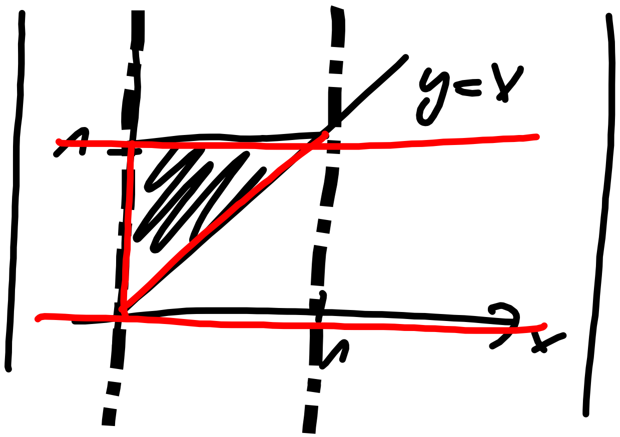
$$\int_0^1 \left(\int_x^1 \cos y^2 \, dy \right) dx =$$



$$=$$

$$= \int_0^1 \left(\int_0^y \cos y^2 \, dx \right) dy = \int_0^1 \cos y^2 \cdot [x]_0^y \, dy =$$

$$= \int_0^1 y \cdot \cos y^2 \, dy = \left| \begin{array}{l} t = y^2 \\ dt = 2y\, dy \\ y\, dy = \frac{1}{2}\, dt \end{array} \right| \begin{array}{l} 1 \Rightarrow 1 \\ 0 \Rightarrow 0 \end{array} \left| = \int_0^1 \cos t \, \frac{1}{2}\, dt = \right.$$

$$= \frac{1}{2} \cdot \left[\sin t \right]_0^1 = \underline{\underline{\frac{\sin 1}{2}}}$$

$$f(x) = \int_0^x \sin t^2 \, dt \quad , \quad f'(x) = \sin x^2$$

MACLAURIN:

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot x^{2n+1}}{(2n+1)!}$$

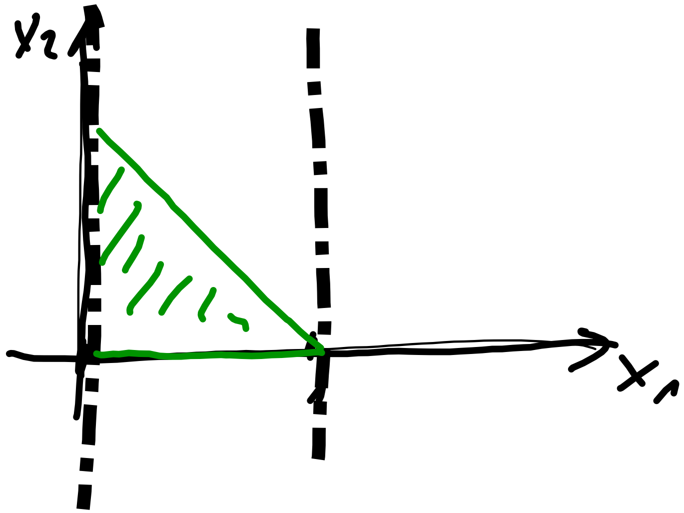
$$\boxed{\sin(t^2) = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot t^{4n+2}}{(2n+1)!}}$$

$\mathbb{R}$

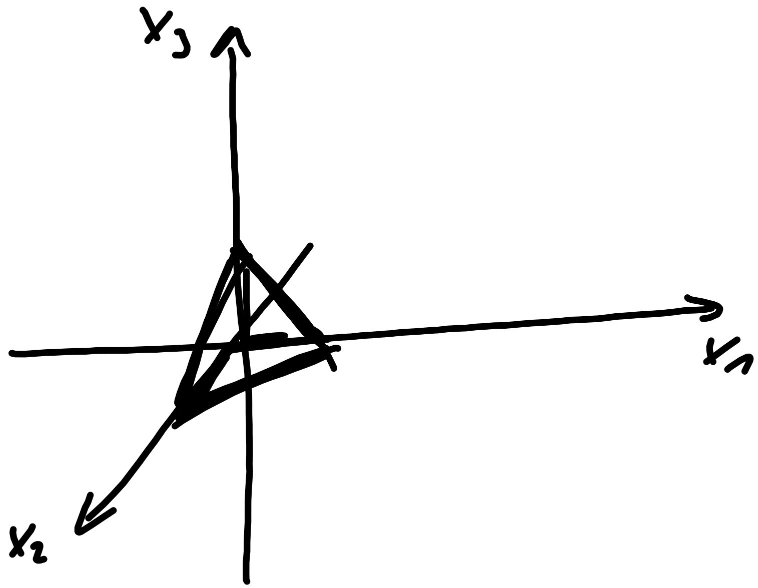
$$f(x) = \int_0^x \sum_{n=0}^{\infty} \frac{(-1)^n \cdot t^{4n+2}}{(2n+1)!} \, dt = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \cdot \int_0^x t^{4n+2} \, dt =$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \cdot \left[\frac{t^{4n+3}}{4n+3} \right]_0^x = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot x^{4n+3}}{(2n+1)! \cdot (4n+3)} =$$

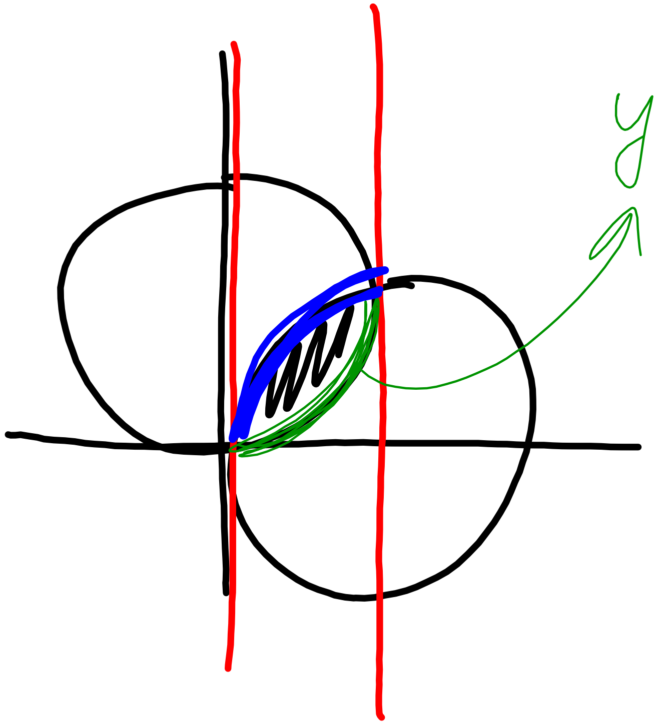
$$= \frac{x^3}{3} - \frac{x^7}{3! \cdot 7} + \dots$$



x_2
x_1



x_3
x_1
x_2

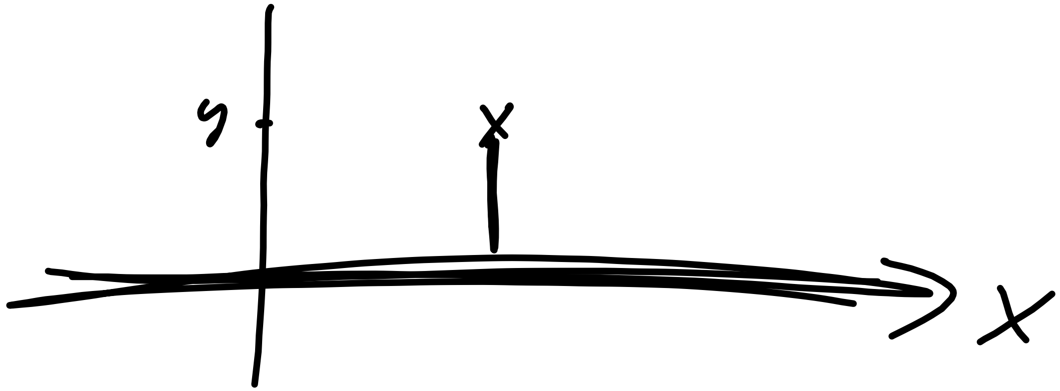


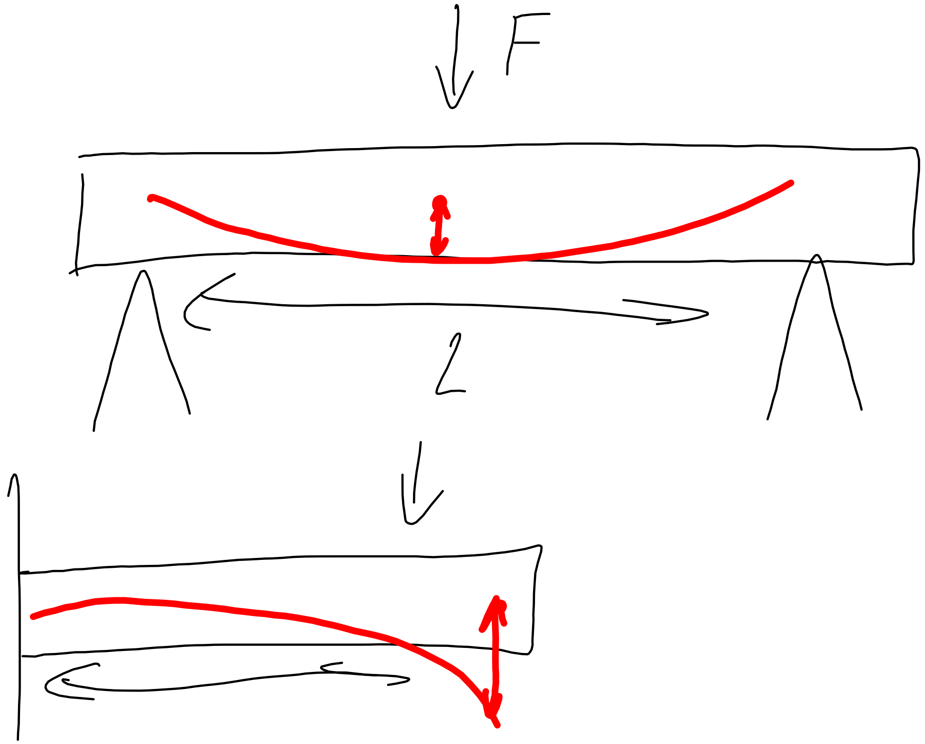
$$t = \sin x$$

$$\sqrt{1-\sin^2 x} = \sqrt{\cos^2 x} =$$

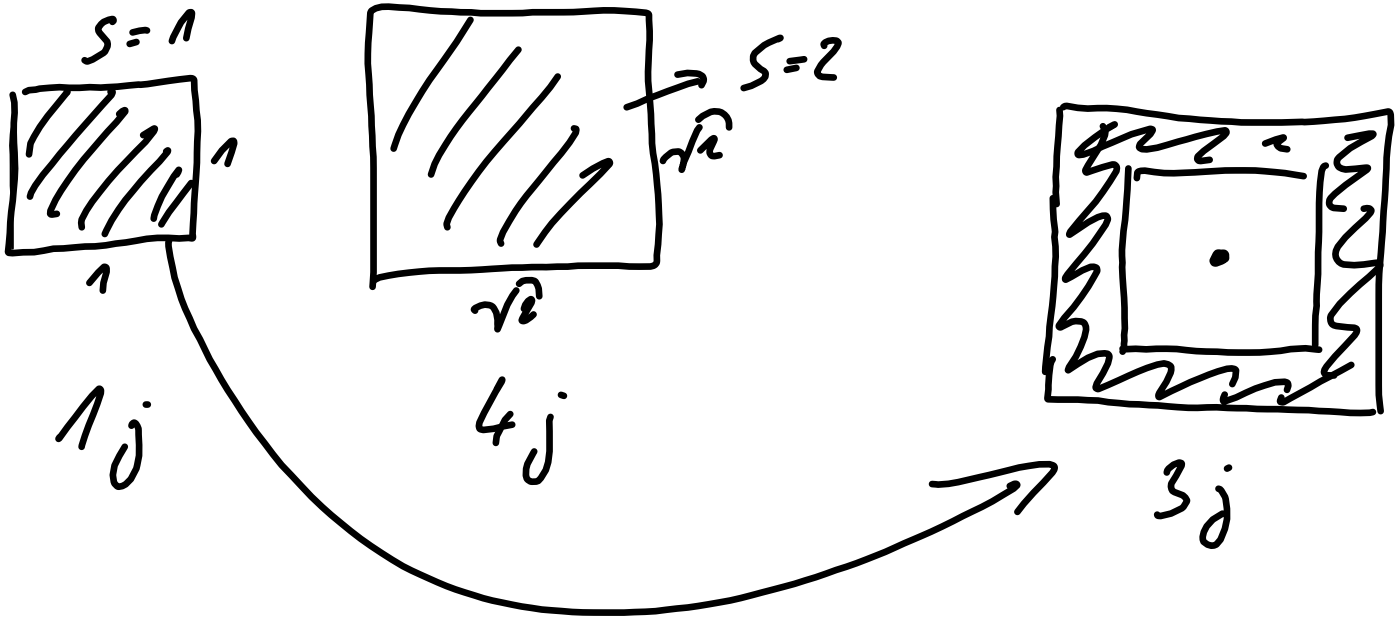
$$= \cos x$$

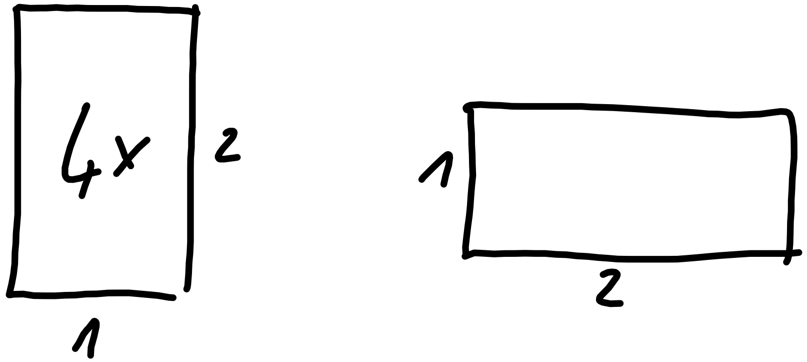
$$av(f) = \frac{1}{b-a} \cdot \int_a^b f(x)\,dx$$



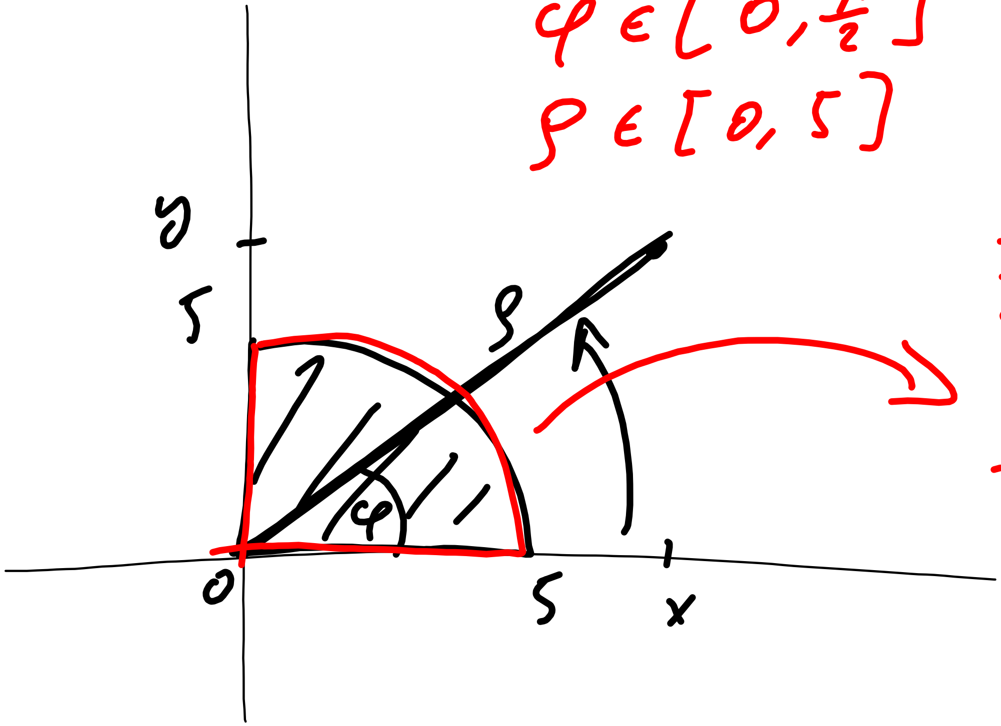


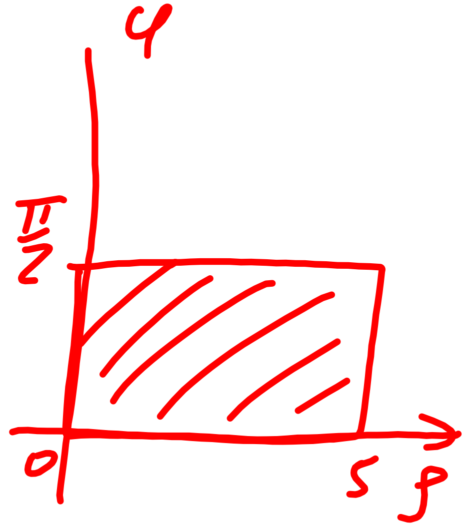
$$\frac{1}{I}$$

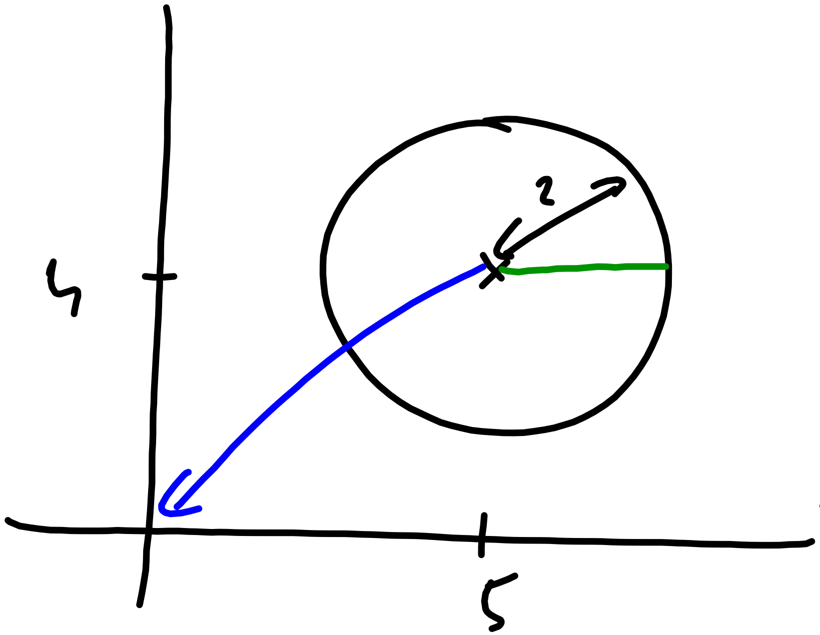

s = 1
1
1
1 j
s = 2
√2
√2
4 j
3 j


4x
2
1
1
2

$\varphi \in [0, \frac{\pi}{2}]$
$\rho \in [0, 5]$







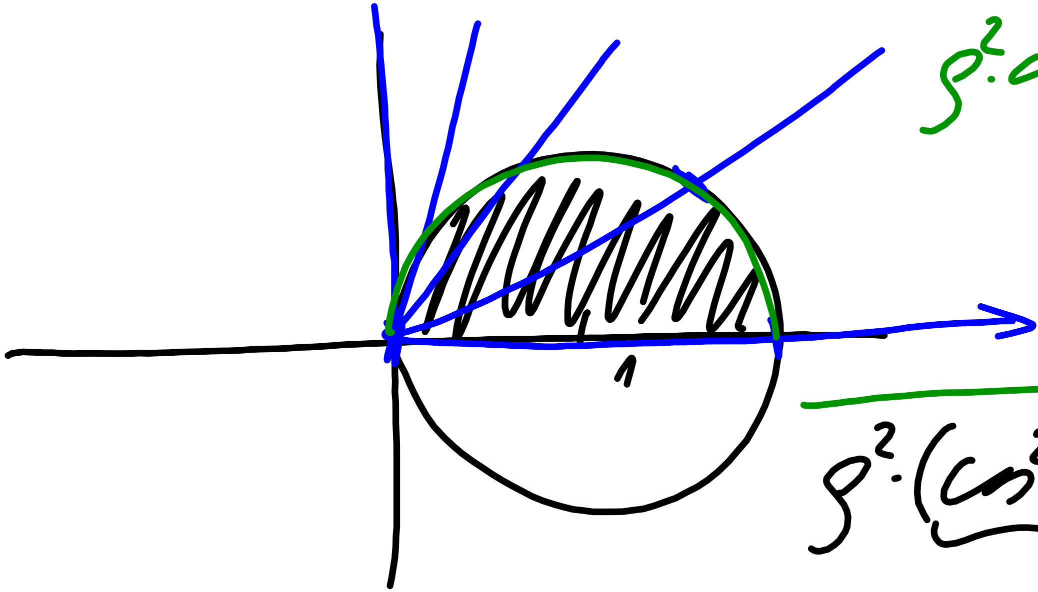
$x - 5 = \rho \cdot \cos\varphi$

$y - 4 = \rho \cdot \sin\varphi$

$x = 5 + \rho \cdot \cos\varphi$

$y = 4 + \rho \cdot \sin\varphi$

$\rho \in [0, 2], \ \varphi \in [0, 2\pi]$

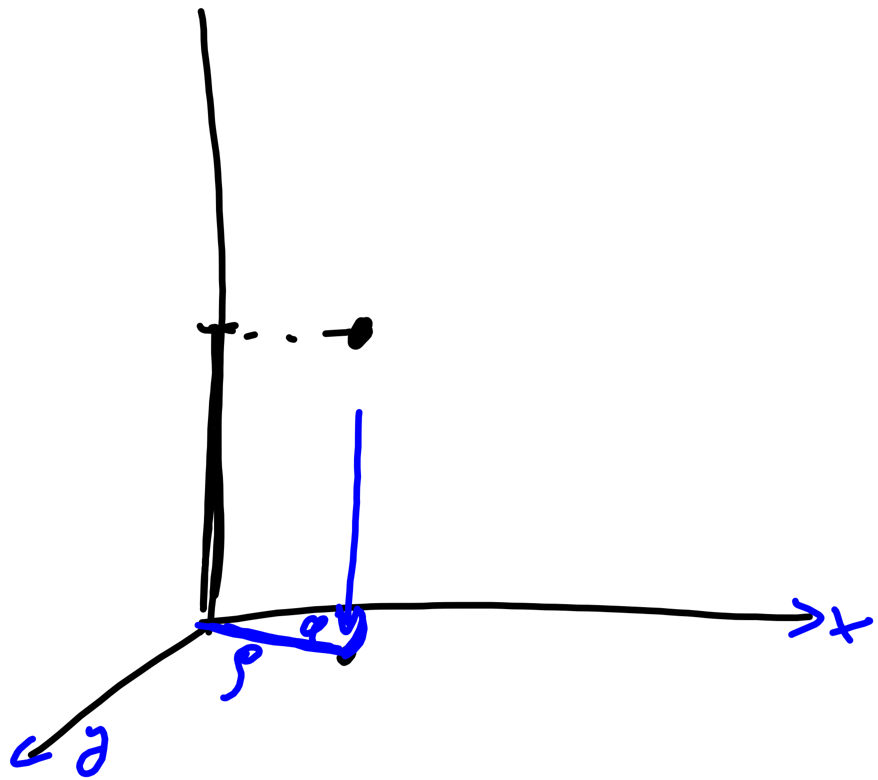


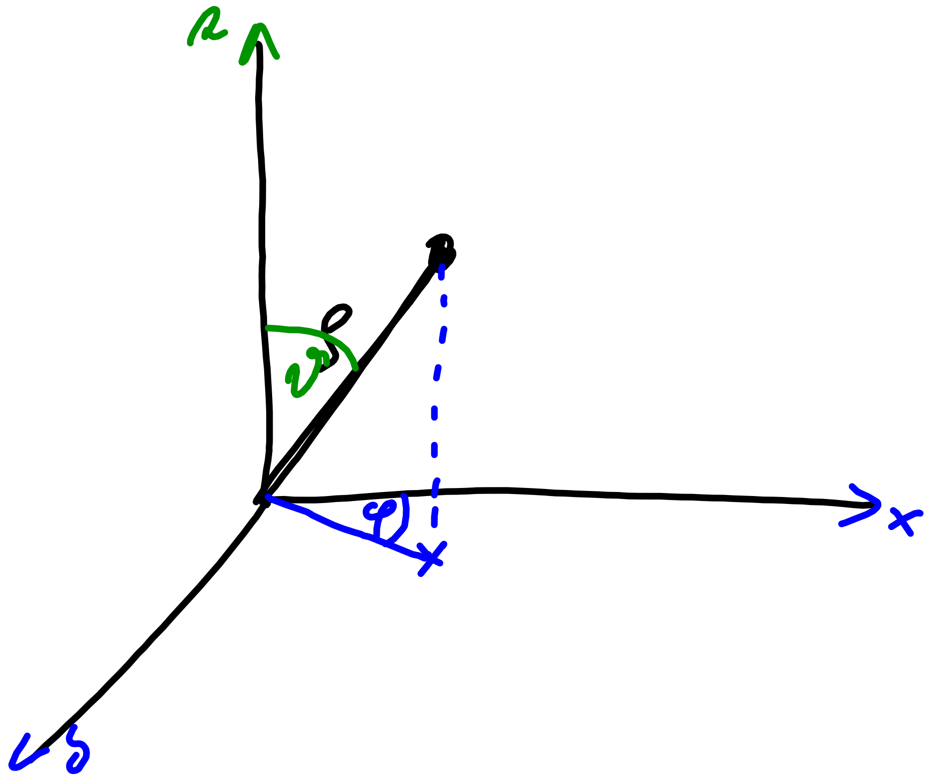
$\rho^2 \cdot \cos^2\varphi + \rho^2 \cdot \sin^2\varphi$

$\leq 2 \cdot \rho \cdot \cos\varphi$

$\rho^2 \cdot \underbrace{(\cos^2\varphi + \sin^2\varphi)}_{=1} \leq 2 \cdot \rho \cdot \cos\varphi$

$\rho \leq 2 \cdot \cos\varphi$

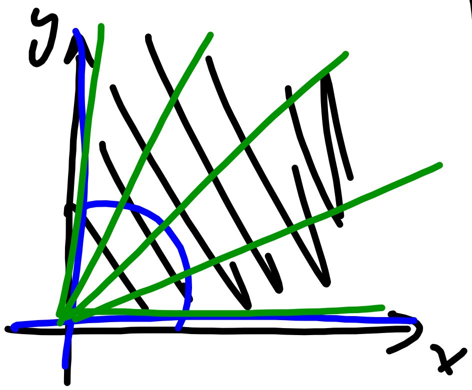

x
y
ρ
φ



z
ϱ
ϑ
φ
x
x
y

$$\int_0^\infty \int_0^\infty e^{-x^2} \cdot e^{-y^2}\, dy\, dx = I^2 =$$

$$= \int_0^\infty \int_0^\infty e^{-x^2-y^2}\, dy\, dx =$$



$$-x^2-y^2 =$$
$$= -(x^2+y^2) =$$
$$= -(\rho^2 \cdot \cos^2\varphi + \rho^2 \cdot \sin^2\varphi) =$$
$$= -\rho^2$$

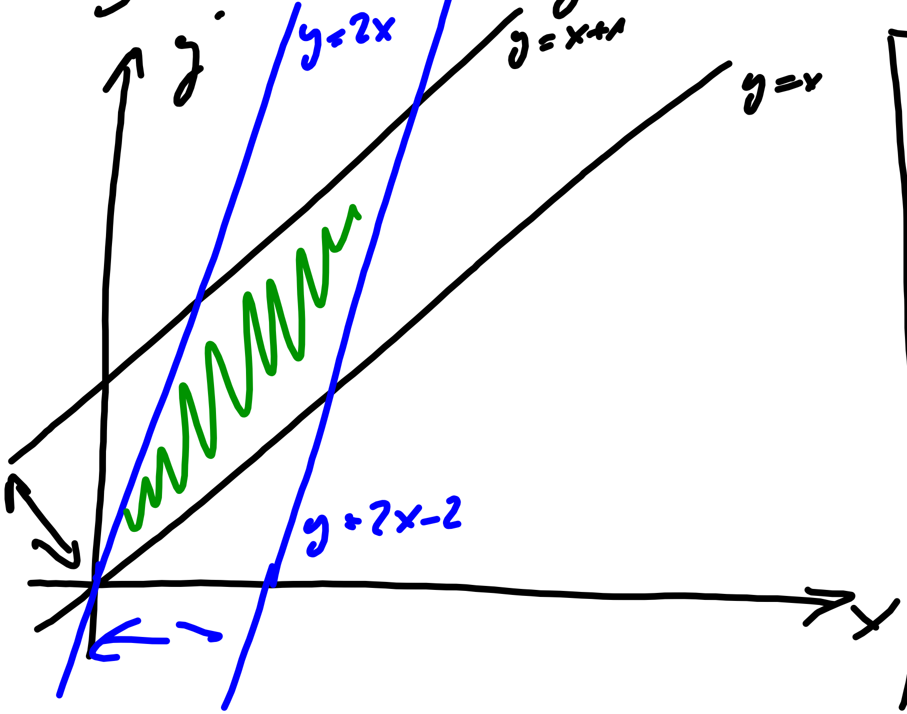
$$\left| \begin{array}{l} \varphi \in [0, \frac{\pi}{2}] \\ \rho \in (0, \infty) \end{array} \right| = \int_0^\infty \int_0^{\frac{\pi}{2}} e^{-\rho^2} \cdot \rho \; d\varphi\, d\rho =$$

$$= \left| \begin{array}{l} t = -\rho^2 \\ dt = -2\rho\, d\rho \\ \rho\, d\rho = \underbrace{-\tfrac{1}{2}\, dt} \end{array} \right| \begin{array}{l} \rho = \infty \Rightarrow t = -\infty \\ \rho = 0 \Rightarrow t = 0 \end{array} \Bigg| = \frac{\pi}{2} \cdot \int_0^{-\infty} e^t \cdot \left(-\frac{1}{2}\right) \cdot dt =$$

$$= \frac{\pi}{4} \cdot \int_{-\infty}^{0} e^t\, dt = \frac{\pi}{4} \cdot \left[e^t\right]_{-\infty}^{0} = \frac{\pi}{4} \cdot (1 - 0) = \frac{\pi}{4}$$

$$I^2 = \frac{\pi}{4} \Rightarrow I = \frac{\sqrt{\pi}}{2}$$

$S = ?$ $\quad x \le y \le x+1, \quad 2x-2 \le y \le 2x$



$x \le y \le x+1$

$0 \le \underbrace{y-x}_{u} \le 1$

$u \in [0,1]$

$u = y - x$

$2x-2 \le y \le 2x$

$-2 \le \underbrace{y-2x}_{} \le 0$

$v \in [-2,0]$

$v = y - 2x$

$$\Rightarrow \quad \begin{array}{l} x = u - v \\ y = 2u - v \end{array} \quad \Bigg| \quad J(u,v) = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = -1+2 = 1$$

$$S = \iint_M 1 \, dx\, dy = \int_0^1 \int_{-2}^0 1 \cdot 1 \, dv\, du = 2$$

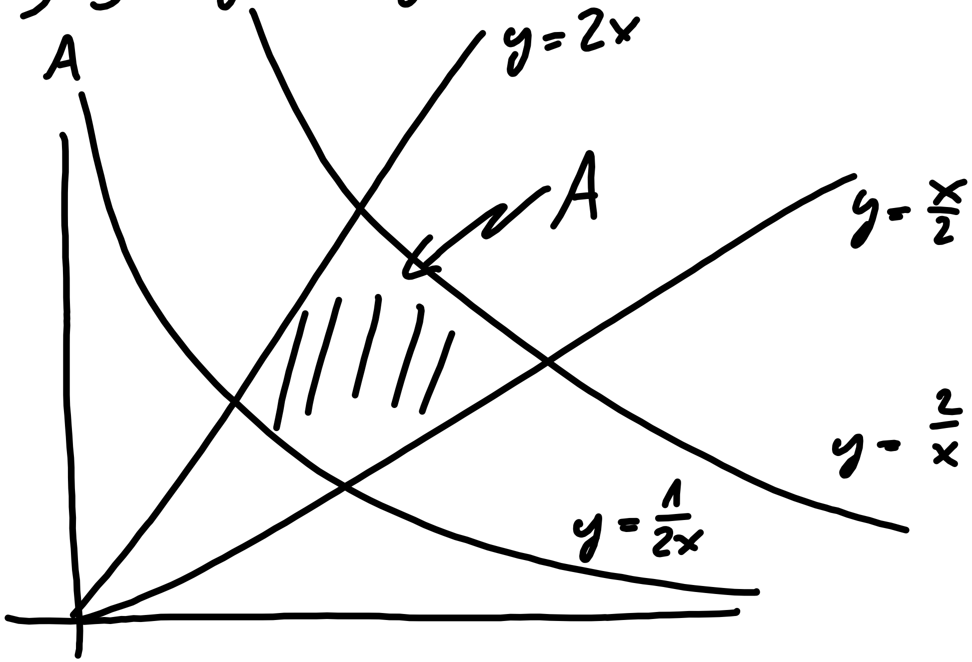
$$L = 0{,}000013\, m\, v^2$$

$$m \in [3000, 4000]\ ,\quad v \in [50, 60]$$

$$\operatorname{av}(f) = \frac{\iint_D f(x,y)\, dx\, dy}{m(D)} = \frac{\int_{3000}^{4000} \int_{50}^{60} 0{,}000013\, m\, v^2\, dv\, dm}{(60-50)\cdot(4000-3000)} =$$

$$\doteq 138{,}01667$$

$$\iint_A x^2 y^2 \, dx \, dy = ?$$



$y = \frac{1/2}{x} \; \dots \; \frac{2}{x}$

$y = \frac{u}{x}, \; u \in [\frac{1}{2}, 2]$

$y = \frac{1}{2}x \; \dots \; 2x$

$y = v x, \; v \in [\frac{1}{2}, 2]$

$u = x \cdot y$

$v = \frac{y}{x}$

$$\tilde{J} = \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix} = \begin{vmatrix} y & x \\ -\frac{y}{x^2} & \frac{1}{x} \end{vmatrix} = \frac{y}{x} + \frac{y}{x} = 2 \cdot \frac{y}{x}$$

$$J = \tilde{J}^{-1} = \frac{1}{2} \cdot \frac{x}{y} = \frac{1}{2}\,\frac{x}{v \cdot x} = \frac{1}{2v}$$

$x = \sqrt{\frac{u}{v}}$

$y = \sqrt{u \cdot v}$

$$I = \int_{1/2}^{2} \int_{1/2}^{2} \frac{u}{\cancel{v}} \cdot u \cdot \cancel{v}\, \frac{1}{2v}\, dv\, du = \int_{1/2}^{2} \int_{1/2}^{2} \frac{u^2}{2v}\, dv\, du =$$

$$= \ldots = \frac{63}{24} \cdot \ln 2$$