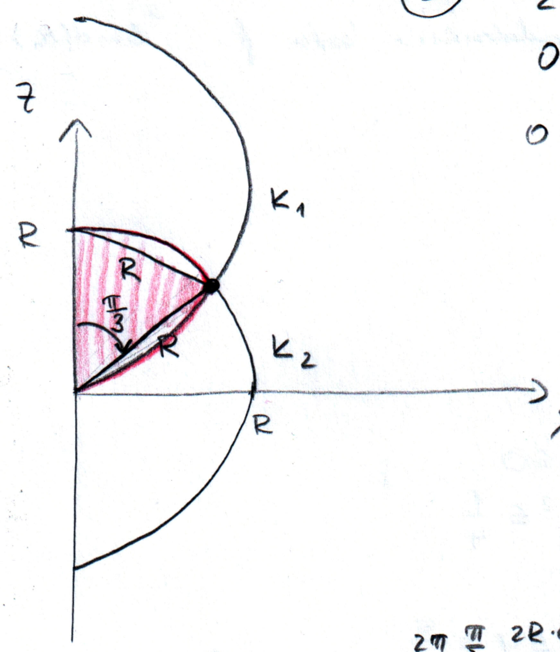
$k_2: \ x^2+y^2+z^2=R^2$

$0 \le \varphi \le 2\pi$

(I) $\frac{\pi}{2} \le \vartheta \le \frac{\pi}{3}$ (II) $\frac{\pi}{3} \le \vartheta \le 0$

$0 \le \rho \le k_1$ $0 \le \rho \le k_2$

$0 \le \rho \le 2R\cdot\cos\vartheta$ $0 \le \rho \le R$



k_2: $x^2+y^2+z^2=R^2$

$\rho^2\cos^2\varphi\cdot\sin^2\vartheta+\rho^2\sin^2\varphi\sin^2\vartheta+\rho^2\cos^2\vartheta=R^2$

$\rho^2\sin^2\vartheta+\rho^2\cos^2\vartheta=R^2$

$\rho^2=R^2$

$\rho=R$

$$\iiint_\Omega f(x,y,z)\,dx\,dy\,dz=\int_0^{2\pi}\int_{\frac{\pi}{2}}^{\frac{\pi}{3}}\int_0^{2R\cdot\cos\vartheta} f(\rho\cdot\cos\varphi\sin\vartheta,\ \rho\cdot\sin\varphi\sin\vartheta,\ \rho\cdot\cos\vartheta)(-\rho^2\sin\vartheta)\,d\rho\,d\vartheta\,d\varphi$$

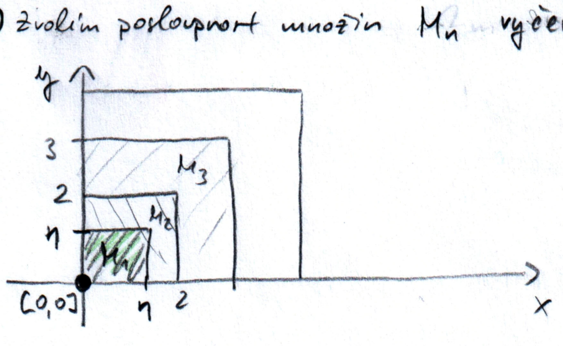
$$+\int_0^{2\pi}\int_{\frac{\pi}{3}}^{0}\int_0^{R} f(-\text{"}-)(-\rho^2\sin\vartheta)\,d\rho\,d\vartheta\,d\varphi.$$

NEVLASTNÍ INTEGRÁL ?

(A) Z OHRANIČENÉ FCE PŘES NEOHRANIČENOU MNOŽINU

(1) $\iint_\Omega xy\,e^{-x^2-y^2}\,dx\,dy$ $\Omega=[0,\infty)^2$

(1) zvolím posloupnost množin M_n vyčerpávajících množinu Ω (→ omezených) $\|\ M_n=[0,n]^2\ \|$. ($\to [\infty,\infty]$)



(2) Určím $\iint_{M_n} f(x,y)\,dx\,dy$

(3) $\iint_\Omega f(x,y)\,dx\,dy=\lim_{n\to\infty}\iint_{M_n} f(x,y)\,dx\,dy$

(2) $\iint_{M_n} xy\,e^{-x^2-y^2}dx\,dy=\int_0^n\int_0^n xy\,e^{-x^2}\cdot e^{-y^2}dx\,dy=\int_0^n x e^{-x^2}dx\cdot\int_0^n y\cdot e^{-y^2}dy=\left|\begin{matrix}\text{subst.}\\ x^2=t\\ 2x\,dx=dt\end{matrix}\right|=$

$=\left[\int_0^{n^2}\frac{1}{2}e^{-t}dt\right]^2=\left[\frac{1}{2}\left[-e^{-t}\right]_0^{n^2}\right]^2=\left[\frac{1}{2}\cdot(-e^{-n^2}+1)\right]^2=+\frac{1}{4}(e^{-n^2}+1)^2$

(3) $\lim_{n\to\infty}\iint_{M_n} 1=\frac{1}{4}(0+1)^2=\underline{\underline{\frac{1}{4}}}$

(B) Z NEOHRANIČENÉ FCE

(2) $\displaystyle\iint_\Omega \frac{dx\,dy}{\sqrt{x^2+y^2}}$ $\qquad \Omega : 0 < x^2+y^2 \le x$

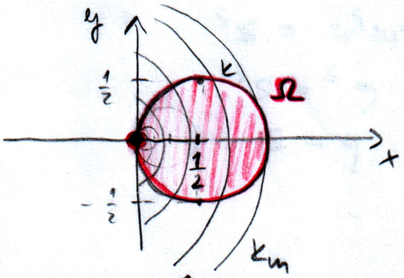
→ libovolná měřitelná MNOŽINA obsahující ve svém vnitřku A

(1) Zvolíme posloupnost M_n smršťujících se k singulárnímu bodu f a $\lim d(M_n)=0$

(2) Určíme $\displaystyle\iint_{\Omega\setminus M_n} f(x,y)\,dx\,dy$

(3) $\displaystyle\iint_\Omega f(x,y)\,dx\,dy = \lim_{n\to\infty} \iint_{\Omega\setminus M_n} f(x,y)\,dx\,dy$

(1)



$$0 < x^2+y^2 \le x$$
$$x^2 - x + y^2 \le 0$$
$$\left(x-\tfrac{1}{2}\right)^2 + y^2 \le \tfrac{1}{4}$$

$x^2+y^2 \le \frac{1}{m^2}$ $\qquad x = \rho\cdot\cos\varphi$ $\qquad -\frac{\pi}{2} \le \varphi \le \frac{\pi}{2}$ $\qquad J=\rho$

$y = \rho\cdot\sin\varphi$ $\qquad K_m \le \rho \le K$

$K_m : \rho^2 = \frac{1}{m^2} \Rightarrow \rho = \frac{1}{m}$ $\qquad \frac{1}{m} \le \rho \le \cos\varphi$

$K : \rho = \cos\varphi$

(2) $$\iint_{\Omega\setminus M_n} \frac{dx}{\sqrt{x^2+y^2}} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{\frac{1}{m}}^{\cos\varphi} \frac{\rho}{\sqrt{\rho^2}}\,d\rho\,d\varphi = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{\frac{1}{m}}^{\cos\varphi} d\rho\,d\varphi = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \Big[\rho\Big]_{\frac{1}{m}}^{\cos\varphi} d\varphi =$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\cos\varphi - \frac{1}{m}\right) d\varphi = \left[\sin\varphi - \frac{\varphi}{m}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 1 - \frac{\pi}{2m} - \left(-1 + \frac{\pi}{2m}\right) =$$

$$= 2 - \frac{\pi}{4m}$$

(3) $$\iint_\Omega f(x,y)\,dx\,dy = \lim_{n\to\infty}\left(2 - \frac{\pi}{4m}\right) = \underline{\underline{2}}$$