Population Growth

“Populační ekologie živočichů“

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Ecological Models

- aim: to simulate (predict) what can happen
- model is tested by comparison with observed data

- realistic models - complex (many parameters), realistic, used to simulate real situations
- strategic models - simple (few parameters), unrealistic, used for understanding the model behaviour

- a model should be:

1. a satisfactory description of diverse systems
2. an aid to enlighten aspects of population dynamics
3. a system that can be incorporated into more complex models

- deterministic models - everything is predictable
- stochastic models - including random events

- discrete models:

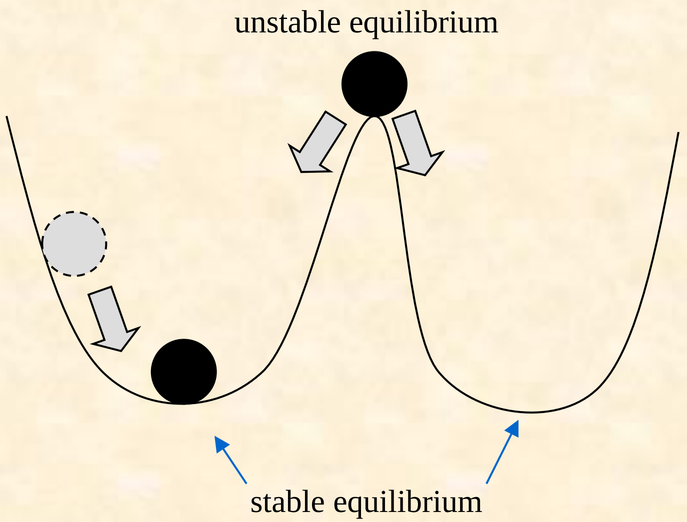
- time is composed of discrete intervals or measured in generations
- used for populations with synchronised reproduction (annual species)
- modelled by difference equations

- continuous models:

- time is continual (very short intervals) thus change is instantaneous
- used for populations with asynchronous and continuous overlapping reproduction
- modelled by differential equations

STABILITY

- stable equilibrium is a state (population density) to which a population will move after a perturbation



Population processes

- focus on rates of population processes

- number of cockroaches in a living room increases:

- influx of cockroaches from adjoining rooms → immigration [***I***]
- cockroaches were born → birth [***B***]

- number of cockroaches declines:

- dispersal of cockroaches → emigration [***E***]
- cockroaches died → death [***D***]

Blatta orientalis

$$N_{t+1} = N_t + I + B - D - E$$

- population increases if $I + B > E + D$

- rate of increase is a summary of all events ($I + B - E - D$)

- growth models are based on ***B*** and ***D***
- spatial models are based on ***I*** and ***E***

Density-independent population increase

Population processes are independent of its density

Assumptions:

- immigration and emigration are none or ignored
- all individuals are identical
- natality and mortality is constant
- all individuals are genetically similar
- reproduction is asexual
- population structure is ignored
- resources are infinite
- population change is instant, no lags

Used only for

- relative short time periods
- closed and homogeneous environments (experiemental chambers)

Discrete (difference) model

- for population with discrete generations (annual reproduction), no generation overlap
- time (t) is discrete, equivalent to generation
- exponential (geometric) growth
- Malthus (1834) realised that any species can potentially increase in numbers according to a geometric series

N_0 .. initial density

b .. birth rate (per capita),

$$b = \frac{B}{N}$$

d .. death rate (per capita)

$$d = \frac{D}{N}$$

$$\Delta N = bN_{t-1} - dN_{t-1}$$

$$N_t - N_{t-1} = (b-d)N_{t-1}$$

$$N_t = (1+b-d)N_{t-1}$$

where $1+b-d=\lambda$

$$N_t = N_{t-1}\lambda$$

$$b-d=R$$

$$\lambda = 1+R$$

- if λ is constant, population number in generations ***t*** is equal to

$$N_2 = N_1\lambda = N_0\lambda\lambda$$

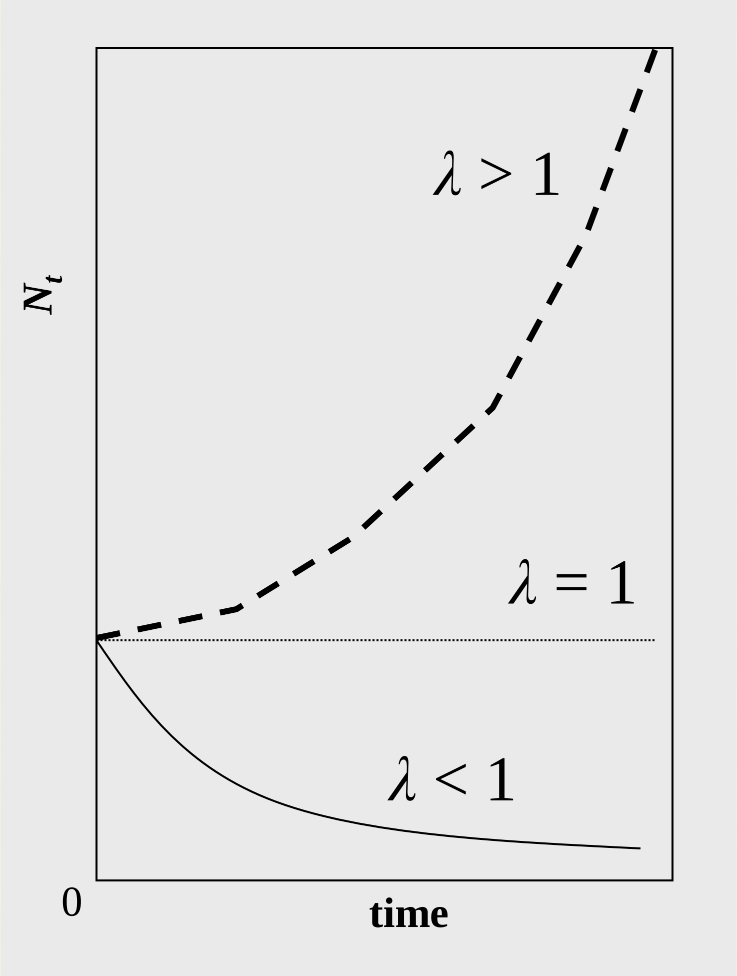
$$N_t = N_0\lambda^t$$

- number of individuals is multiplied each time - the larger the population the larger the increase

λ = finite growth-rate, per capita rate of growth
λ = 1.23 .. 23% increase

average of finite growth rates:

$$\bar{\lambda} = \left(\prod_{i=1}^{t}\lambda_i\right)^{\frac{1}{t}} = (\lambda_1\lambda_2\ldots\lambda_t)^{\frac{1}{t}}$$



Continuous (differential) model

- populations that are continuously reproducing, with overlapping generations
- when change in population number is permanent
- derived from the discrete model

$$N_t = N_0 \lambda^t$$

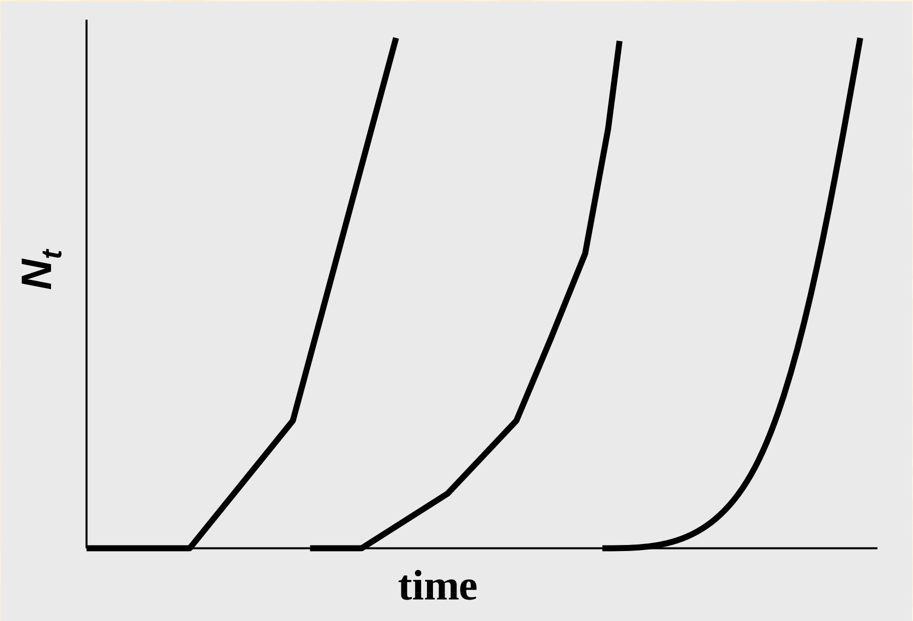
$$\ln(N_t) = \ln(N_0) + t\ln(\lambda)$$

$$\ln(N_t) - \ln(N_0) = t\ln(\lambda)$$

$$\frac{\mathrm{d}N}{\mathrm{d}t}\frac{1}{N} = \ln(\lambda)$$

$$\frac{\mathrm{d}N}{\mathrm{d}t} = N\ln(\lambda)$$

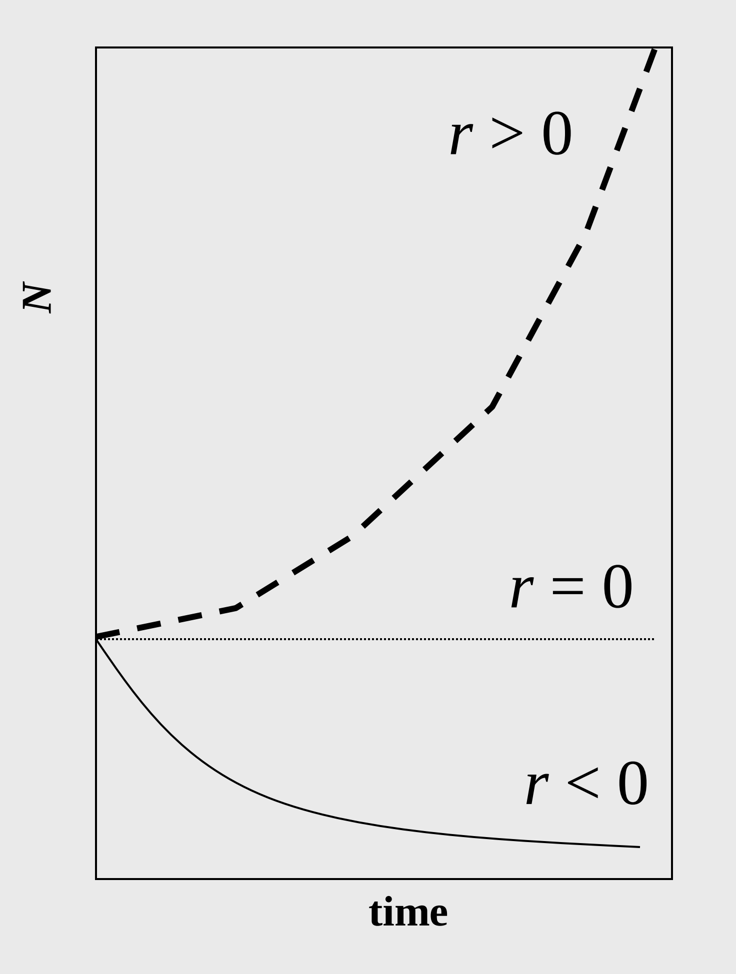
Comparison of discrete and continuous generations



if $r = \ln(\lambda)$

r - intrinsic rate of natural increase, instantaneous per capita growth rate

$$\frac{dN}{dt} = Nr$$



<u>Solution of the differential equation:</u>

- analytical or numerical

- at each point it is possible to determine the rate of change by differentiation (slope of the tangent)
- when *t* is large it is approximated by the exponential function

$$\frac{dN}{dt} = Nr$$

$$\frac{dN}{dt}\frac{1}{N} = r$$

$$\int_0^T \frac{1}{N} dN = \int_0^T r dt$$

$$\ln(N_T) - \ln(N_0) = r(T - 0)$$

$$\ln\left(\frac{N_T}{N_0}\right) = rT$$

$$\frac{N_T}{N_0} = e^{rT}$$

$$N_t = N_0 e^{rt}$$

- doubling time: time required for a population to double

$$t = \frac{\ln(2)}{r}$$

r versus λ

$$N_t = N_0 \lambda^t \qquad N_t = N_0 e^{rt}$$

$$\lambda^t = e^{rt}$$

$$r = \ln(\lambda)$$

- r is symmetric around 0, λ is not

  r = 0.5 ... λ = 1.65

  r = -0.5 ... λ = 0.61