$$\sum_{n=1}^{\infty} \frac{5^{4n-1}}{4^{5n+1}}$$

$$\frac{a_{n+1}}{a_n} = \frac{\dfrac{5^{4(n+1)-1}}{4^{5\cdot(n+1)+1}}}{\dfrac{5^{4n-1}}{4^{5n+1}}} =$$

$$= \frac{5^{\cancel{4n}+3}\cdot 4^{\cancel{5n}+1}}{4^{\cancel{5n}+6}\cdot 5^{\cancel{4n}-1}} = \frac{5^4}{4^5} = \frac{625}{1024} < 1 \Rightarrow K$$

$$\lim_{n\to\infty}$$

$$\sum_{n=1}^{\infty} \underbrace{\left(1+\frac{1}{n}\right)^{n^2}} \underbrace{(x+3)^{n}} \longrightarrow x_0 = -3$$

$$a_n = \left(1+\frac{1}{n}\right)^{n^2}, \quad \sqrt[n]{|a_n|} = \left(1+\frac{1}{n}\right)^{n} \xrightarrow{n\to\infty} e$$

$$\Rightarrow \frac{1}{R} = e \Rightarrow R = \frac{1}{e} \Rightarrow \underline{\underline{x \in \left(-3-\frac{1}{e}, -3+\frac{1}{e}\right)}}$$

$$\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n^2}\cdot\underbrace{(2x+3)}_{}^{n} =$$

$$2x+3=0$$

$$\boxed{x_0=-\frac{3}{2}}$$

$$=\sum_{n=1}^{\infty}\underbrace{\left(1+\frac{1}{n}\right)^{n^2}\cdot 2^n}_{a_n}\cdot\underbrace{\left(x+\frac{3}{2}\right)}_{}^{n}$$

!

$$\sum_{n=1}^{\infty} \frac{(x-5)^n}{(n^4+3n^2+2)\cdot 5^n} \Rightarrow x_0 = 5$$

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{\frac{1}{[(n+1)^4+3\cdot(n+1)^2+2]\cdot 5^{n+1}}}{\frac{1}{(n^4+3n^2+2)\cdot 5^n}} = \frac{\cancel{5^n}\cdot(n^4+3n^2+2)}{5^{\cancel{n+1}}\cdot[(n+1)^4+\dots]}$$

$$\xrightarrow{n\to\infty} \lim_{n\to\infty} \frac{n^4}{5n^4} = \frac{1}{5} \Rightarrow \frac{1}{R} = \frac{1}{5} \Rightarrow R = 5$$

$\Rightarrow$ ABS. K. PRO $x \in (0,10)$

$$x=0 \Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n \cdot \cancel{5^n}}{(n^4+3n^2+2)\cdot \cancel{5^n}} \quad \ldots \text{ ALT. ŘADA}$$

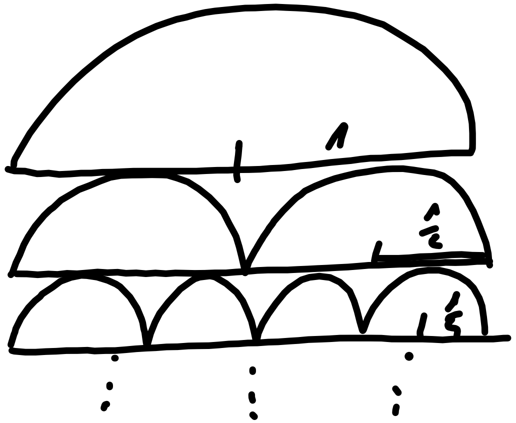
$$\sum (-1)^n \cdot a_n, \quad a_n = \frac{1}{n^4+3n^2+2} \searrow 0$$

LEIBN. KR. $\Rightarrow$ K

$$\underline{x=10} \quad \sum_{1}^{\infty} \frac{\cancel{5^n}}{(n^4+3n^2+2)\cancel{5^n}} \leq \sum_{1}^{\infty} \frac{1}{n^4} \quad K$$

K ABS.

ABS. K. $x \in [0, 10]$



S půlkruhu $= \frac{1}{2}\pi r^2$

ŘÁDEK	r	POČET	S
1	$1 = \frac{1}{2^0}$	$1 = 2^0$	$1 \cdot \frac{1}{2}\pi \cdot 1^2 = \frac{\pi}{2}$
2	$\frac{1}{2} = \frac{1}{2^1}$	$2 = 2^1$	$2 \cdot \frac{1}{2}\pi \cdot \left(\frac{1}{2}\right)^2 = \frac{\pi}{4}$
3	$\frac{1}{4} = \frac{1}{2^2}$	$4 = 2^2$	$4 \cdot \frac{1}{2}\pi \left(\frac{1}{4}\right)^2 = \frac{\pi}{8}$
$\vdots$	$\vdots$	$\vdots$	
n	$\frac{1}{2^{n-1}}$	2^{n-1}	$\cancel{2^{n-1}} \cdot \frac{1}{2} \cdot \pi \cdot \left(\frac{1}{2^{n-1}}\right)^{\cancel{2}} = \frac{\pi}{2^n}$

$$\sum_{n=1}^{\infty} \frac{\pi}{2^n} = \pi \cdot \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n = \left| \begin{array}{l} a_1 = \frac{1}{2} \\ q = \frac{1}{2} \end{array} \right| = \pi \cdot \frac{\frac{1}{2}}{1 - \frac{1}{2}} = \pi$$

$$\sum_{n=0}^{\infty} (-1)^n \cdot \frac{n^2+3n+2}{4^n} = \sum_{n=0}^{\infty} (n+2)\cdot(n+1)\cdot\left(-\frac{1}{4}\right)^n = \sum_{n=0}^{\infty} \underbrace{(n+2)\cdot(n+1)}_{a_n}\cdot \underbrace{x^n}_{(x-0)^n \Rightarrow \boxed{x_0 = 0}}$$

$$n^2+3n+3 = (n+2)\cdot(n+1)$$

$$\frac{a_{n+1}}{a_n} = \frac{(n+3)\cdot\cancel{(n+2)}}{\cancel{(n+2)}\cdot(n+1)} \xrightarrow{n\to\infty} 1 = \frac{1}{R}$$

$$R = 1$$

$\Rightarrow$ KONV. PRO $x \in (-1,1)$, $-\frac{1}{4} \in (-1,1) \Rightarrow$ <u>OK</u>

MÁM STEJN. KONV.

$$\sum_{n=0}^{\infty} x^n = \left| \begin{matrix} a_0 = 1 \\ q = x \end{matrix} \right| = \frac{1}{1-x} \Big/ \cdot x^2 \qquad \sum (n+2)(n+1) \cdot x^n$$

$$\sum_{n=0}^{\infty} x^{n+2} = \frac{x^2}{1-x} \Big/ \frac{d}{dx}$$

$$\sum_0^{\infty} (n+2) \cdot x^{n+1} = \frac{2x \cdot (1-x) - x^2 \cdot (-1)}{(1-x)^2} = \frac{-x^2+2x}{(1-x)^2} \Big/ \frac{d}{dx}$$

$$\sum_0^{\infty} (n+2) \cdot (n+1) \cdot x^n = \boxed{\frac{2}{(1-x)^3}}$$

ZAD. ŘADA $= \dfrac{2}{\left(1-\left(-\frac{1}{4}\right)\right)^3} = \dfrac{2}{\left(\frac{5}{4}\right)^3} = \dfrac{128}{125}$

f 2π PER., na $[-\pi, \pi)$ PO ČÁSTECH SPOJ.
A PO ČÁSTECH MONOT. PAK

(i) $\forall x_0 \in (-\pi, \pi)$ KDE JE f SPOJ. PLATÍ, ŽE
SOUČET F. Ř. $= f(x_0)$

(ii) $\forall x_0$ ... SKOK, SOUČET $= \frac{1}{2}\left(\lim_{x \to x_0^+} f(x) + \lim_{x \to x_0^-} f(x)\right)$

(iii) $x_0 \in \{-\pi, \pi\}$, SOUČET $= \frac{1}{2}\left(\lim_{x \to -\pi^+} f(x) + \lim_{x \to \pi^-} f(x)\right)$

A, B měř. $\|$ $m(A\cap B)=0 \Rightarrow m(A\cup B) = m(A) + m(A)$

(i) $A\cap B=\emptyset \Rightarrow m_*(A) + m_*(B) \leq m_*(A\cup B) \leq$

$\leq m^*(A\cup B) \leq m^*(A) + m^*(B)$

$m_*(A) = m^*(A)$ & $m_*(B) = m^*(B) \Rightarrow m_*(A\cup B) = m^*(A\cup B)$

$\Rightarrow m(A\cup B)$ JE MĚŘ. &

$m(A\cup B) = m(A) + m(B)$

(ii) POKUD $m(A\cap B)=0$ ... PŘEVEDEME NA (i)

$$A\cup B = A \cup (B\setminus A)$$

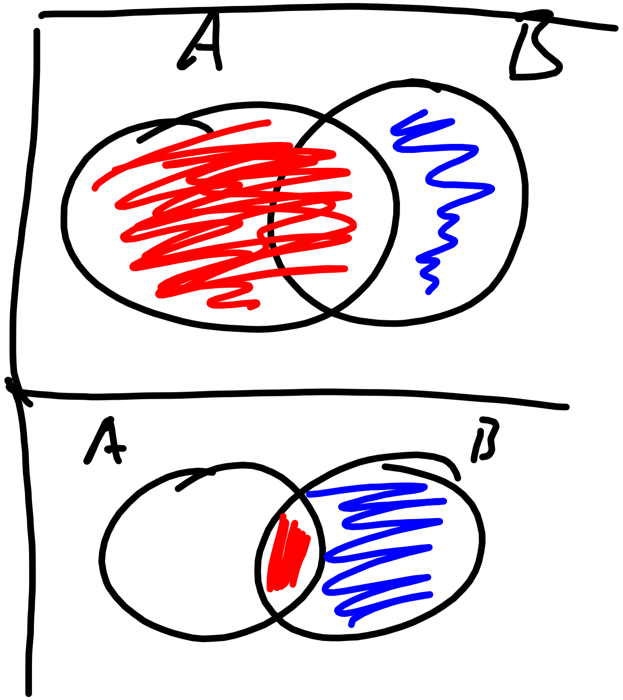
DISJ. ⇒ MÁM (i)

$$\Rightarrow m(A\cup B) = m(A) + m(B\setminus A)$$

$$B\setminus A = B\setminus (A\cap B)$$

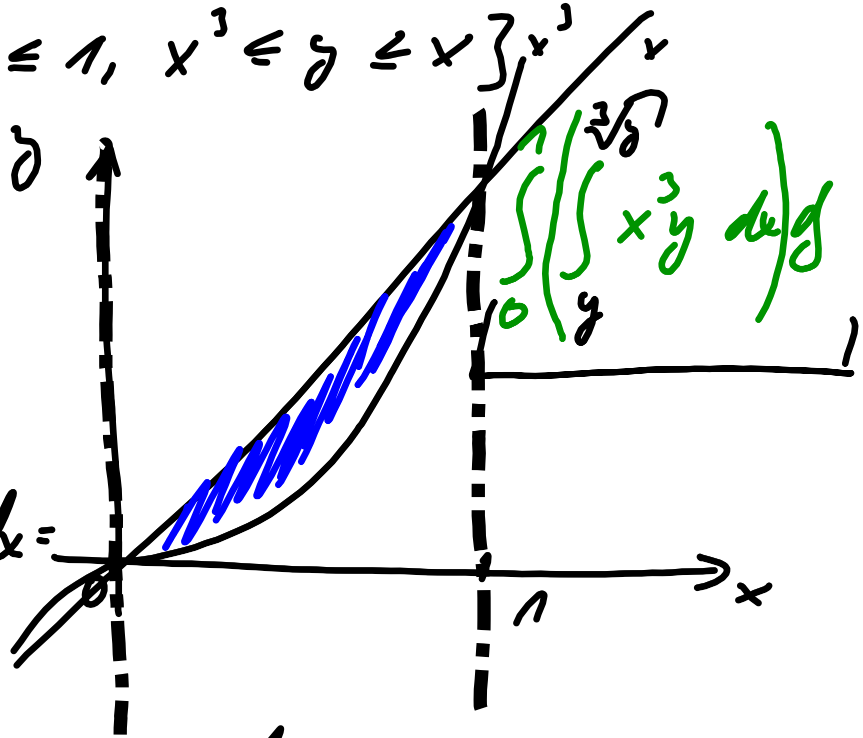
DISJ. ⇒ MÁM (i)

$$\Rightarrow m(B\setminus A) = m(B) - m(A\cap B)$$



$$\Rightarrow m(A\cup B) = m(A) + m(B) - \underbrace{\cancel{m(A\cap B)}}_{=0}$$

$$\iint_A x^3 y \, dx\, dy = I, \quad A=\{0\le x\le 1,\ x^3\le y\le x\}$$



$$\int_0^1\left(\int_y^{\sqrt[3]{y}} x^3 y \, dx\right)dy$$

$$I=\int_0^1\left(\int_{x^3}^{x} x^3 y \quad dy\right)dx =$$

$$=\int_0^1 x^3\cdot\left[\frac{y^2}{2}\right]_{x^3}^{x} dx = \frac{1}{2}\int_0^1 x^3\cdot(x^2-x^6)\,dx =$$

$$=\frac{1}{2}\cdot\int_0^1 x^5-x^9 \, dx = \frac{1}{2}\cdot\left[\frac{x^6}{6}-\frac{x^{10}}{10}\right]_0^1 = \frac{1}{2}\cdot\left(\frac{1}{6}-\frac{1}{10}\right)=$$

$$=\frac{1}{2}\,\frac{-3+5}{30} = +\frac{1}{30}$$

$x^2+y^2+z^2 \le 2z, \quad z \le 2-x^2-y^2$

$x^2+y^2+z^2-2z \le 0$

$x^2+y^2+(z-1)^2-1 \le 0$

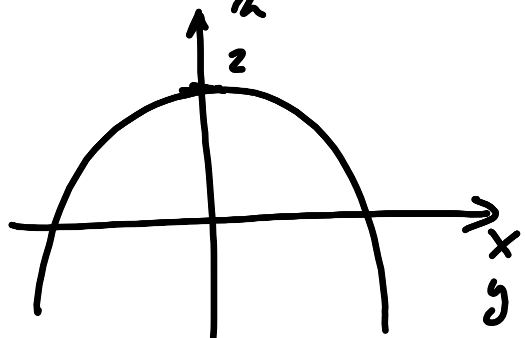
$x^2+y^2+(z-1)^2 \le 1$

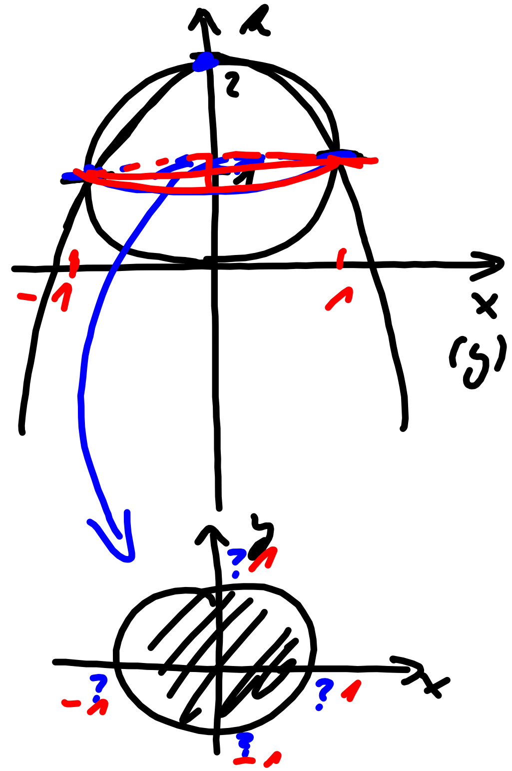
$S=[0,0,1], \ r=1$

REZ $xz \Rightarrow y=0$

$z=2-x^2$

$z-2=-x^2$





$x^2+y^2+z^2=2z, \quad z=2-x^2-y^2 \Rightarrow x^2+y^2=2-z$

$\Rightarrow 2-z+z^2=2z \Rightarrow z^2-3z+2=0$

$D=9-8=1$

$z_{1,2}=\frac{3\pm1}{2}=2, \ =1$

VÁLC. SOUŘ.

$x = \rho \cdot \cos\varphi$

$y = \rho \cdot \sin\varphi$

$z = z$

$|J| = \rho$

$\varphi \in [0, 2\pi]$ | $z \in [1-\sqrt{1-\rho^2}, 2-\rho^2]$

$\rho \in (0, 1]$

$z = 2 - x^2 - y^2 = 2 - \rho^2(\cos^2\varphi + \sin^2\varphi)$
$= 2 - \rho^2$

$x^2 + y^2 + (z-1)^2 = 1$

$(z-1)^2 = 1 - (x^2 + y^2) = 1 - (\rho^2\cos^2\varphi + \rho^2\sin^2\varphi) = 1 - \rho^2(\overbrace{\cos^2\varphi + \sin^2\varphi}^{=1})$
$= 1 - \rho^2$

$z - 1 = \pm\sqrt{1-\rho^2}$

$z = 1 - \sqrt{1-\rho^2}$

$$V = \int_0^{2\pi} \left(\int_0^1 \left(\int_{1-\sqrt{1-\rho^2}}^{2-\rho^2} 1 \cdot \rho \, dz \right) d\rho \right) d\varphi =$$

$$= 2\pi \cdot \int_0^1 \rho \cdot [z]_{1-\sqrt{1-\rho^2}}^{2-\rho^2} \, d\rho = 2\pi \cdot \int_0^1 \rho \cdot \Big(\underbrace{2-\rho^2-1}_{1-\rho^2} + \sqrt{1-\rho^2} \Big) d\rho$$

$$= \left| \begin{array}{l} t = 1-\rho^2 \\ dt = -2\rho \, d\rho \end{array} \right| \left| \begin{array}{l} \rho = 1 \Rightarrow t = 0 \\ \rho = 0 \Rightarrow t = 1 \end{array} \right| = -\pi \int_1^0 t + \sqrt{t} \, dt =$$

$$= \pi \cdot \left[\frac{t^2}{2} + \frac{t^{3/2}}{3/2} \right]_0^1 = \pi \cdot \left(\frac{1}{2} + \frac{2}{3} \right) = \pi \cdot \frac{3+4}{6} = \frac{7}{6}\pi$$

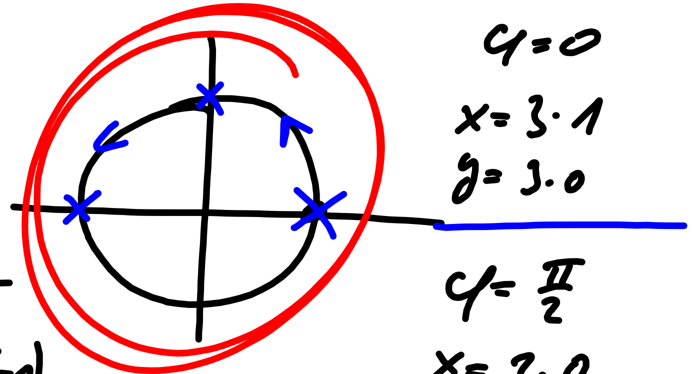
$$\int_C \frac{x+y}{x^2+y^2}\,dx - \frac{x-y}{x^2+y^2}\,dy = I$$

$C: x^2+y^2 = 9 = 3^2$

$$x = 3\cdot\cos\varphi$$
$$y = 3\cdot\sin\varphi \quad , \varphi \in [0, 2\pi)$$

$$dx = 3(-\sin\varphi)\,d\varphi$$

$$dy = 3\cdot\cos\varphi\,d\varphi$$



$\varphi = 0$

$x = 3\cdot 1$

$y = 3\cdot 0$

$\varphi = \frac{\pi}{2}$

$x = 3\cdot 0$

$y = 3\cdot 1$

$\varphi = \pi$

$x = 3\cdot(-1)$

$y = 3\cdot 0$

$$I = -\int_0^{2\pi} \frac{3\cdot\cos\varphi + 3\cdot\sin\varphi}{\underbrace{9\cdot\cos^2\varphi + 9\cdot\sin^2\varphi}_{=9\cdot 1}}\cdot\underline{(-3)\cdot\sin\varphi} - \frac{3\cdot\cos\varphi - 3\cdot\sin\varphi}{\underbrace{9\cdot\cos^2\varphi + 9\cdot\sin^2\varphi}_{=9\cdot 1}}\, 3\cdot\cos\varphi\, d\varphi$$

$$= -\int_0^{2\pi} \frac{1}{9}\cdot\left(-9\sin\varphi\cdot\cos\varphi - 9\sin^2\varphi\right) - \frac{1}{9}\cdot\left(9\cdot\cos^2\varphi - 9\cdot\sin\varphi\cdot\cos\varphi\right)d\varphi$$

$$= +\int_0^{2\pi} \cancel{-\sin\varphi\cos\varphi} + \sin^2\varphi + \cos^2\varphi + \cancel{\sin\varphi\cos\varphi}\, d\varphi$$

$$= \int_0^{2\pi} 1\, d\varphi = 2\pi$$