c=5$$

ŘEŠ. POČ. PR. = $y = e^{x^3}\cdot\left(\frac{x^2}{2}+2x+5\right)$

$$y(1)=3 \Rightarrow 3 = e\cdot\left(\frac{1}{2}+2+c\right)$$

$$\frac{3}{e} = \frac{5}{2}+c \Rightarrow c = \frac{3}{e}-\frac{5}{2}$$

$$y = e^{x^3}\cdot\left(\frac{x^2}{2}+2x+\frac{3}{e}-\frac{5}{2}\right)$$