$$\sum_{n=0}^{\infty} \frac{3}{5^n} = \left| \begin{array}{l} q = \frac{1}{5} < 1 \\ a_0 = 3 \end{array} \right| = \frac{3}{1-\frac{1}{5}} = \frac{15}{4}$$

$$\sum_{n=1}^{\infty} \frac{n}{2^n} = \ldots = 2$$

$$\ln x + \ln \sqrt{x} + \ln \sqrt[4]{x} + \dots = 2, \quad x = ?$$

$$\ln x + \frac{1}{2} \ln x + \frac{1}{4} \ln x + \dots = 2$$

$$\ln x \cdot \left(1 + \underbrace{\frac{1}{2} + \frac{1}{4} + \dots}\right) = 2$$

$$1 \qquad 2$$

$$\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1 + 1 = 2$$

$$n = 0$$

$$2 \cdot \ln x = 2, \quad \ln x = 1, \quad x = e$$

$$\sum_{1}^{\infty}\left(\sqrt{n}-2\sqrt{n+1}+\sqrt{n+2}\right)= \;\ldots\; = 1-\sqrt{2}$$

$$\sum_{1}^{\infty} \frac{2^n \cdot n!}{n^n}$$

$$\frac{a_{n+1}}{a_n} = \frac{\dfrac{2^{n+1} \cdot (n+1)!}{(n+1)^{n+1}}}{\dfrac{2^n \cdot n!}{n^n}} = \frac{2 \cdot \cancel{2^n} \cdot \cancel{(n+1)} \cdot \cancel{n!} \cdot n^n}{\cancel{2^n} \cdot \cancel{n!} \cdot (n+1)^{n+\cancel{1}}} =$$

$$= 2 \cdot \underbrace{\left(\frac{n}{n+1}\right)^n} \xrightarrow{n \to \infty} 2 \cdot \frac{1}{e} = \frac{2}{e} < 1$$

$$\left(\frac{n+1}{n}\right)^{-n} = \left[\left(1 + \frac{1}{n}\right)^n\right]^{-1}$$

KONV.

$$\sum_{1}^{\infty} \left(\arccos\frac{1}{n}\right)^{n^2}$$

$$\sqrt[n]{a_n} = \left(\arccos\frac{1}{n}\right)^{n} \xrightarrow{n\to\infty} (\arccos 0)^{\infty} =$$

$$= \left(\frac{\pi}{2}\right)^{\infty} = \underline{\infty > 1}$$

DIV.

$$\sum_{1}^{\infty} \operatorname{arctg}^n \frac{1}{n}$$

$$\sqrt[n]{a_n} = \operatorname{arctg} \frac{1}{n} \xrightarrow{n \to \infty} \operatorname{arctg} 0 = 0 < 1$$

K ZAK.

$$\sum_{1}^{\infty} \frac{(-2)^n}{\left[5+(-1)^{n+1}\right]^n}$$

ABS. K.

$$\sum_{1}^{\infty} \frac{2^n}{\left[5+(-1)^{n+1}\right]^n} \leq \sum_{1}^{\infty} \frac{2^n}{4^n} = \sum_{1}^{\infty} \frac{1}{2^n} = 1$$

K. ABS.

$$\sum_{1}^{\infty} \frac{2^n}{n^2} x^n , \quad x_0 = 0 , \quad a_n = \frac{2^n}{n^2}$$

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim \frac{2^{n+1} \cdot n^2}{(n+1)^2 \cdot 2^n} = \lim_{n\to\infty} \frac{2 n^2}{(n+1)^2} = \frac{2}{1} = 2$$

$$\Rightarrow R = \frac{1}{2} \Rightarrow x \in \left(-\frac{1}{2}, \frac{1}{2}\right)$$

$$x = -\frac{1}{2} \Rightarrow \sum_{1}^{\infty} \frac{\cancel{2^n}}{n^2} \cdot \left(-\frac{1}{\cancel{2}}\right)^n = \sum_{1}^{\infty} \frac{(-1)^n}{n^2} \quad \text{KONV.}$$

$$x = \frac{1}{2} \Rightarrow \sum_{1}^{\infty} \frac{\cancel{2^n}}{n^2} \cdot \left(\frac{1}{\cancel{2}}\right)^n = \sum_{1}^{\infty} \frac{1}{n^2} \quad \text{KONV.}$$

$$\Rightarrow \quad I = \left[-\frac{1}{2}, \frac{1}{2}\right] \text{ .. ABS. KONV.}$$

$$\sum_{1}^{\infty} \frac{(-1)^n \cdot (x+2)^n}{n+\sqrt{n}} , \quad x_0 = -2 , \quad \underline{a_n = \frac{(-1)^n}{n+\sqrt{n}}}$$

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{\frac{1}{n+1+\sqrt{n+1}}}{\frac{1}{n+\sqrt{n}}} = \frac{n+\sqrt{n}}{n+1+\sqrt{n+1}} \xrightarrow{n\to\infty} \frac{1}{1} = 1$$

$$\Rightarrow R = \frac{1}{1} = 1 \quad \Rightarrow \quad \underline{\tilde{I} = (-3,-1)}$$

$$\cancel{x=-3} \Rightarrow \sum_{1}^{\infty} \frac{\cancel{(-1)^n \cdot (-1)^n}}{n+\sqrt{n}} = \sum_{1}^{\infty} \frac{1}{n+\sqrt{n}} \geq \sum_{1}^{\infty} \frac{1}{n+n} = \sum_{1}^{\infty} \frac{1}{2n} = \frac{1}{2} \cdot \sum_{1}^{\infty} \frac{1}{n} = \infty$$

$$x=-1 \Rightarrow \sum_{1}^{\infty} \frac{(-1)^n \cdot 1^n}{n+\sqrt{n}} = \sum_{1}^{\infty} \frac{(-1)^n}{n+\sqrt{n}} \quad \text{ALT. Ř.}$$

$$\frac{1}{n+\sqrt{n}} \searrow 0$$

LEIB. KR. $\Rightarrow$ KONV.

KONV. $x \in (-3,-1]$

(ABS. K. $x \in (-3,-1)$, REL. K. $x=-1$)

$$\sum_{1}^{\infty} \alpha^{n^2} \cdot x^n , \quad 0<\alpha<1 \Rightarrow x_0 = 0$$

$$a_n = \alpha^{n^2}$$

$$\lim_{n\to\infty} \sqrt[n]{|\alpha^{n^2}|} = \lim \alpha^n = 0 \Rightarrow R = \infty$$

$\Rightarrow$ (ABS.) KONV. $x \in \mathbb{R}$

$$\sum \frac{n!}{\beta^{n^2}} \cdot x^n, \quad \beta > 1 \qquad a_n = \frac{n!}{\beta^{n^2}}, \quad x_0 = 0$$

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{(n+1)! \cdot \beta^{n^2}}{\beta^{(n+1)^2} \cdot n!} = \lim_{n\to\infty} \frac{n+1}{\beta^{2n+1}} = 0$$

$$n^2 - (n+1)^2 = \cancel{n^2} - \cancel{n^2} - 2n - 1 =$$
$$= -(2n+1)$$

$$\Rightarrow R = \infty, \quad I = \mathbb{R}$$

$$\sum_{1}^{\infty} \frac{x^{4n-3}}{4n-3}, \quad x=0, \quad R=1$$

$$\left(\sum_{1}^{\infty} \frac{x^{4n-3}}{4n-3}\right)' = \sum_{1}^{\infty} \frac{\cancel{(4n-3)} \cdot x^{4n-4}}{\cancel{4n-3}} = \sum_{n=1}^{\infty} x^{4\cdot(n-1)} =$$

$$= \left| m = n-1 \right| = \sum_{m=0}^{\infty} x^{4m} = \sum_{m=0}^{\infty} \left(x^{4}\right)^{m} = \left| \begin{matrix} q = x^{4} \\ a_0 = 1 \end{matrix} \right| = \frac{1}{1-x^{4}}$$

$$\int \frac{1}{1-x^{4}}\, dx = \int \frac{\frac{1}{2}}{1+x^{2}} + \frac{\frac{1}{4}}{1-x} + \frac{\frac{1}{4}}{1+x}\, dx = \frac{1}{2} \operatorname{arctg} x + \frac{1}{4} \ln\left|\frac{1+x}{1-x}\right|$$

$+C$

$x = x_0 = 0$

$$\sum 0 = 0 \overset{\downarrow}{=} \underbrace{\frac{1}{2}\operatorname{arctg} 0}_{0} + \underbrace{\frac{1}{4}\ln(1)}_{0} + C$$

$$\Rightarrow C = 0$$

$$\sum_{n=1}^{\infty} \frac{x^{4n-3}}{4n-3} = \frac{1}{2}\operatorname{arctg} x + \frac{1}{4}\ln\frac{1+x}{1-x}$$

$x \in (-1, 1)$

$$\sum_{n=3}^{\infty} \frac{x^n}{n\cdot 5^n}, \quad x_0 = 0, \quad \frac{1}{R} = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim \frac{n\cdot 5^n}{(n+1)\cdot 5^{n+1}} =$$

$$= \frac{1}{5}\overbrace{\lim_{n\to\infty} \frac{n}{n+1}}^{=1} = \frac{1}{5} \Rightarrow \boxed{R = 5}$$

$$x = -5 \Rightarrow \sum \frac{(-5)^n}{n\cdot 5^n} = \sum \frac{(-1)^n}{n} \quad K \quad (REL)$$

$$x = 5 \Rightarrow \sum \frac{5^n}{n\cdot 5^n} = \sum \frac{1}{n} \quad D \quad // \quad I = [-5, 5)$$

$$\left(\sum_{n=3}^{\infty}\frac{x^n}{n\cdot 5^n}\right)' = \sum_{n=3}^{\infty}\frac{\cancel{n}\cdot x^{n-1}}{\cancel{n}\cdot 5^n} = \sum_{n=3}^{\infty}\frac{x^{n-1}}{5^n} = \frac{1}{5}\cdot\sum_{n=3}^{\infty}\left(\frac{x}{5}\right)^{n-1} =$$

$$= \left| \begin{array}{l} q = \frac{x}{5} \\ a_3 = \frac{x^2}{25} \end{array} \right| = \frac{1}{5}\cdot\frac{\frac{x^2}{25}}{1-\frac{x}{5}} = \frac{1}{5}\cdot\frac{\frac{x^2}{25}}{\frac{5-x}{5}} =$$

$$= \frac{1}{\cancel{5}}\cdot\frac{\cancel{5}x^2}{25\cdot(5-x)} = \frac{x^2}{25\cdot(5-x)}$$

$$\sum_{3}^{\infty} \frac{x^n}{n \cdot 5^n} = \frac{1}{25} \cdot \int \frac{x^2}{5-x}\,dx = \left| \; x^2 : (5-x) = \dots \; \right| =$$

$$= \frac{1}{25} \cdot \int -x - 5 + \frac{25}{5-x}\,dx = \frac{1}{25} \cdot \left(-\frac{x^2}{2} - 5x + 25 \cdot (-\ln(5-x)) \right) + C$$

$$x = 0 \Rightarrow \sum 0 = 0 = \frac{1}{25} \cdot (0 + 0 - 25 \cdot \ln 5) + C$$

$$0 = -\ln 5 + C \Rightarrow C = \ln 5$$

$$\sum_{3}^{\infty} \frac{x^n}{n \cdot 5^n} = \frac{-x^2}{50} - \frac{x}{5} - \ln|5-x| + \ln 5 =$$

$$= \ln \frac{5}{5-x} - \frac{x^2}{50} - \frac{x}{5} \quad , \quad x \in [-5, 5)$$

$$x = -5 \Rightarrow \sum_{3}^{\infty} \frac{(-1)^n}{n} = \ln \frac{5}{10} - \frac{25}{50} + \frac{5}{5} = \ln \frac{1}{2} - \frac{1}{2} + 1 =$$

$$= \ln \frac{1}{2} + \frac{1}{2} =$$

$$= \frac{1}{2} - \ln 2$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} = \underbrace{-1 + \frac{1}{2}}_{=0} + \frac{1}{2} - \ln 2 = -\ln 2$$

$$\sum_{n=1}^{\infty} \frac{(n-1)\cdot(e-1)^n}{n\cdot e^n + e^n} = \sum_{n=1}^{\infty} \underbrace{\frac{n-1}{n+1}}_{a_n} \cdot \Big(\underbrace{\frac{e-1}{e}}_{x}\Big)^n = \sum_{n=1}^{\infty} \frac{n-1}{n+1}\cdot x^n$$

$$x = \frac{e-1}{e}$$

$$x_0 = 0, \quad R = 1 \Rightarrow I = (-1, 1) \ni \frac{e-1}{e}$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad \Big/ \int dx$$

$$\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1} = \int \frac{1}{1-x}\,dx = -\ln|1-x| + C$$

$x=0 \Rightarrow$

$0 = -\ln 1 + C$

$\boxed{C=0}$

$$\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1} = -\ln(1-x) \quad \Big/ \cdot \frac{1}{x^2}$$

$$\sum_{n=0}^{\infty} \frac{x^{n-1}}{n+1} = \frac{-\ln(1-x)}{x^2} \quad \Big/ \frac{d}{dx}$$

$$\sum_{n=0}^{\infty} \frac{(n-1)\cdot x^{n-2}}{n+1} = \frac{1}{x^2\cdot(1-x)} + \frac{2\cdot\ln(1-x)}{x^3} \quad \Big/ \cdot x^2$$

$$\sum_{n=0}^{\infty} \frac{n-1}{n+1} \cdot x^n = \frac{1}{1-x} + \frac{2}{x} \cdot \ln(1-x)$$

$$\boxed{\sum_{n=1}^{\infty} \frac{n-1}{n+1} x^n = \frac{1}{1-x} + \frac{2}{x} \cdot \ln(1-x) - (-1)}$$

$$x = \frac{e-1}{e}$$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{n-1}{n+1} \cdot \left(\frac{e-1}{e}\right)^n = \frac{1}{1-\frac{e-1}{e}} + \frac{2e}{e-1} \cdot \ln\left(1-\frac{e-1}{e}\right) + 1 =$$

$$= e - \frac{2e}{e-1} + 1$$