Definujme charakter $\omega : \mathbb{F}_q^\times \to \langle \zeta_{q-1} \rangle$ předpisem

$$\omega(\zeta_{q-1}^i + \tilde{\mathfrak{P}}) = \zeta_{q-1}^i,$$

tento charakter se nazývá Teichmüllerův. Dále označme $d = \frac{q-1}{m}$ a $\chi = \omega^{-d}$.

Definujme funkci $G : \mathbb{N}_0 \to \mathbb{N}_0$ takto: $G(a) = \nu_{\tilde{\mathfrak{P}}}(g(\omega^{-a}))$

<u>Lemma 10</u> Platí $G(1) = 1$, neboli $\nu_{\tilde{\mathfrak{P}}}(g(\omega^{-1})) = 1$.

<u>Důkaz</u>

$$g(\omega^{-1}) = -\sum_{t \in \mathbb{F}_q^\times} \omega(t)^{-1} \zeta_p^{Tr(t)} = -\sum_{i=0}^{q-2} \zeta_{q-1}^{-i} \zeta_p^{Tr(\zeta_{q-1}^i + \tilde{\mathfrak{P}})}$$

$$Tr(\zeta_{q-1}^i + \tilde{\mathfrak{P}}) = \sum_{j=0}^{f-1} \zeta_{q-1}^{ip^j} + \tilde{\mathfrak{P}}$$

Nechť m_i je přirozené číslo takové, že $m_i \in Tr(\zeta_{q-1}^i + \tilde{\mathfrak{P}})$.

$$g(\omega^{-1}) = -\sum_{i=0}^{q-2} \zeta_{q-1}^{-i} (1 - \lambda_p)^{m_i}$$

Z Binomické věty plyne

$$(1 - \lambda_p)^{m_i} \equiv 1 - m_i \lambda_p \pmod{\tilde{\mathfrak{P}}^2}$$

Tedy

$$g(\omega^{-1}) \equiv -\sum_{i=0}^{q-2} \zeta_{q-1}^{-i} (1 - m_i \lambda_p) = \lambda_p \sum_{i=0}^{q-2} m_i \zeta_{q-1}^{-i} \pmod{\tilde{\mathfrak{P}}^2}$$

Protože

$$m_i \equiv \sum_{j=0}^{f-1} \zeta_{q-1}^{ip^j} \pmod{\tilde{\mathfrak{P}}},$$

platí

$$\lambda_p m_i \equiv \lambda_p \sum_{j=0}^{f-1} \zeta_{q-1}^{ip^j} \pmod{\tilde{\mathfrak{P}}^2},$$

odkud

$$g(\omega^{-1}) \equiv \lambda_p \sum_{i=0}^{q-2} \sum_{j=0}^{f-1} \zeta_{q-1}^{ip^j - i} = \lambda_p \sum_{j=0}^{f-1} \sum_{i=0}^{q-2} (\zeta_{q-1}^{p^j - 1})^i \pmod{\tilde{\mathfrak{P}}^2}$$

Geometrická řada je pro $j \neq 0$ nulová, a proto

$$g(\omega^{-1}) \equiv \lambda_p (q-1) \equiv -\lambda_p \pmod{\tilde{\mathfrak{P}}^2}$$

neboť $q \in \tilde{\mathfrak{P}}$. Ovšem $\nu_{\tilde{\mathfrak{P}}}(\lambda_p) = 1$, neboť se $(1-\zeta_p)$ v $\mathbb{Q}(\zeta_{q-1}, \zeta_p)/\mathbb{Q}(\zeta_p)$ nerozkládá! ■