

LECTURE 4

Pulse: frequency

B_0 : 9 T–27 T

^1H : 400 MHz–1200 MHz

^{13}C : 100 MHz–300 MHz

^{15}N : 40 MHz–120 MHz

Pulse: phase

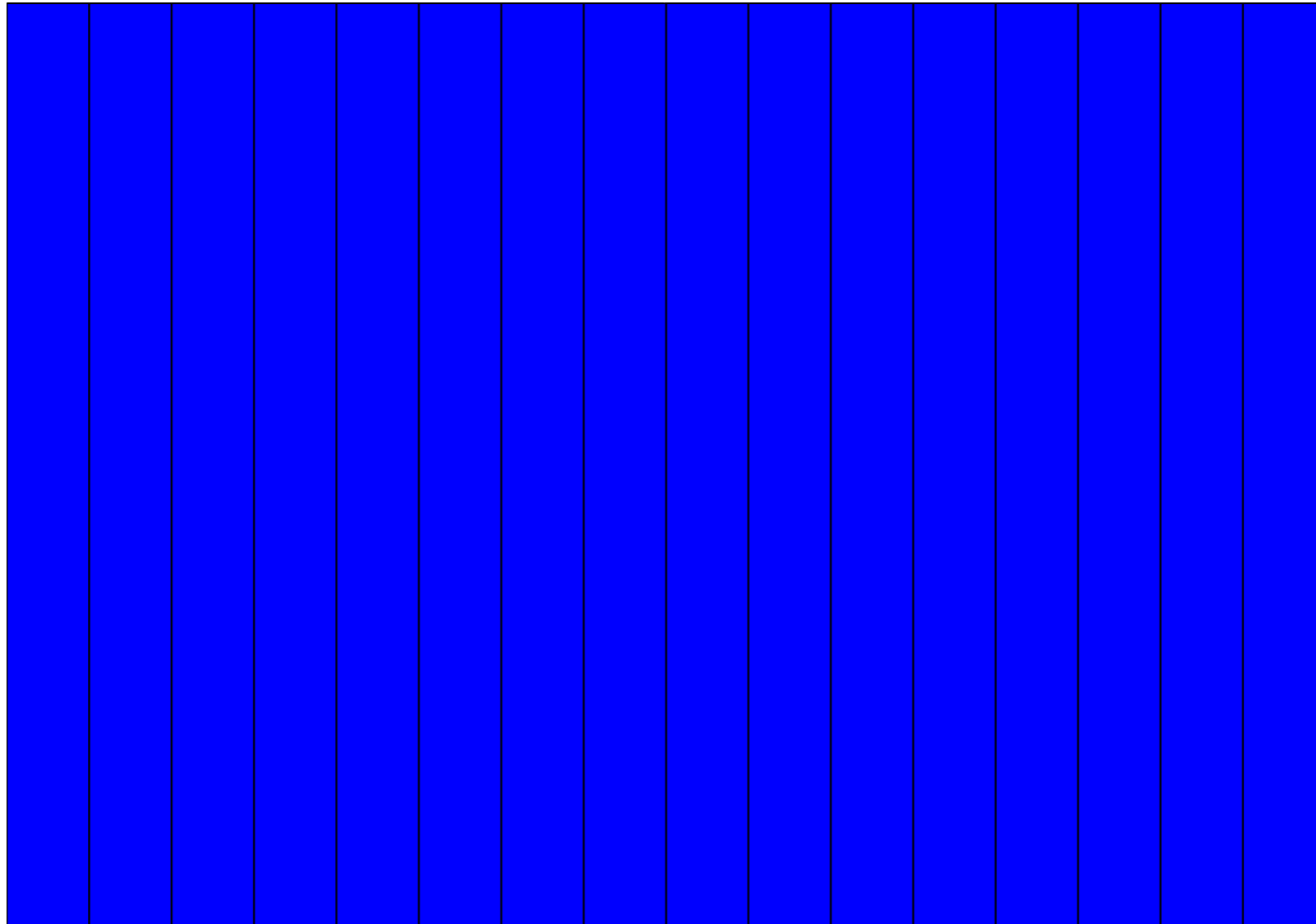
first pulse:

convention: 180° for $\gamma > 0$
convention: 0° for $\gamma < 0$

other pulses:

0°	x
90°	y
180°	$-x$
270°	$-y$

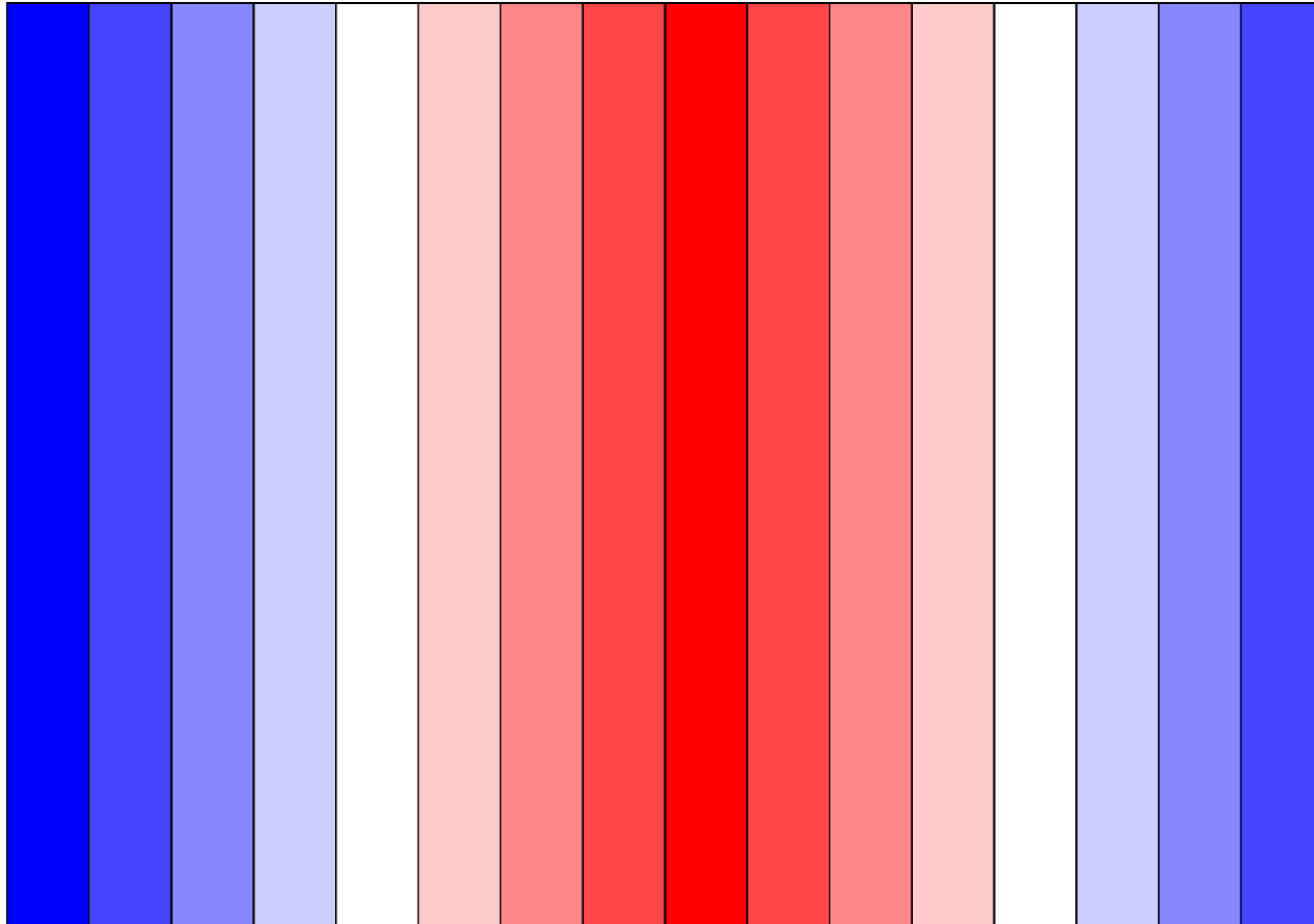
Pulse: phase modulation



0 cycle per pulse

100 μs pulse: +0 kHz

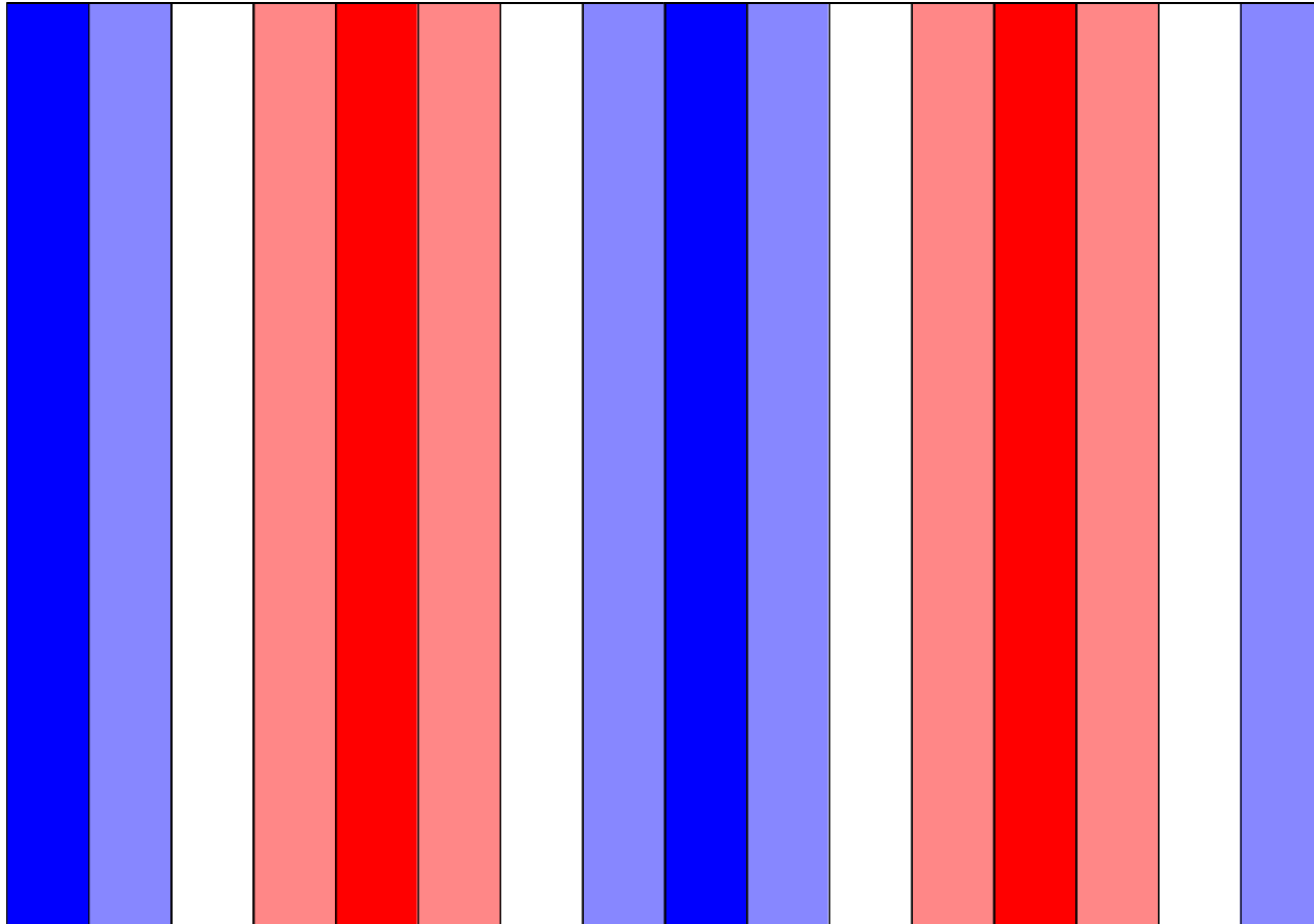
Pulse: phase modulation



1 cycle per pulse

100 μs pulse: +10 kHz

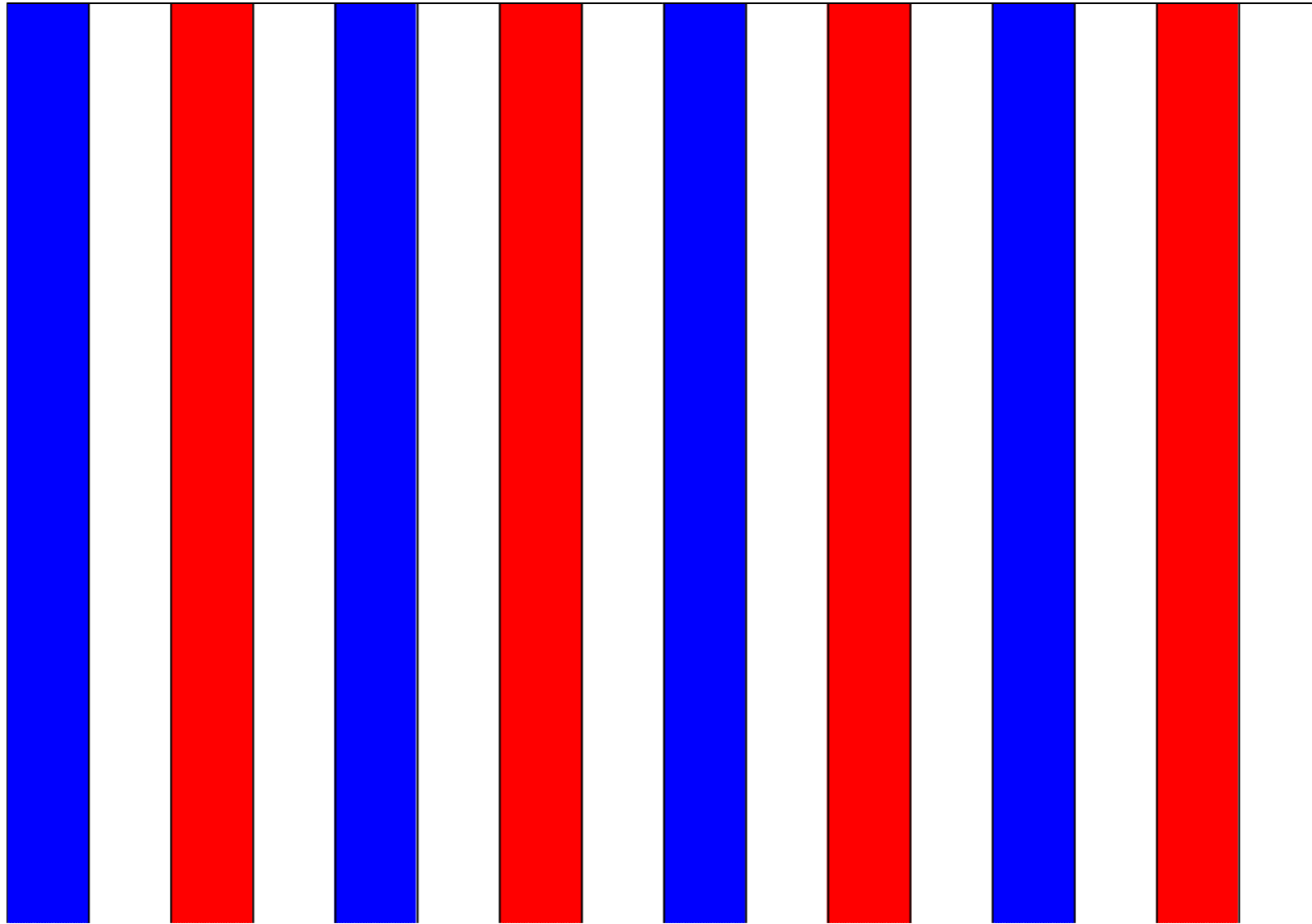
Pulse: phase modulation



2 cycles per pulse

100 μs pulse: +20 kHz

Pulse: phase modulation



4 cycles per pulse

100 μs pulse: +40 kHz

Pulse: power / amplitude

field strength	$ \omega_1 = \gamma B_1 $	kHz
power	$P \propto B_1^2$	W
attenuation	$10 \log_{10} \frac{P_2}{P_1}$	dB

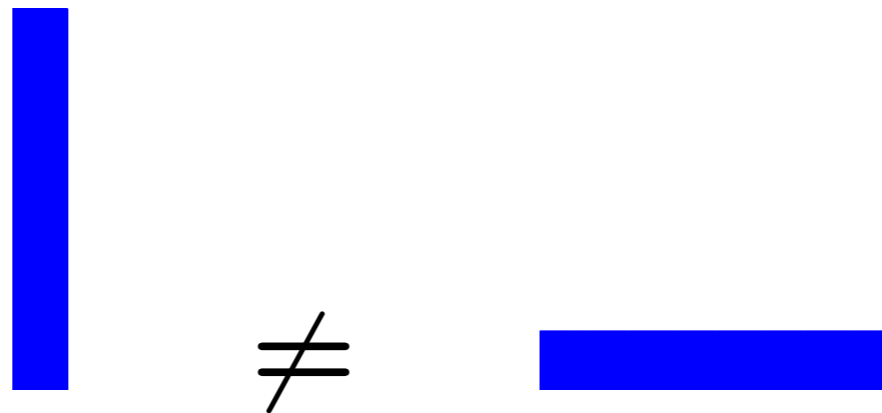
$$20 \log_{10} \frac{|\vec{B}_1|_2}{|\vec{B}_1|_1} = 10 \log_{10} \frac{|\vec{B}_1|_2^2}{|\vec{B}_1|_1^2} = 10 \log_{10} \frac{P_2}{P_1}$$

typical: $P < 1000 \text{ W}$, $|\omega_1| < 50 \text{ kHz}$

limit: sample/coil heating

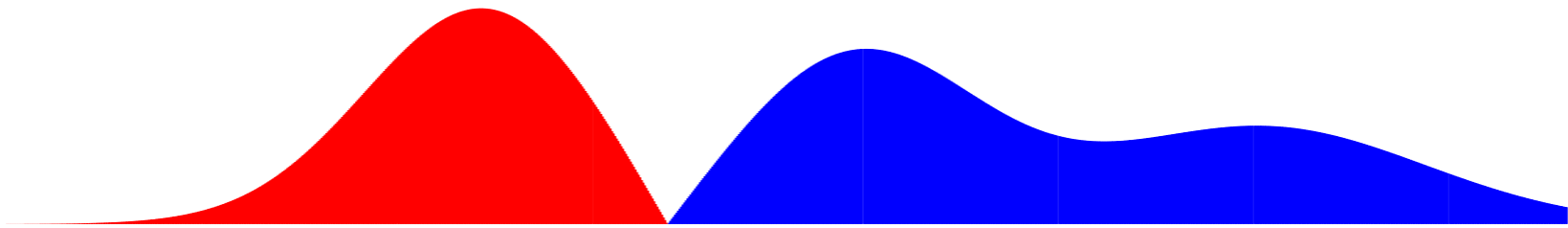
Pulse: length

calibrate so that $\alpha = |\omega_1|t_p$
where $\alpha = 90^\circ, 180^\circ, \dots$



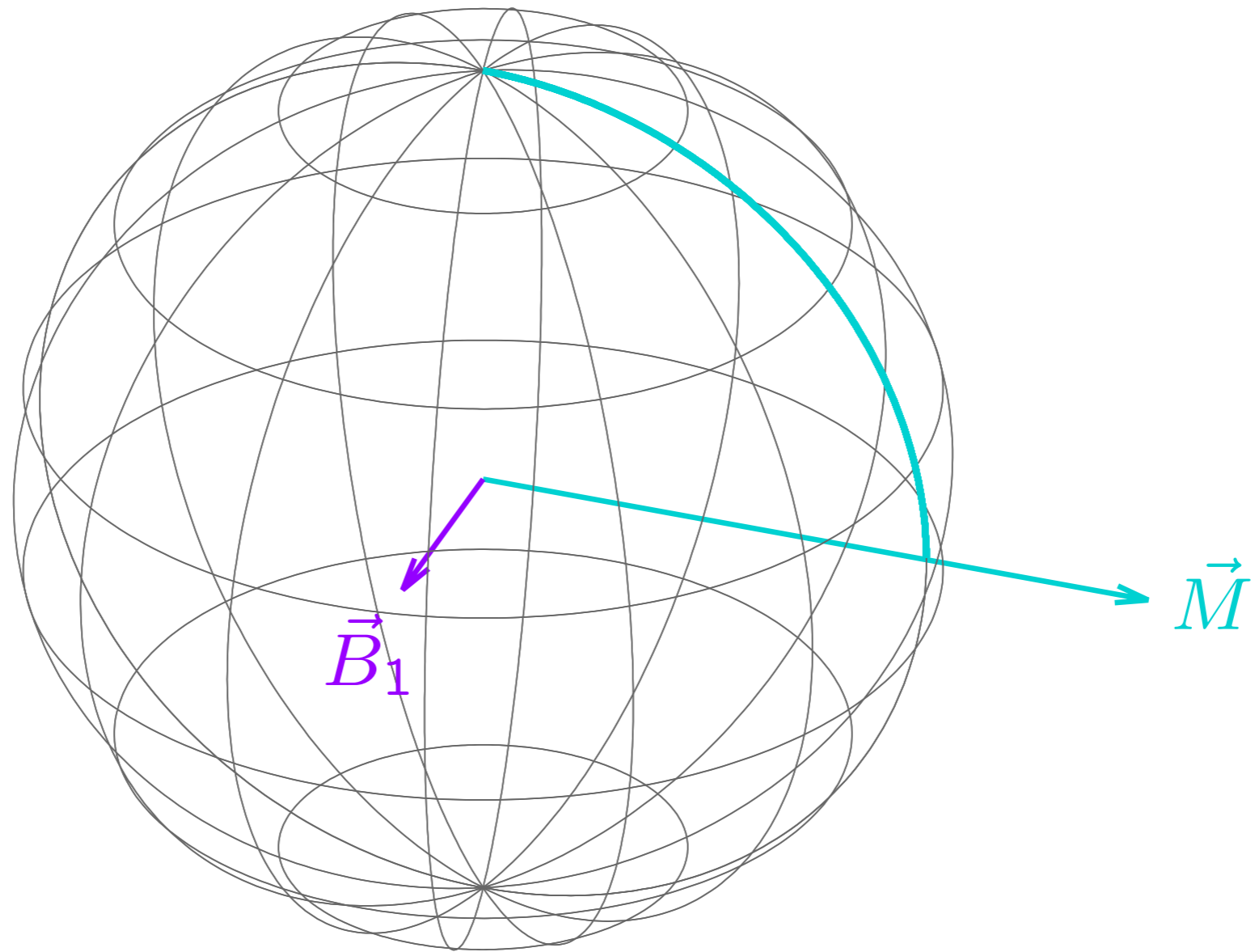
OFFSET EFFECTS!

Pulse: amplitude modulation



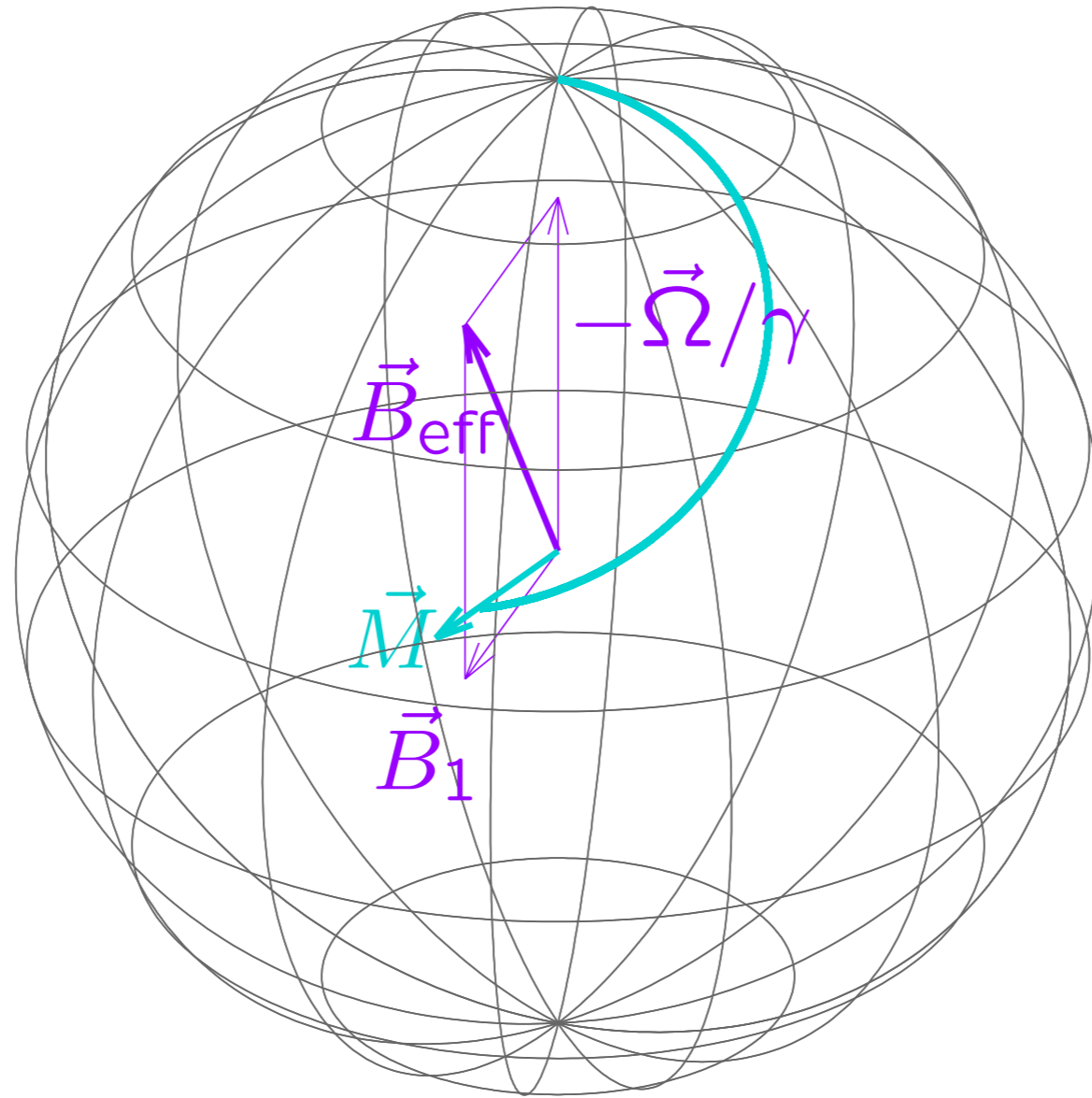
calibrate/calculate power so that $\alpha = |\omega_1|t_p$
where $\alpha = 90^\circ, 180^\circ, \dots$

Offset effects / selectivity



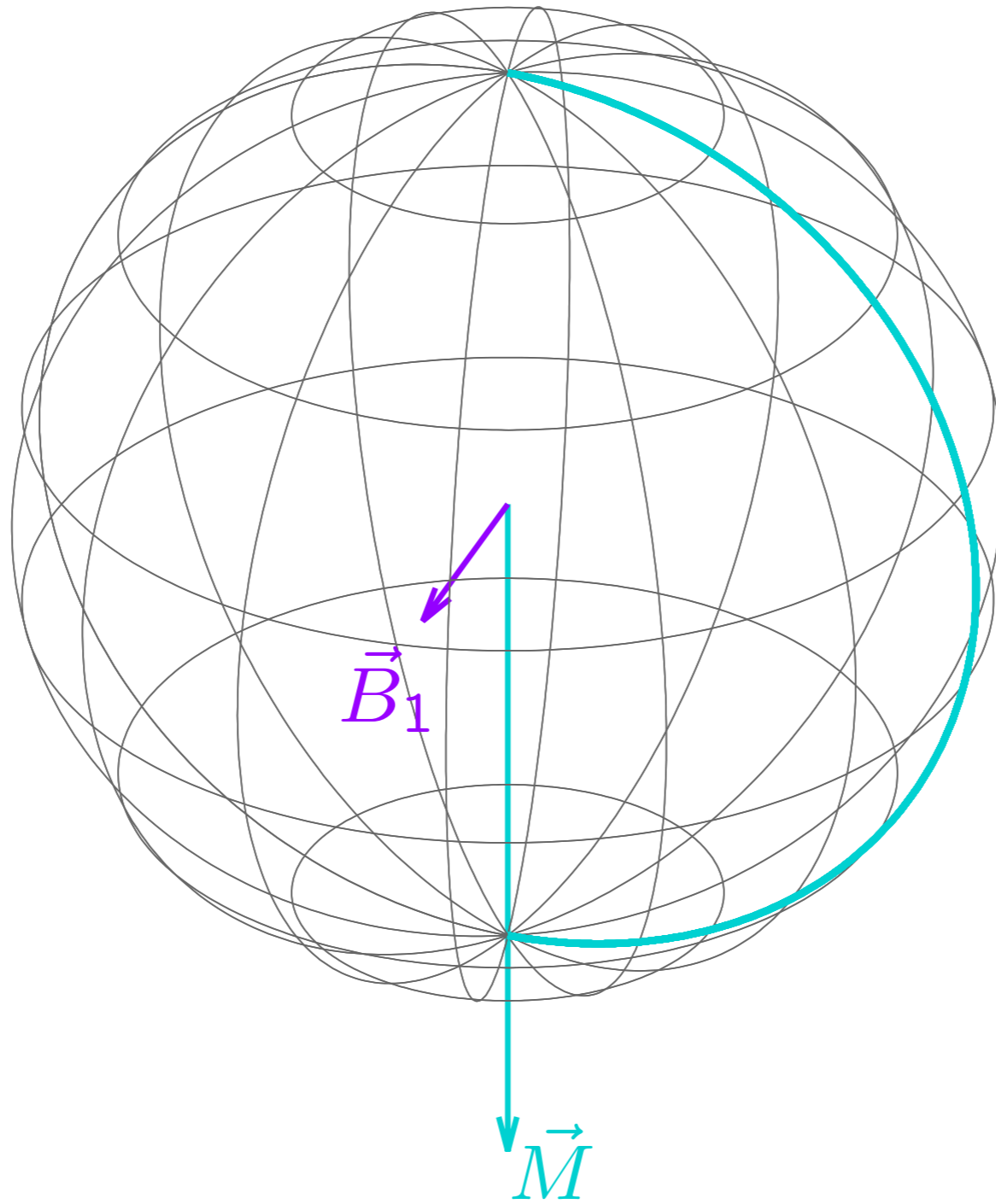
$\Omega = 0$ on resonance

Offset effects / selectivity



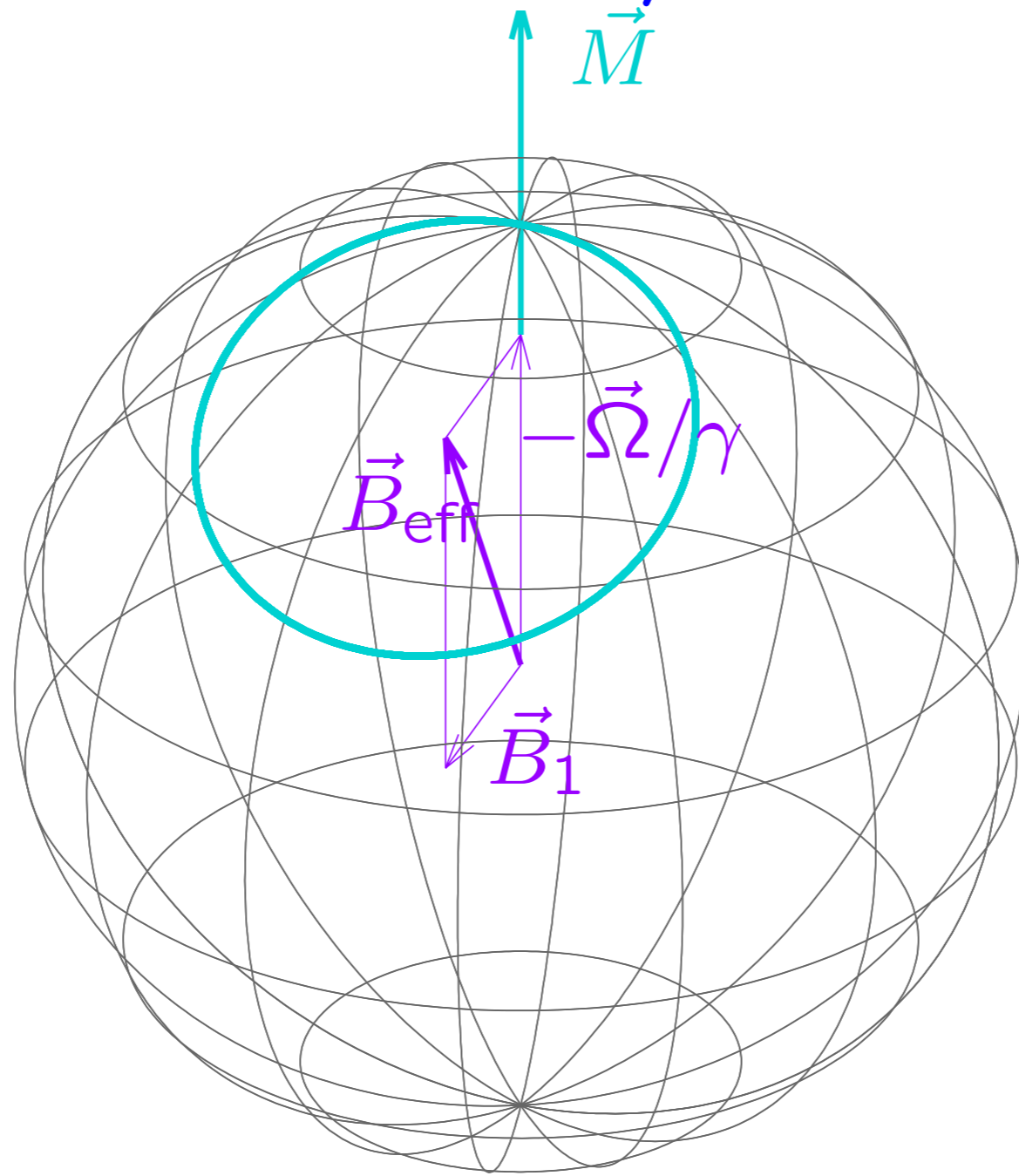
$\Omega \neq 0$ off resonance

Offset effects / selectivity



$\Omega = 0$ on resonance

Offset effects / selectivity



$$\omega_1 = \Omega/\sqrt{3}; \quad \omega_{\text{eff}} = \sqrt{1 + 3}\omega_1 = 2\omega_1$$

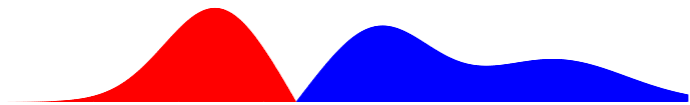
hard



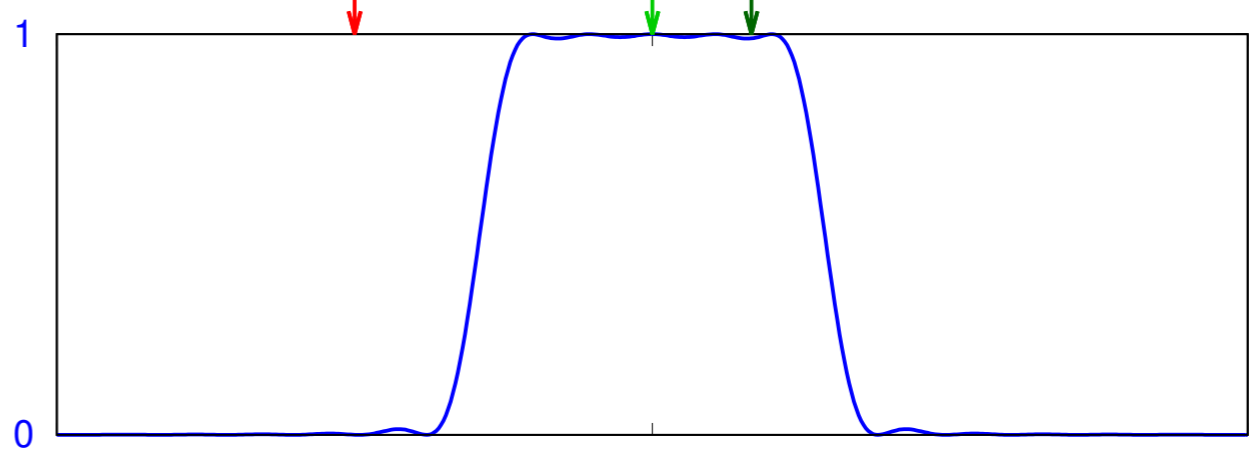
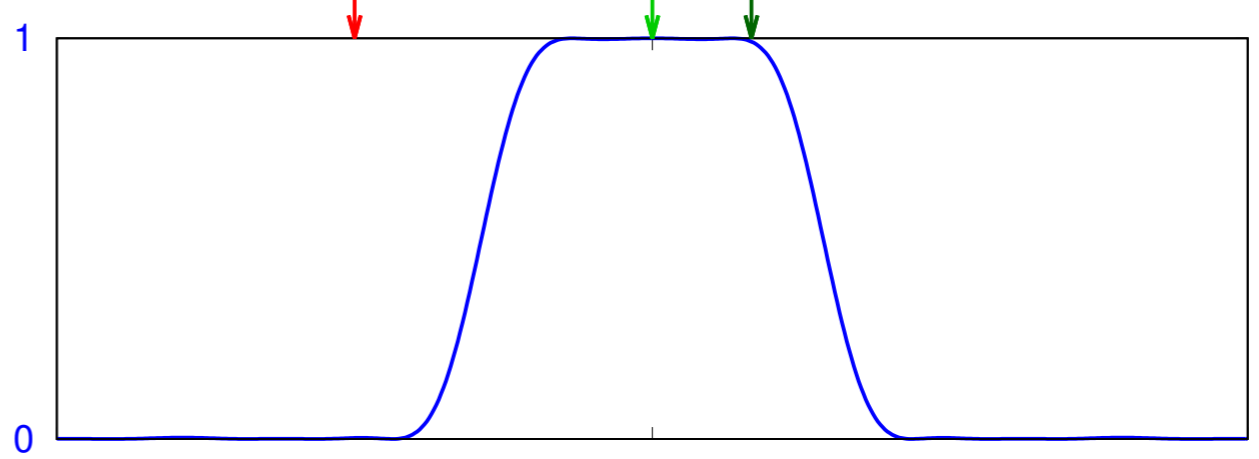
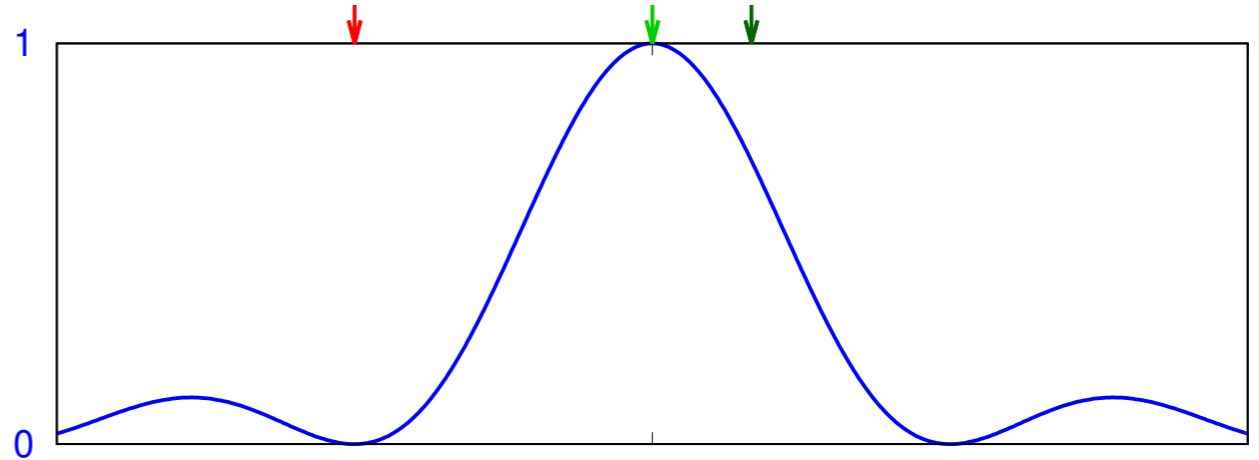
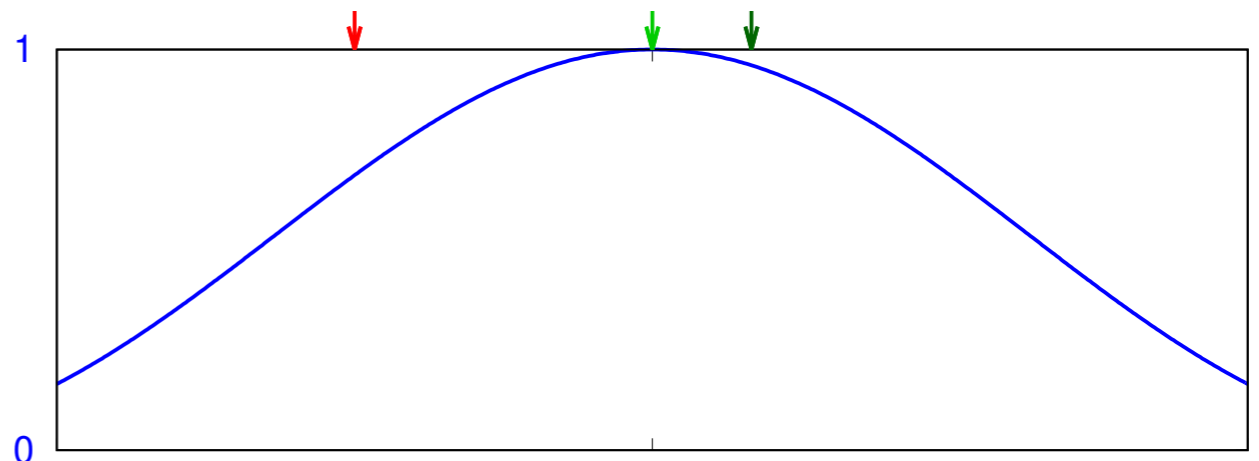
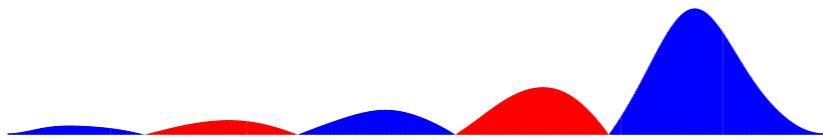
selective



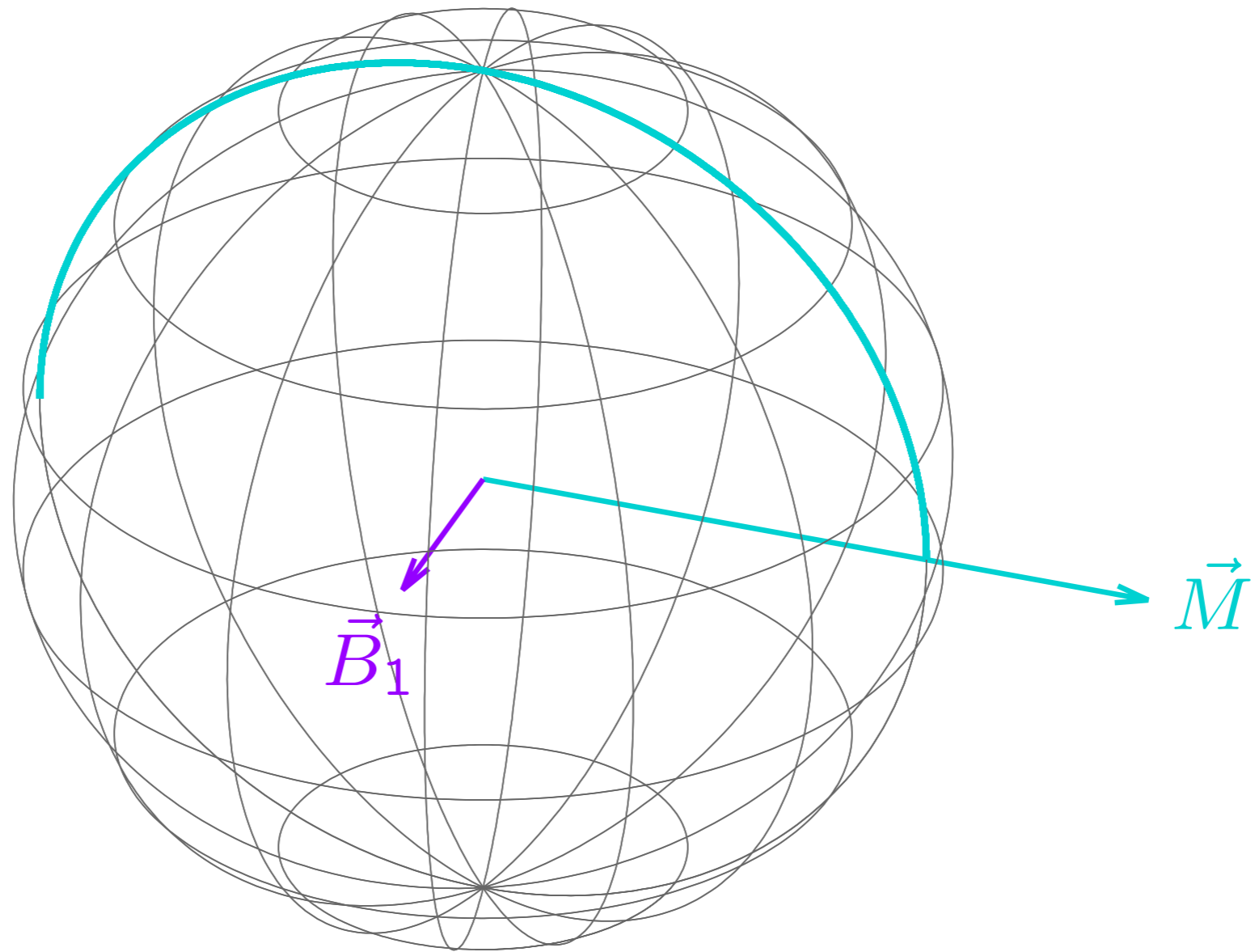
Q3



IBURP2

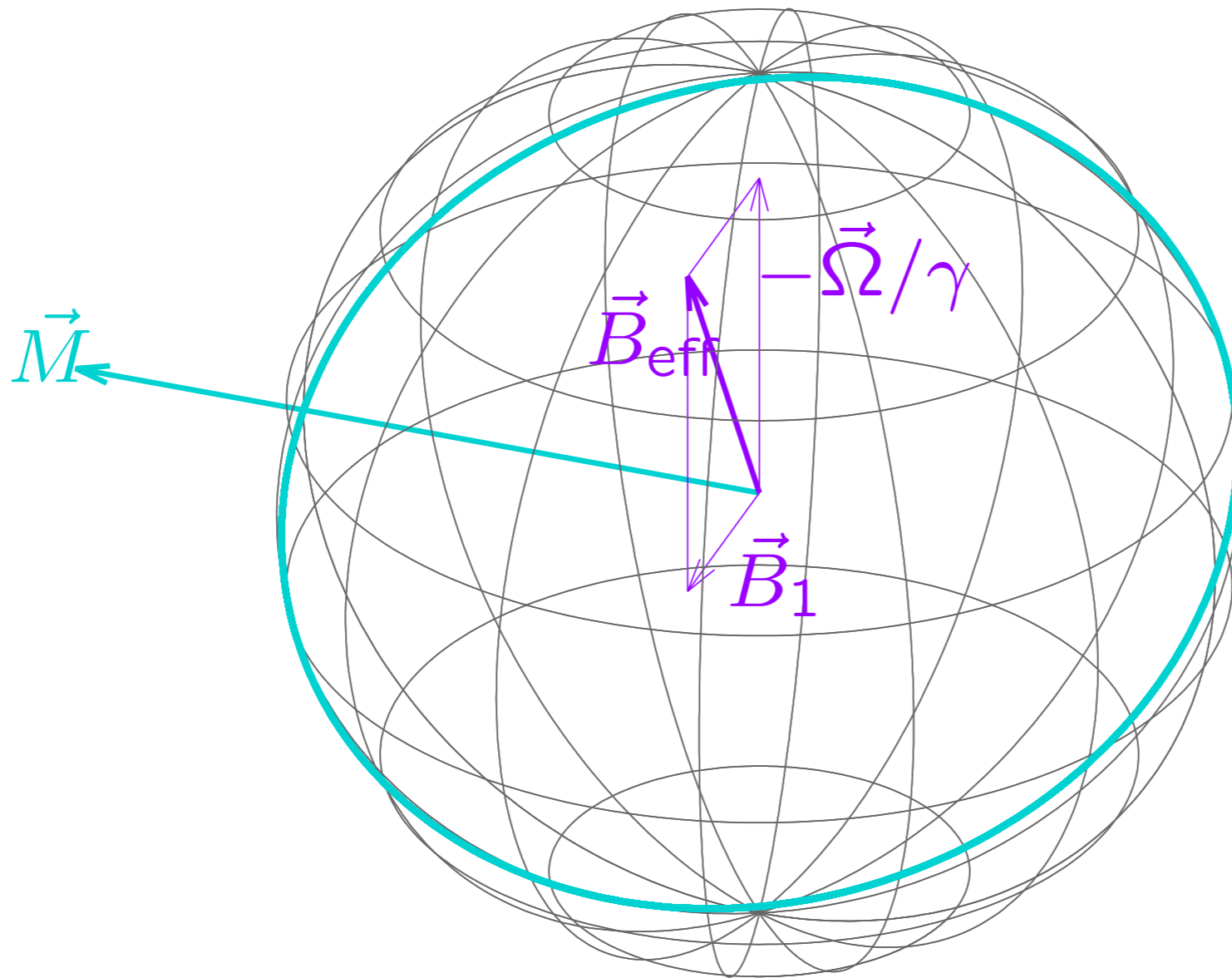


Offset effects / selectivity



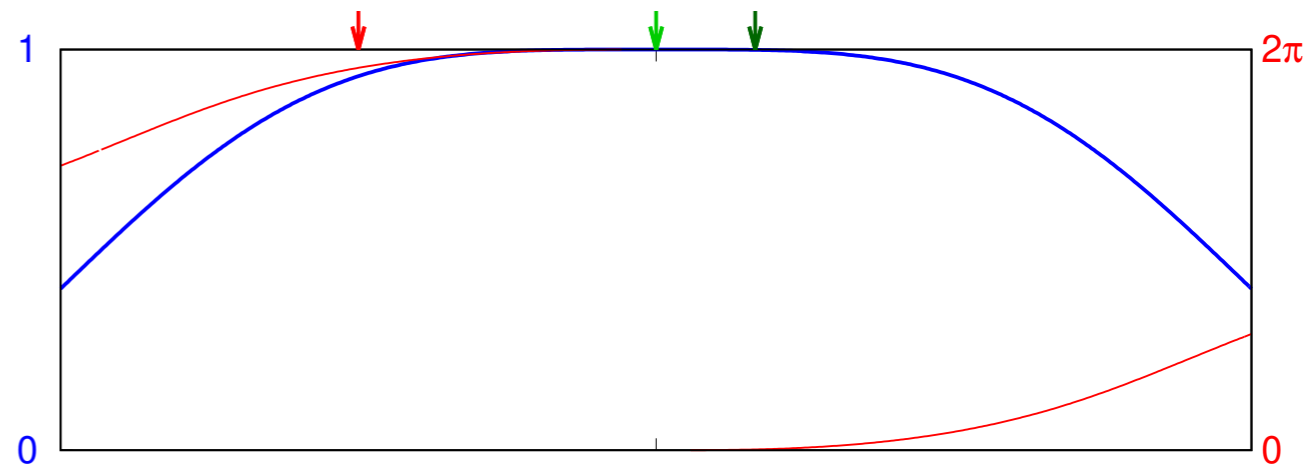
$\Omega = 0$ on resonance

Offset effects / selectivity

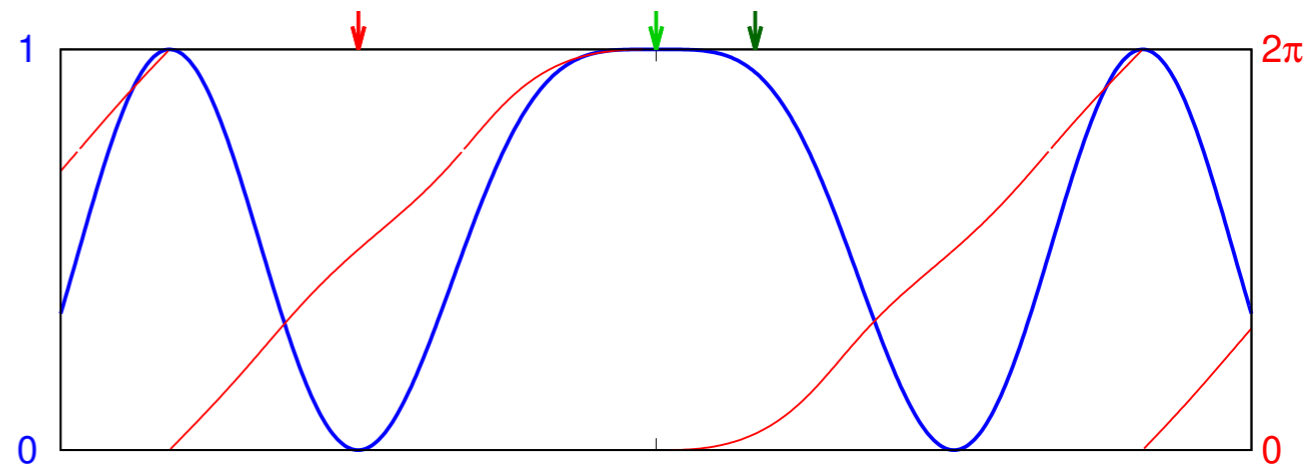


$$\omega_1 = \Omega/\sqrt{3}; \quad \omega_{\text{eff}} = \sqrt{1 + 3\omega_1} = 2\omega_1$$

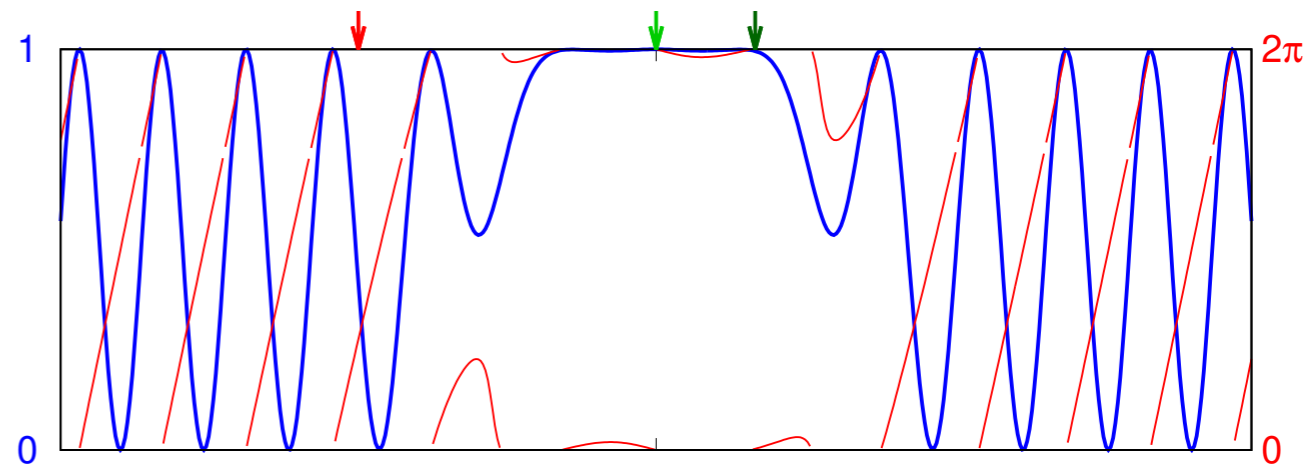
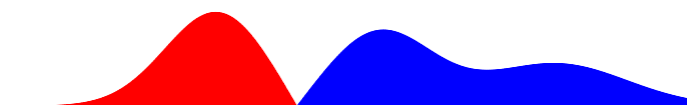
hard



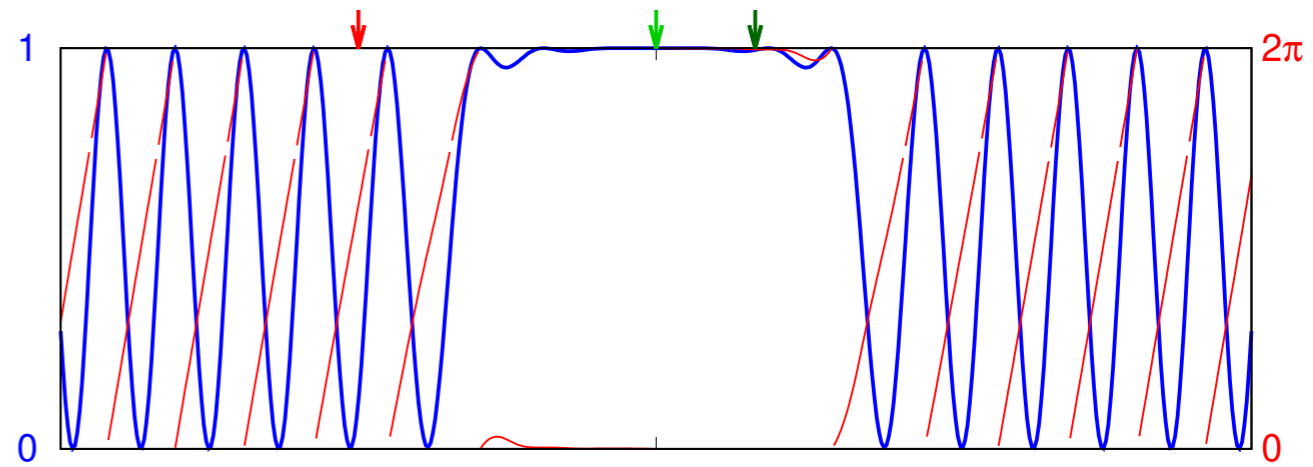
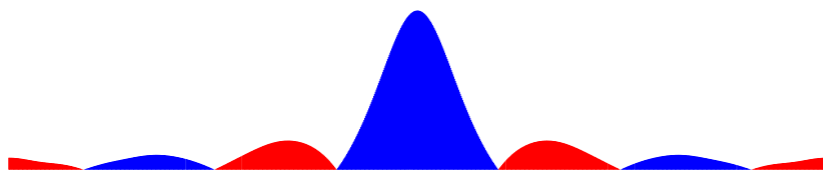
selective



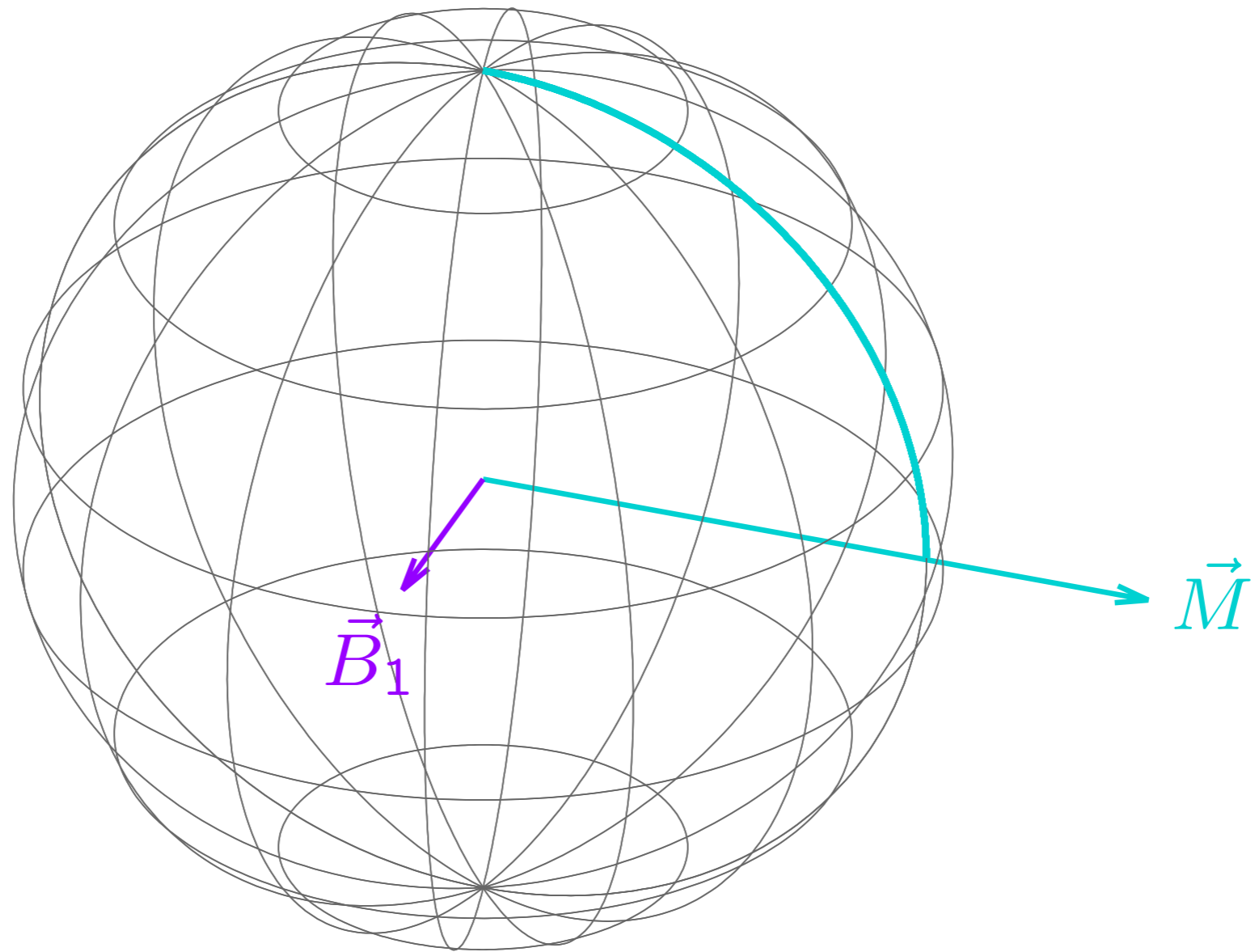
Q3



REBURP

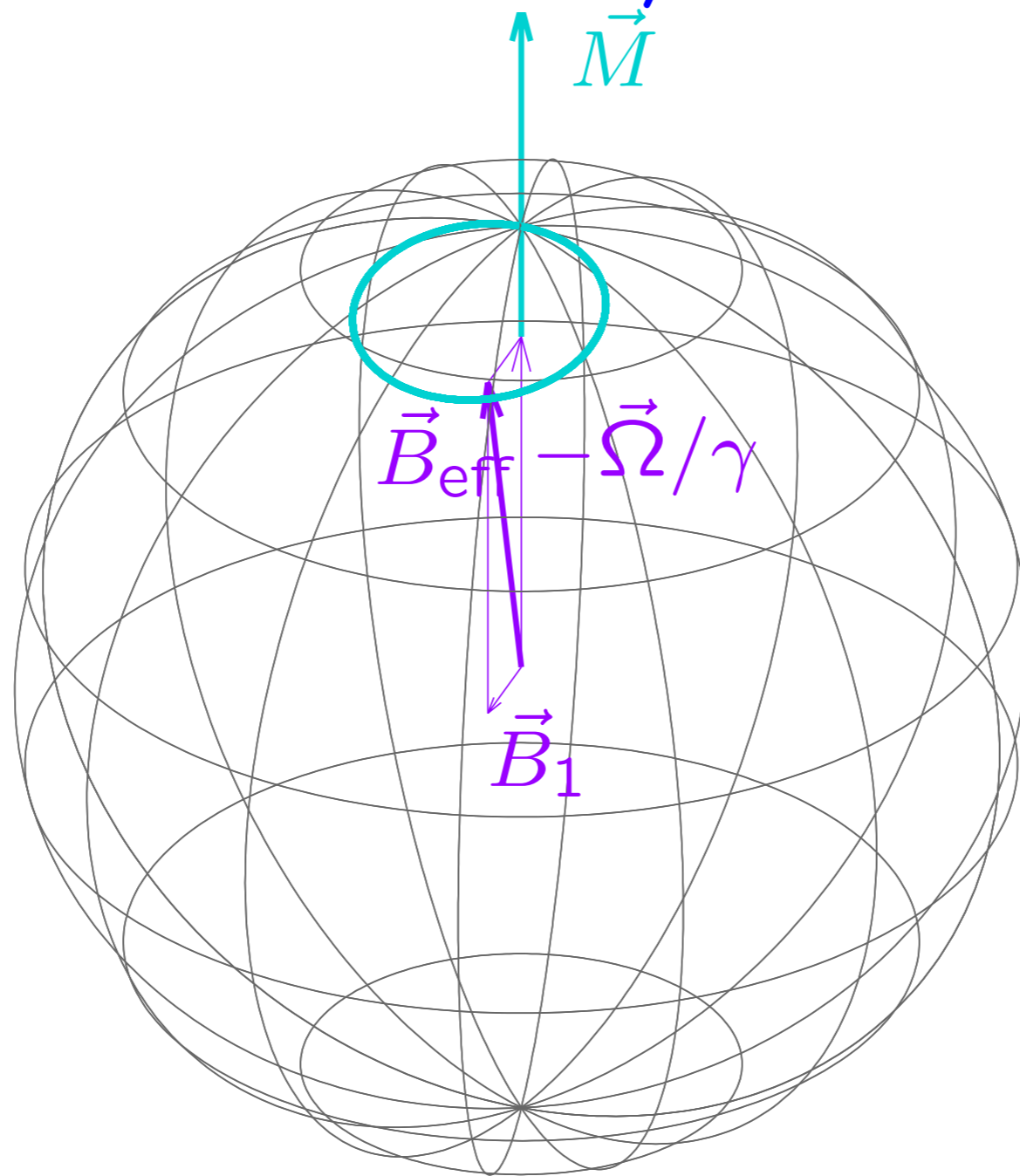


Offset effects / selectivity



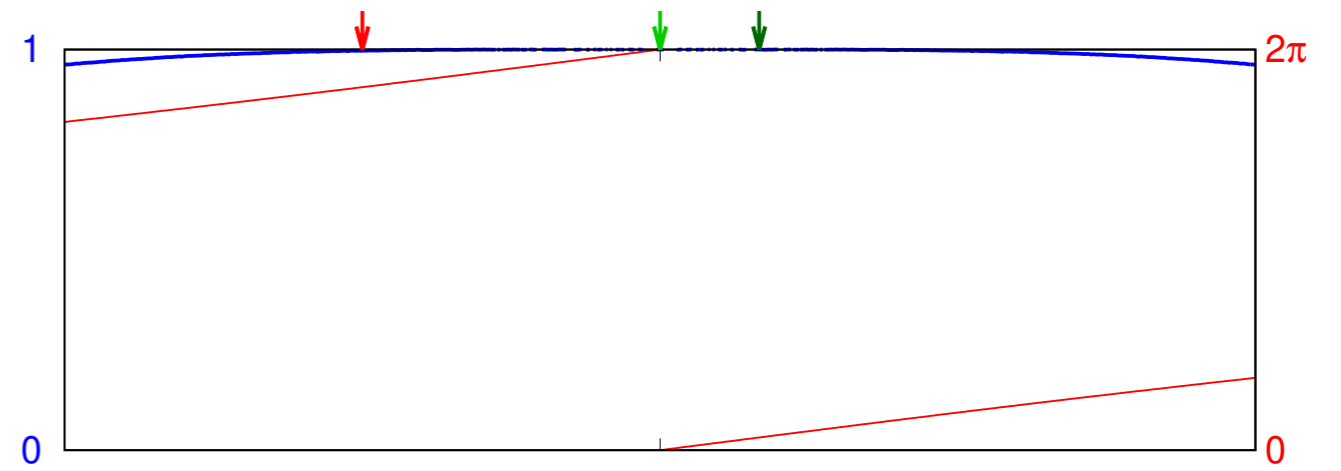
$\Omega = 0$ on resonance

Offset effects / selectivity

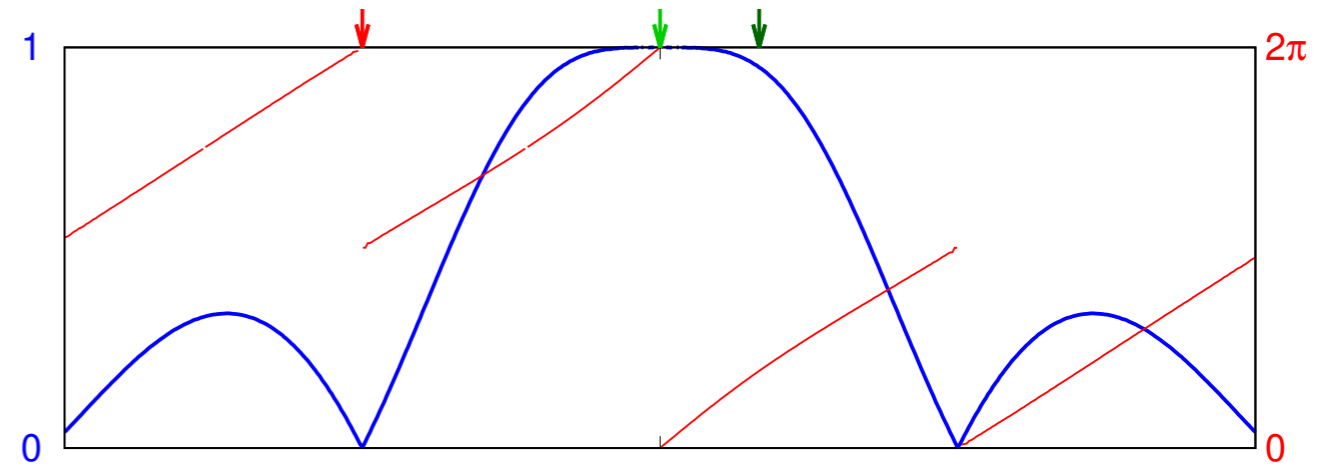


$$\omega_1 = \Omega / \sqrt{15}; \quad \omega_{\text{eff}} = \sqrt{1 + 15\omega_1} = 4\omega_1$$

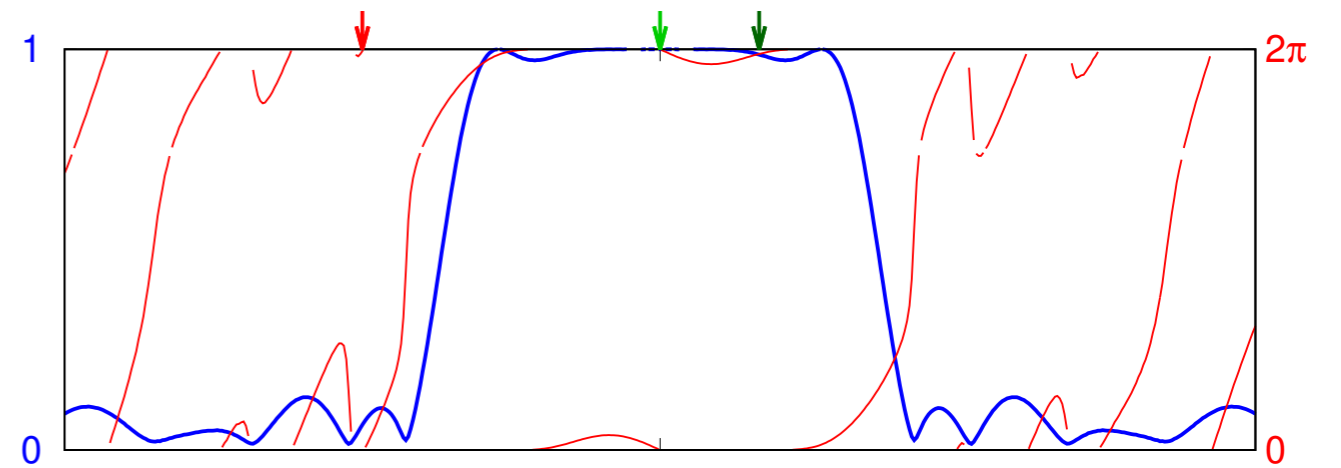
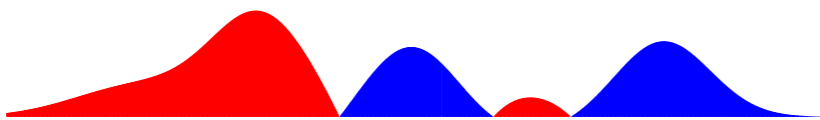
hard



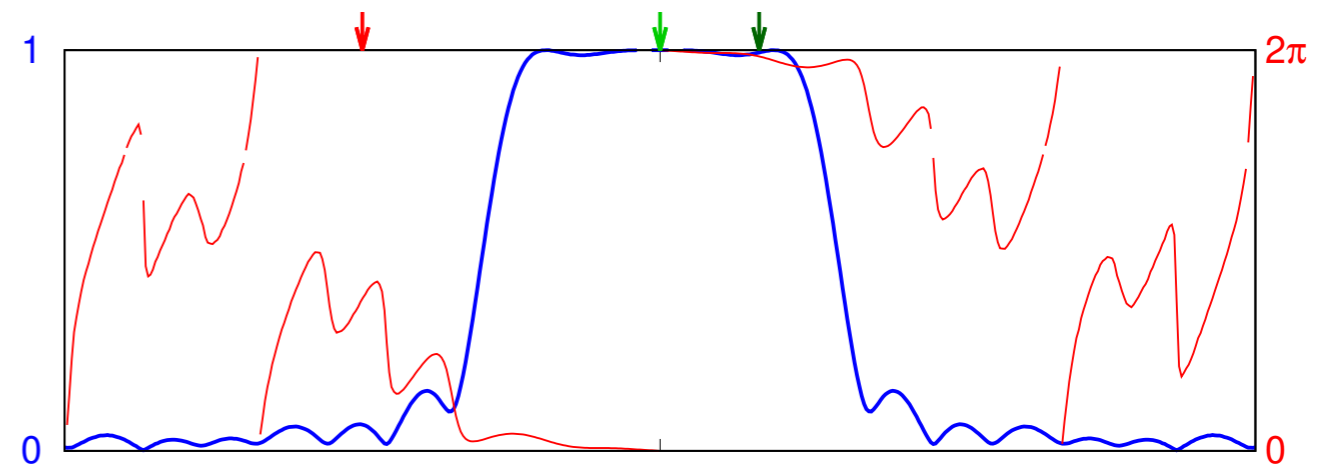
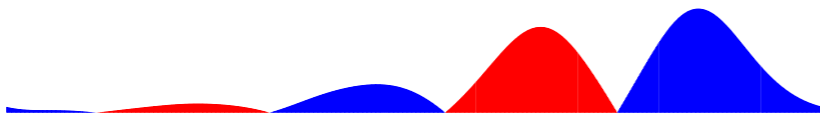
selective



Q5



EBURP2



Quadrature detection / frequency discrimination

$$\cos(\omega_0 t) \rightarrow \begin{cases} \frac{1}{2} \cos(\omega_0 t) \rightarrow \frac{1}{2} \cos(\omega_0 t) \cos(-\omega_{\text{radio}} t) & \text{channel } a \\ \frac{1}{2} \cos(\omega_0 t) \rightarrow \frac{1}{2} \cos(\omega_0 t) \sin(-\omega_{\text{radio}} t) & \text{channel } b \end{cases}$$

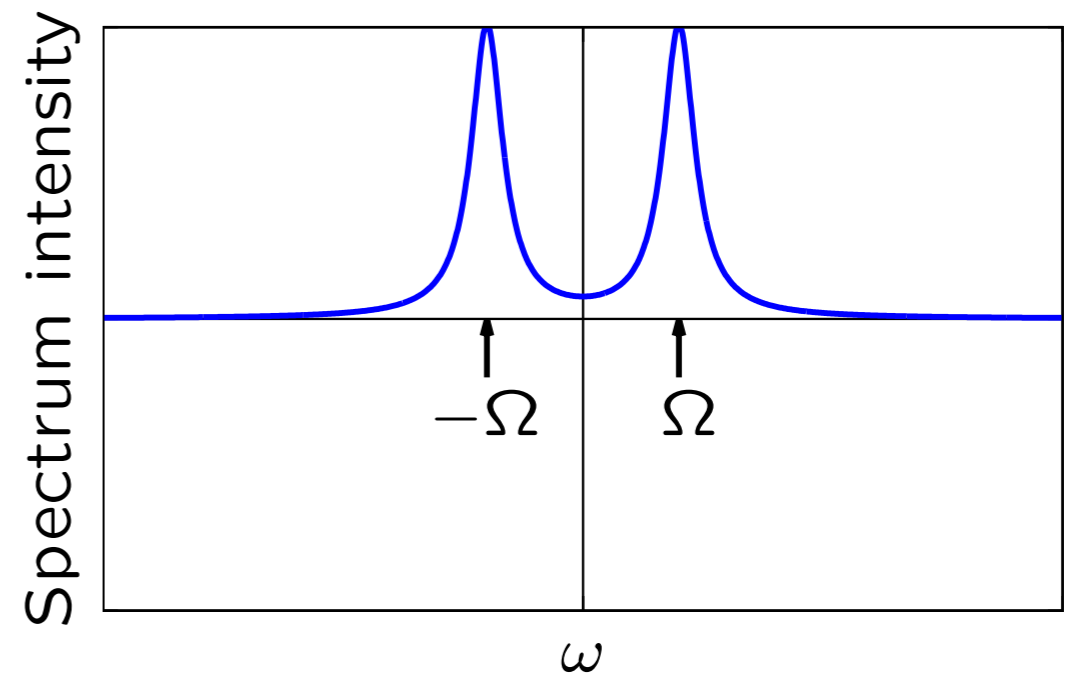
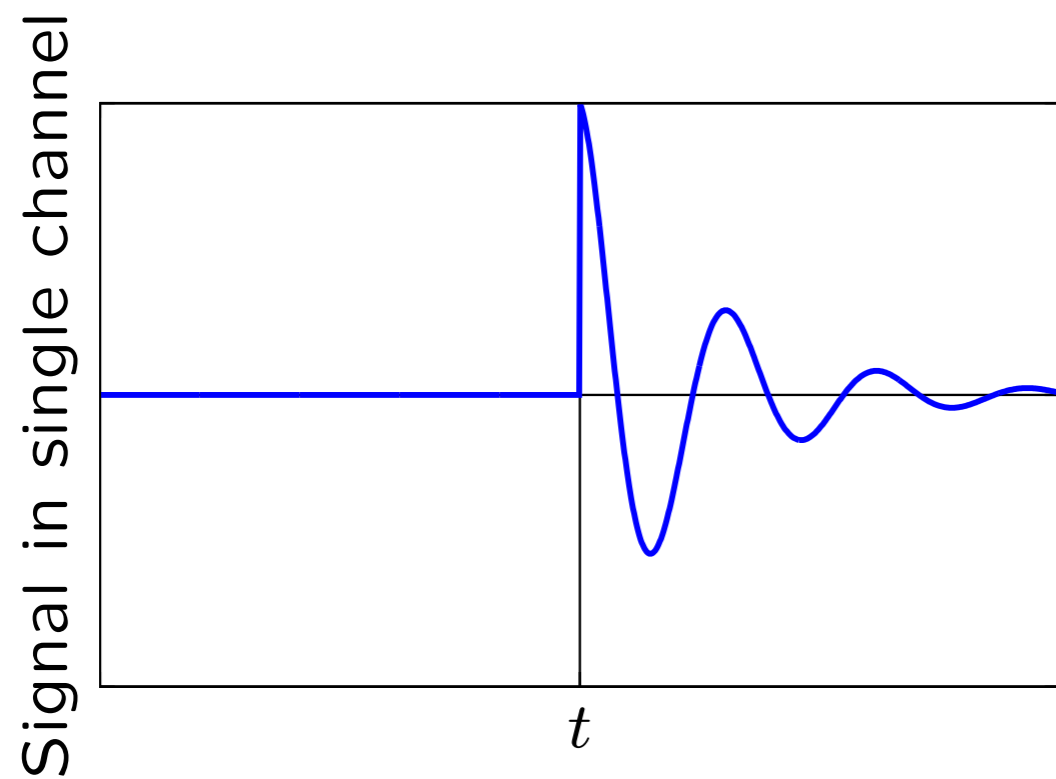
$$\begin{aligned} \frac{1}{2} \cos(\omega_0 t) \cos(-\omega_{\text{radio}} t) &= \frac{1}{4} \cos(\overbrace{(\omega_0 - \omega_{\text{radio}})}^{\text{high}} t) + \frac{1}{4} \cos(\overbrace{(\omega_0 + \omega_{\text{radio}})}^{\Omega \text{ low}} t) \\ \frac{1}{2} \cos(\omega_0 t) \sin(-\omega_{\text{radio}} t) &= \frac{1}{4} \sin(\overbrace{(\omega_0 - \omega_{\text{radio}})}^{\text{high}} t) - \frac{1}{4} \sin(\overbrace{(\omega_0 + \omega_{\text{radio}})}^{\Omega \text{ low}} t) \end{aligned}$$

$$\cos(\omega_0 t) \rightarrow \begin{cases} \frac{1}{4} \cos(\Omega t) & \text{channel } a & a = A \cos(\Omega t) \\ \frac{1}{4} \sin(\Omega t) & \text{channel } b & b = A \sin(\Omega t) \end{cases}$$

data storing option:

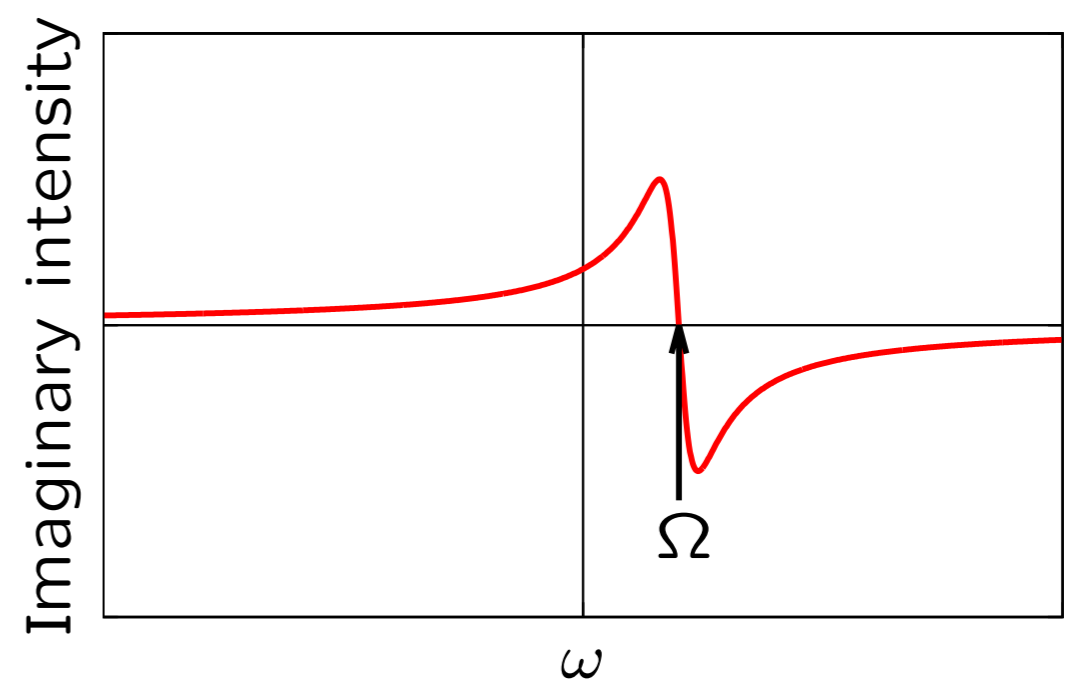
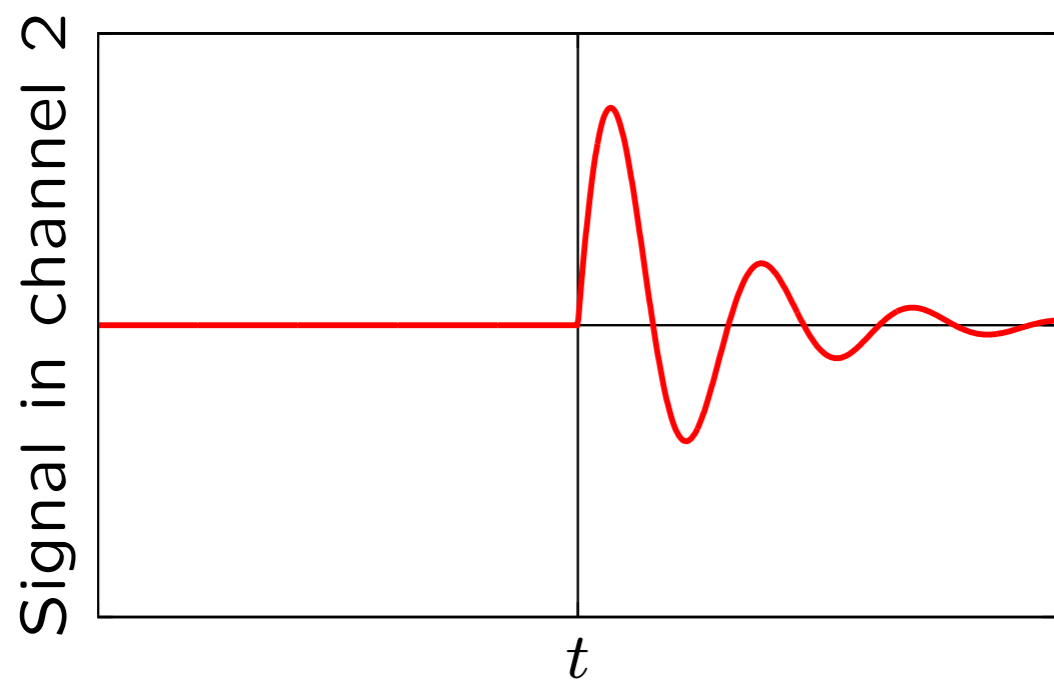
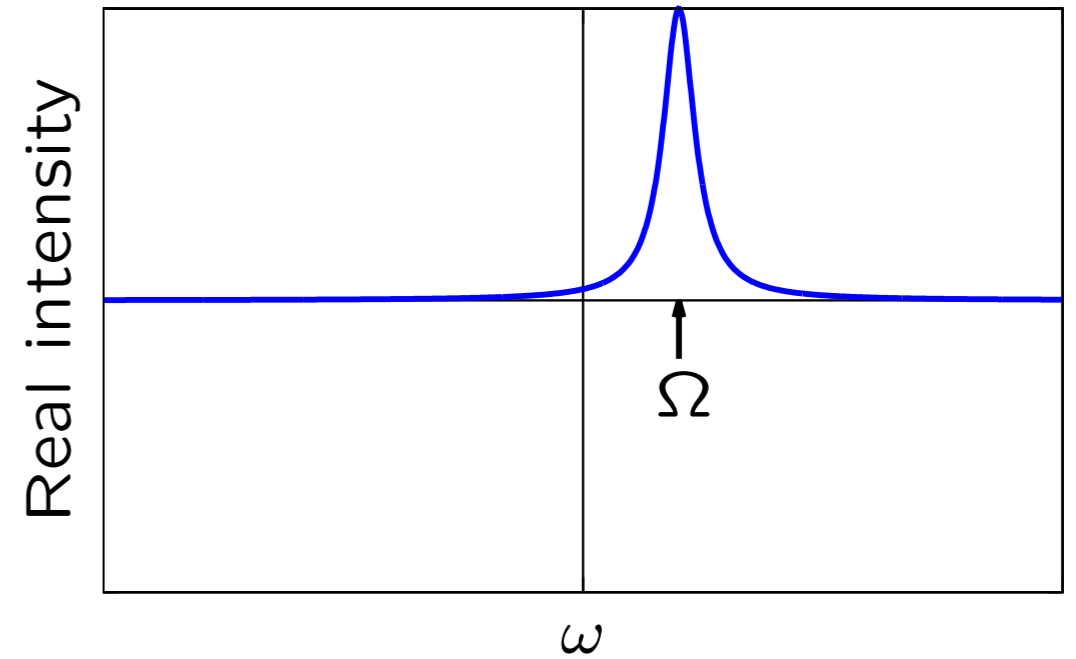
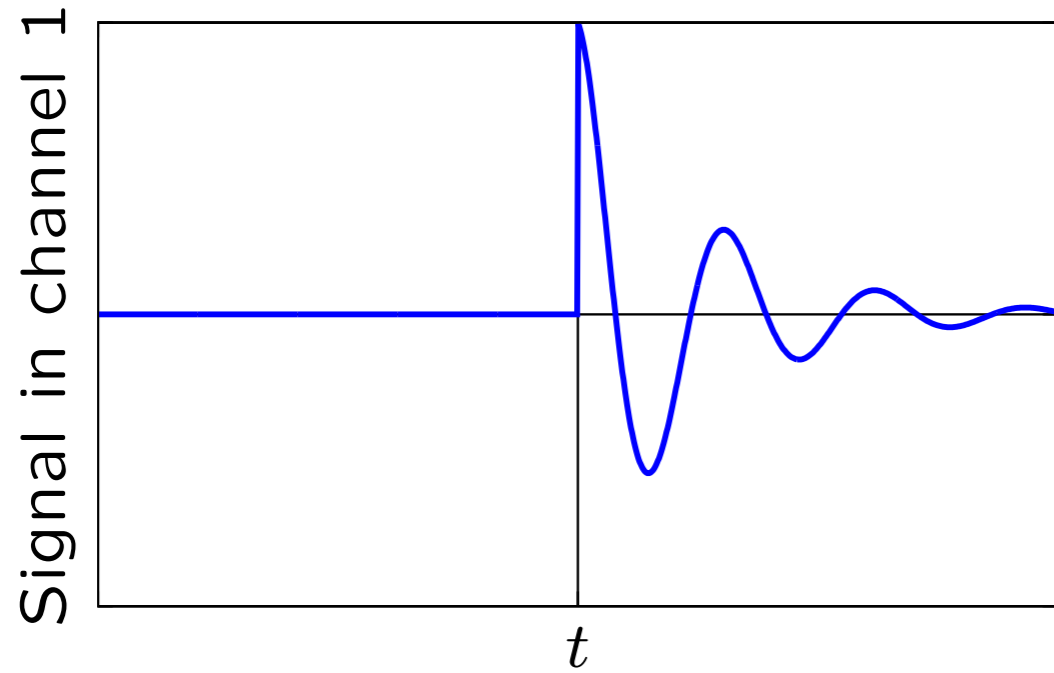
a, b	conventionally labeled:	x
$b, -a$		y
$-a, -b$		$-x$
$-b, a$		$-y$

Single channel, real Fourier transformation

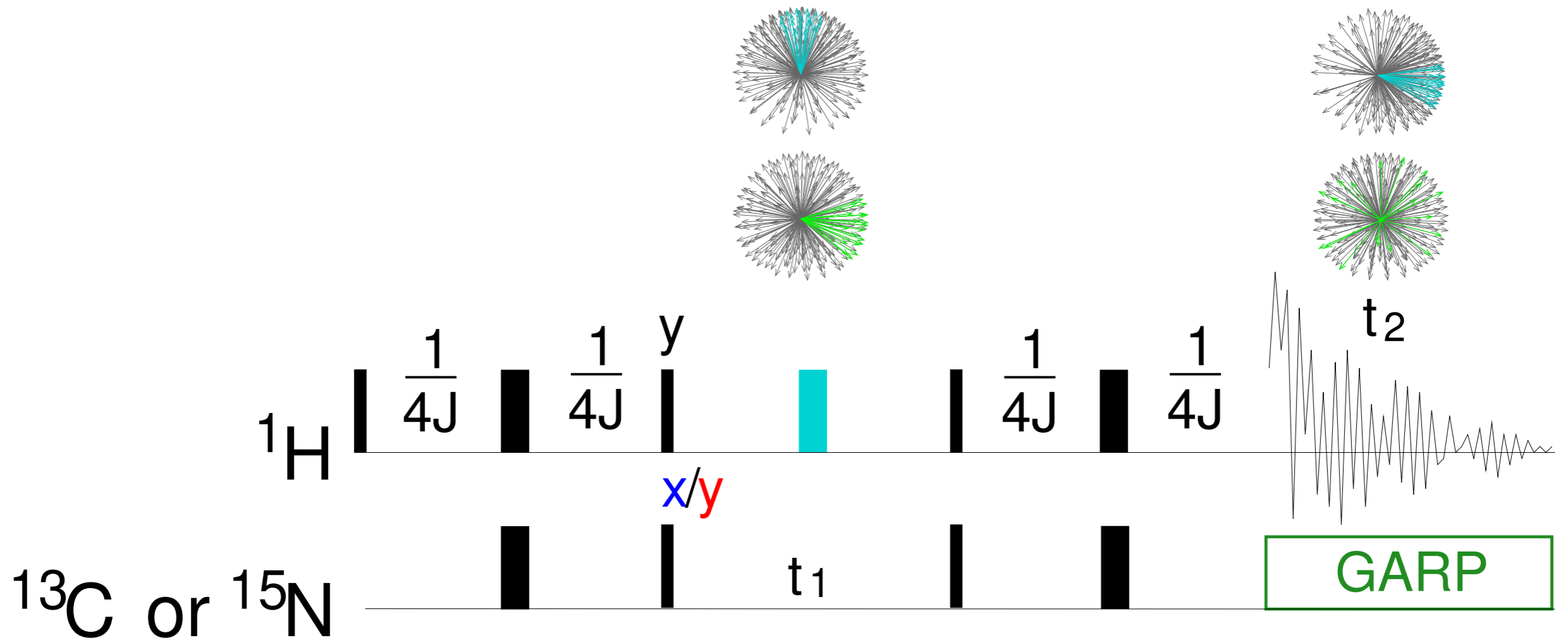


Two channels, complex Fourier transformation

$$a + ib \longrightarrow X + iY$$



Complex signal in indirect dimension



Repeat with y and y

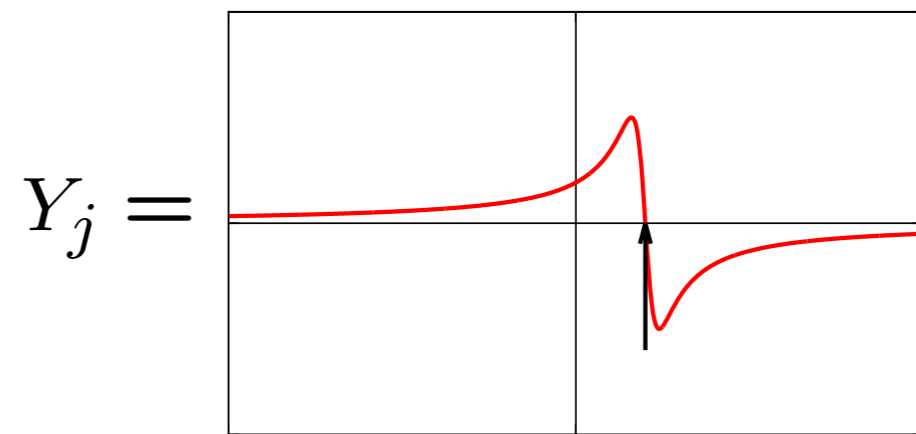
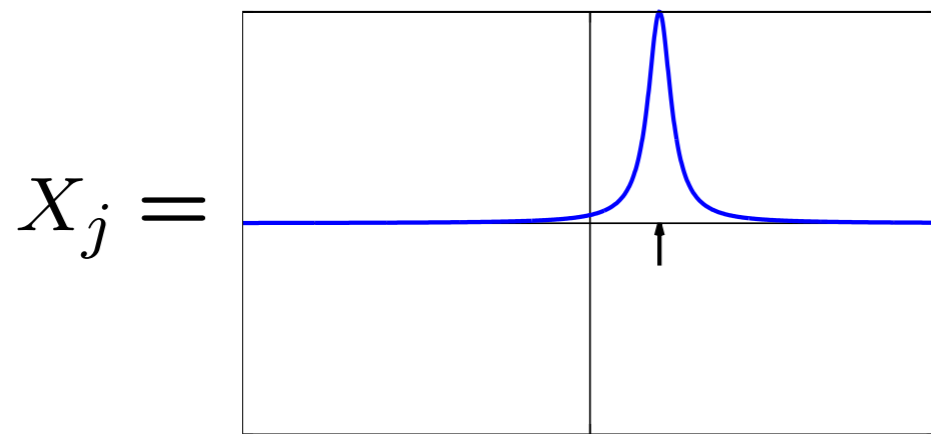
Complex signal in indirect dimension

$$\left. \begin{aligned}
 r_1 &= A_c a = \cos(\Omega_C t_1) \cos(\Omega_H t_2) && \text{channel } a, \text{ pulse } x \\
 r_2 &= A_c b = \cos(\Omega_C t_1) \sin(\Omega_H t_2) && \text{channel } b, \text{ pulse } x \\
 r_3 &= A_s a = \sin(\Omega_C t_1) \cos(\Omega_H t_2) && \text{channel } a, \text{ pulse } y \\
 r_4 &= A_s b = \sin(\Omega_C t_1) \sin(\Omega_H t_2) && \text{channel } b, \text{ pulse } y
 \end{aligned} \right\} (A_c + iA_s)(a + ib)$$

x/y	receiver	acquired as	stored as records r_j
$+x$	$+x$	$a : A_c \sin(\Omega_H t_2)$ $b : A_c \cos(\Omega_H t_2)$	$r_1, r_2 = A_c \sin(\Omega_H t_2), A_c \cos(\Omega_H t_2)$
$+y$	$+x$	$a : A_s \sin(\Omega_H t_2)$ $b : A_s \cos(\Omega_H t_2)$	$r_3, r_4 = A_s \sin(\Omega_H t_2), A_s \cos(\Omega_H t_2)$

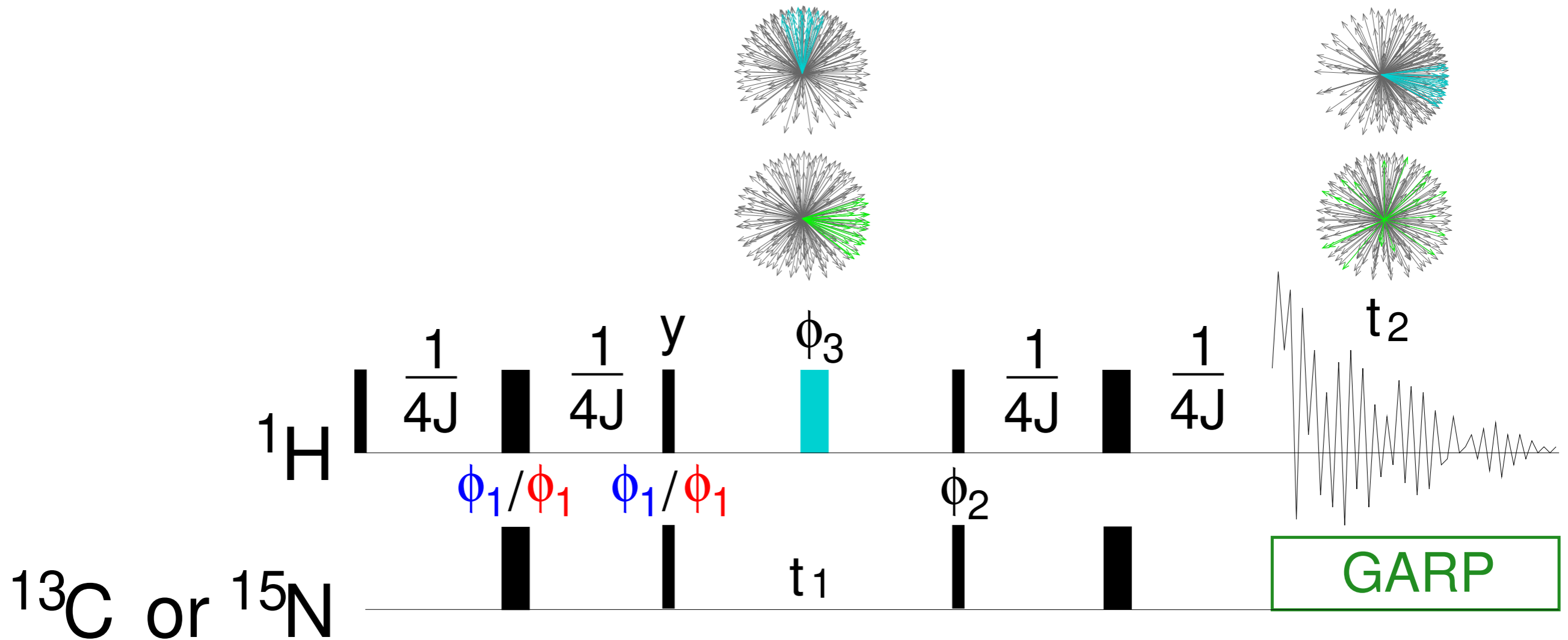
Complex signal in indirect dimension

$$(A_c + iA_s)(a + ib) \longrightarrow (A_c + iA_s)(X_2 + iY_2) \longrightarrow (X_1 + iY_1)(X_2 + iY_2) = X_1X_2 - Y_1Y_2 + i(X_1Y_2 + Y_1X_2)$$



$$(A_c + iA_s)(a + ib) \longrightarrow (A_c + iA_s)(X_2 + iY_2) \longrightarrow (A_c + iA_s)X_2 \longrightarrow (X_1 + iY_1)X_2 = X_1X_2 + iY_1X_2$$

Phase cycling

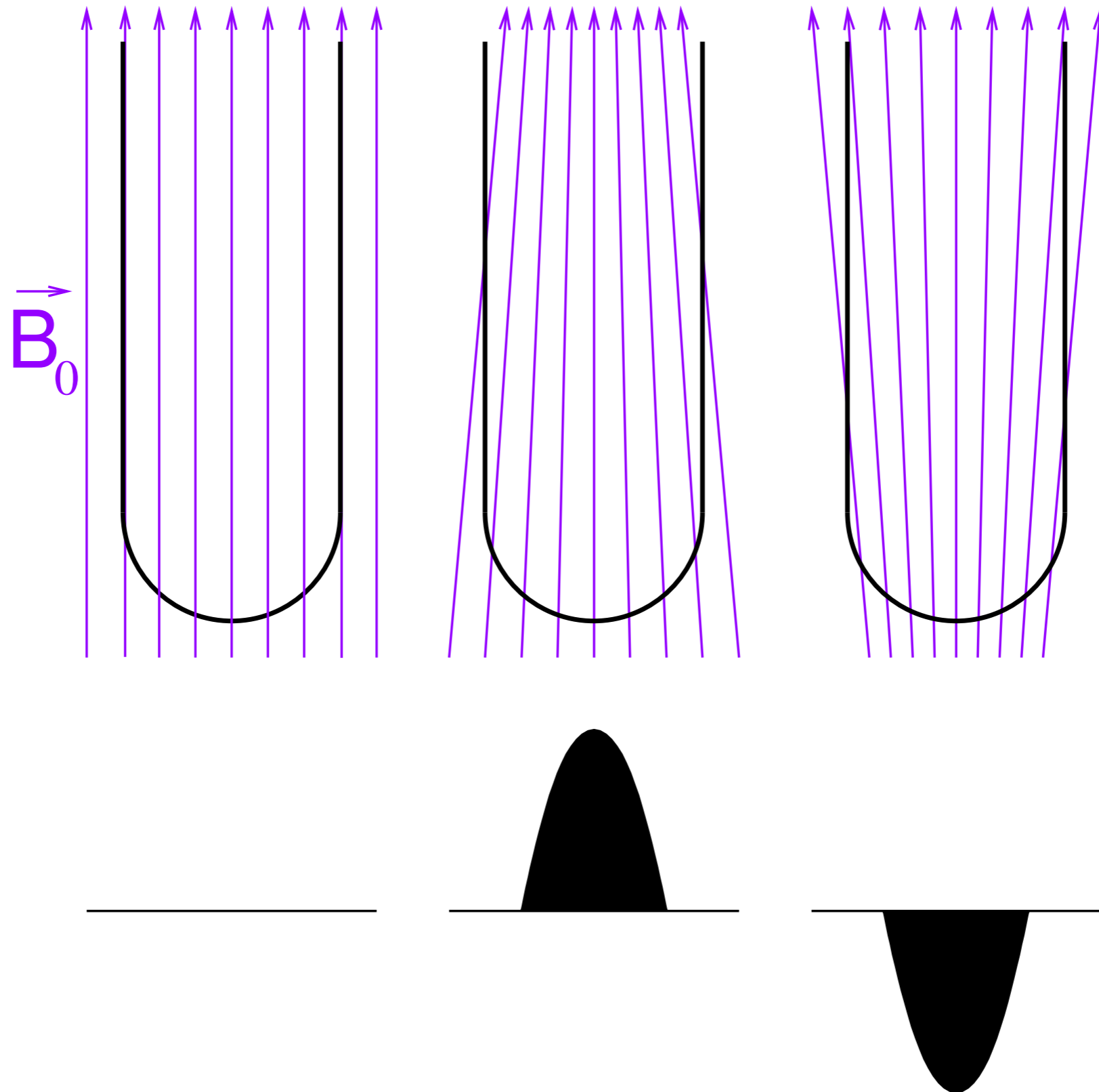


$$\phi_1 = x, -x, x, -x, x, -x, x, -x \quad \phi_2 = x, x, -x, -x, x, x, -x, -x$$

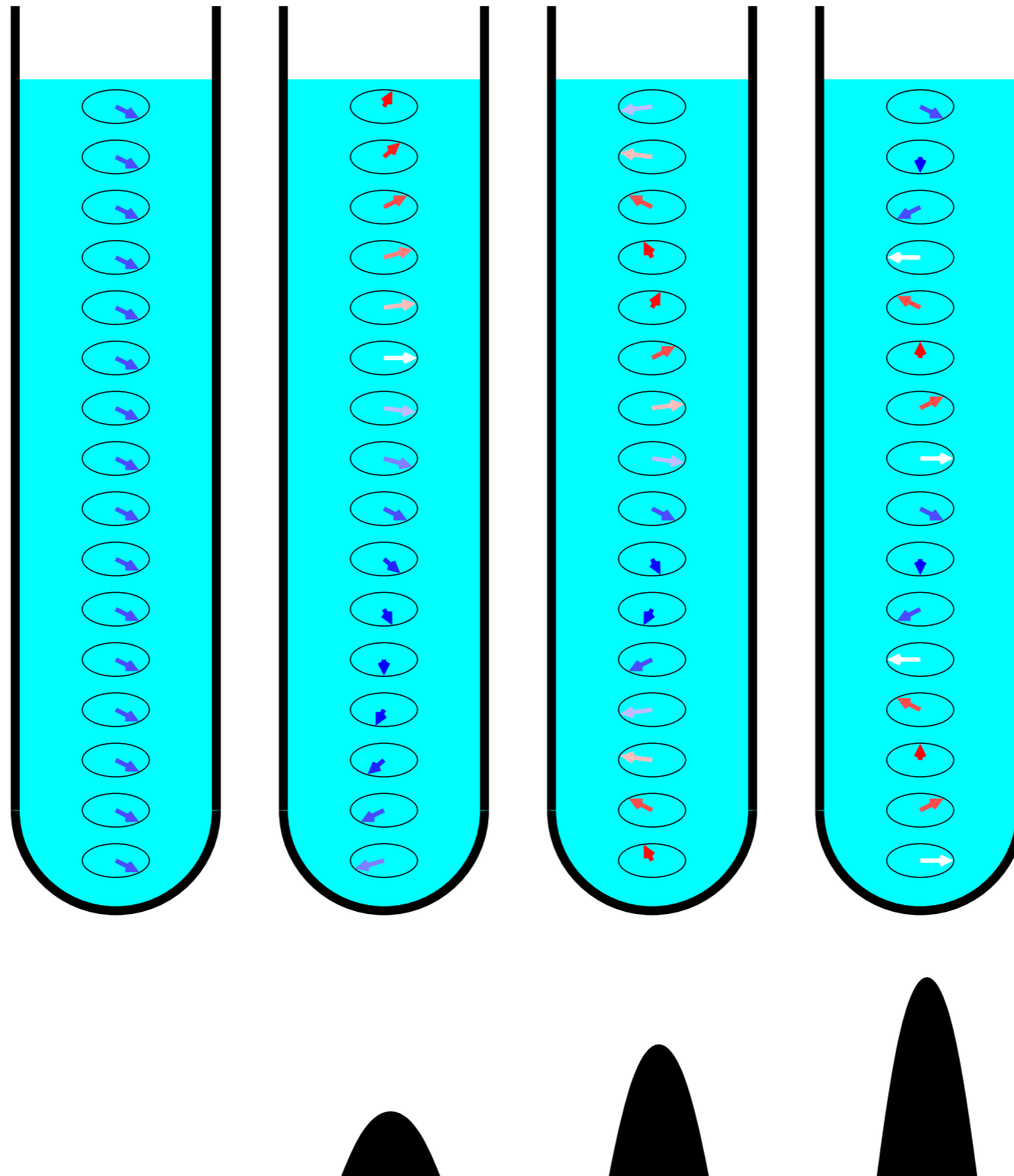
$$\phi_1 = y, -y, y, -y, y, -y, y, -y \quad \phi_3 = x, x, x, x, -x, -x, -x, -x$$

receiver phase: $x, -x, -x, x, x, -x, -x, x$

Pulsed-field gradients

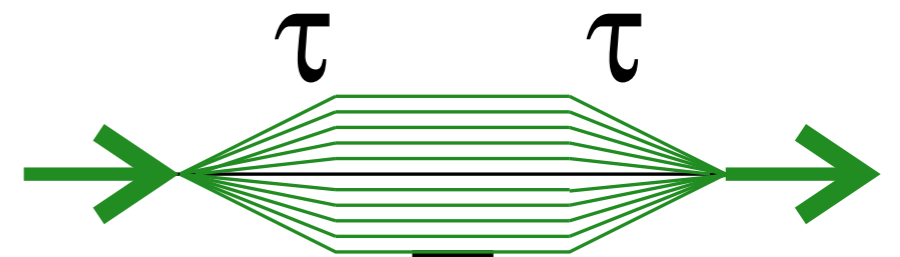
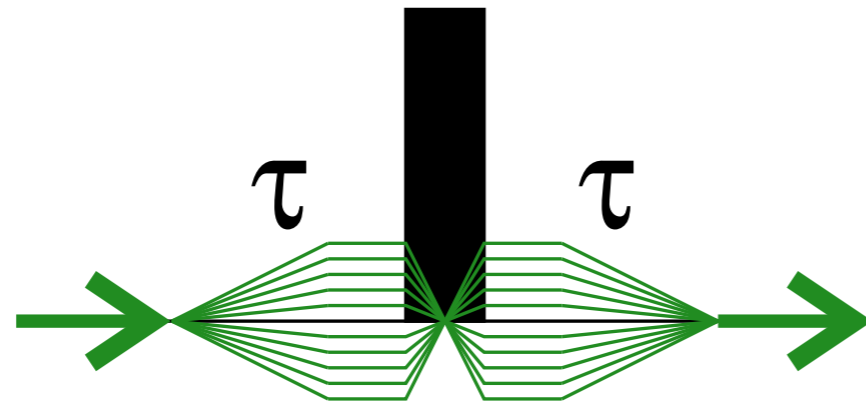


Pulsed-field gradients

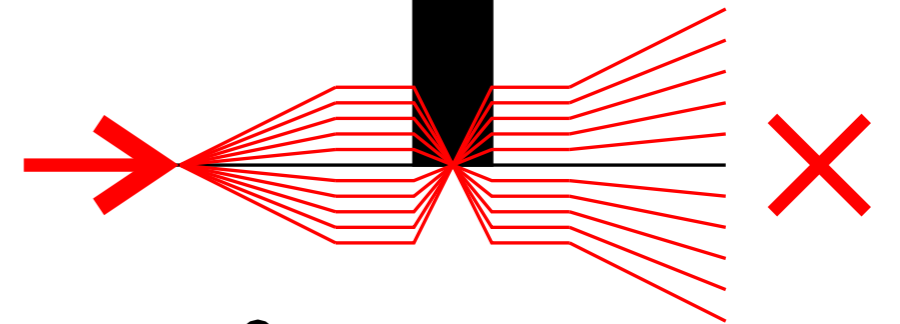
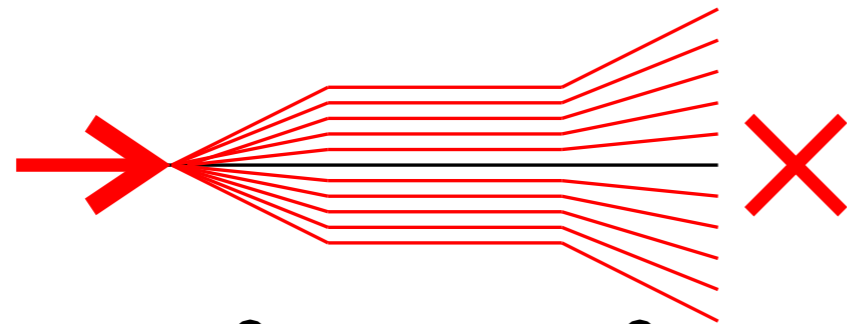


Gradient echoes

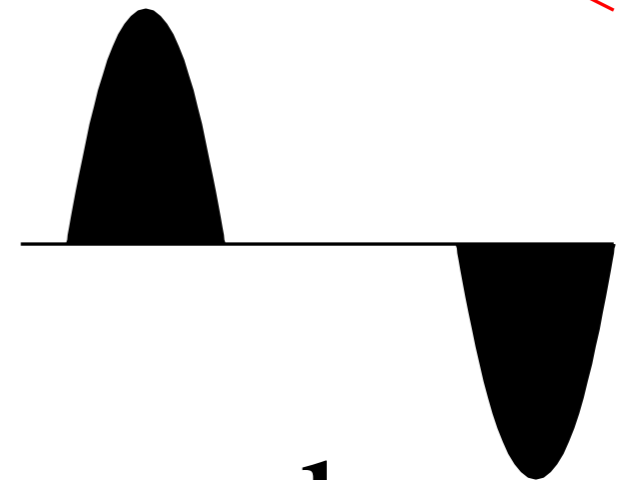
Wanted



Unwanted



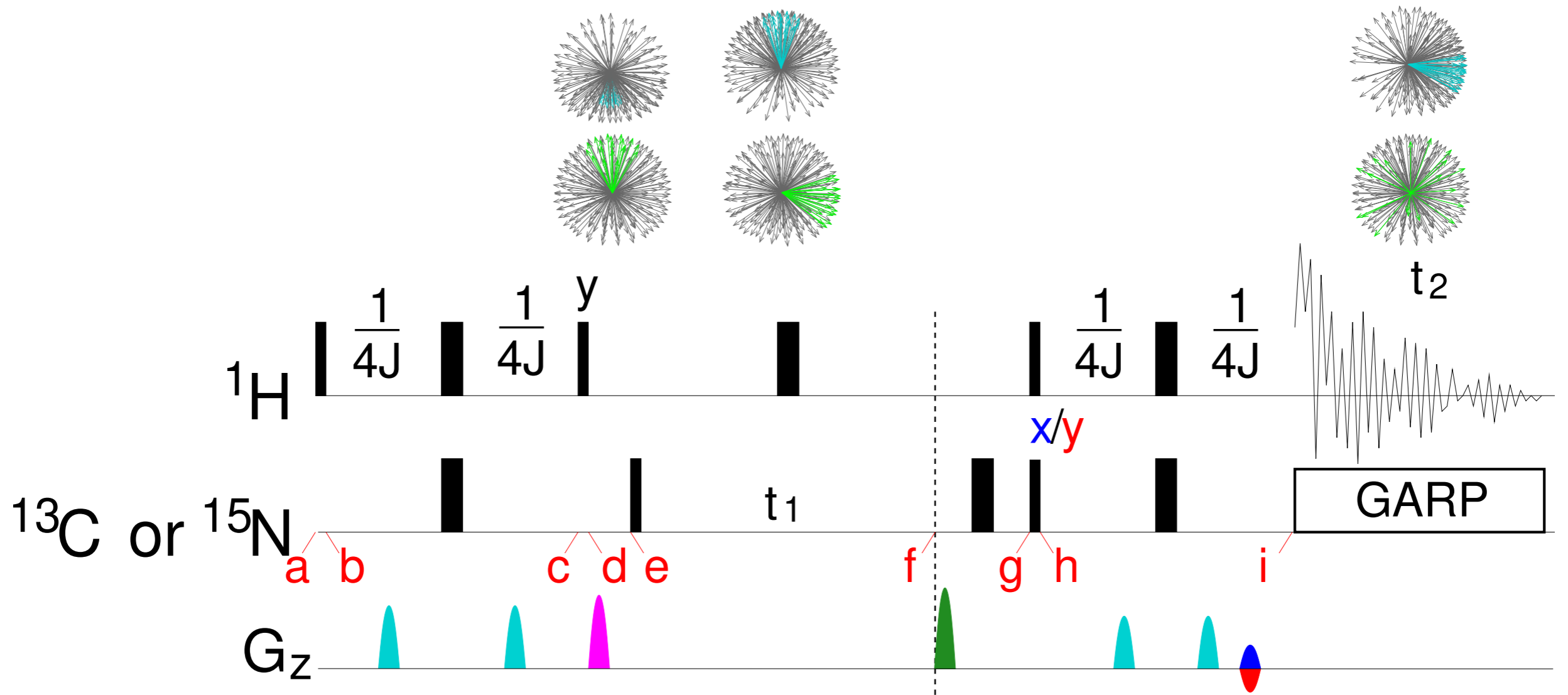
G_z



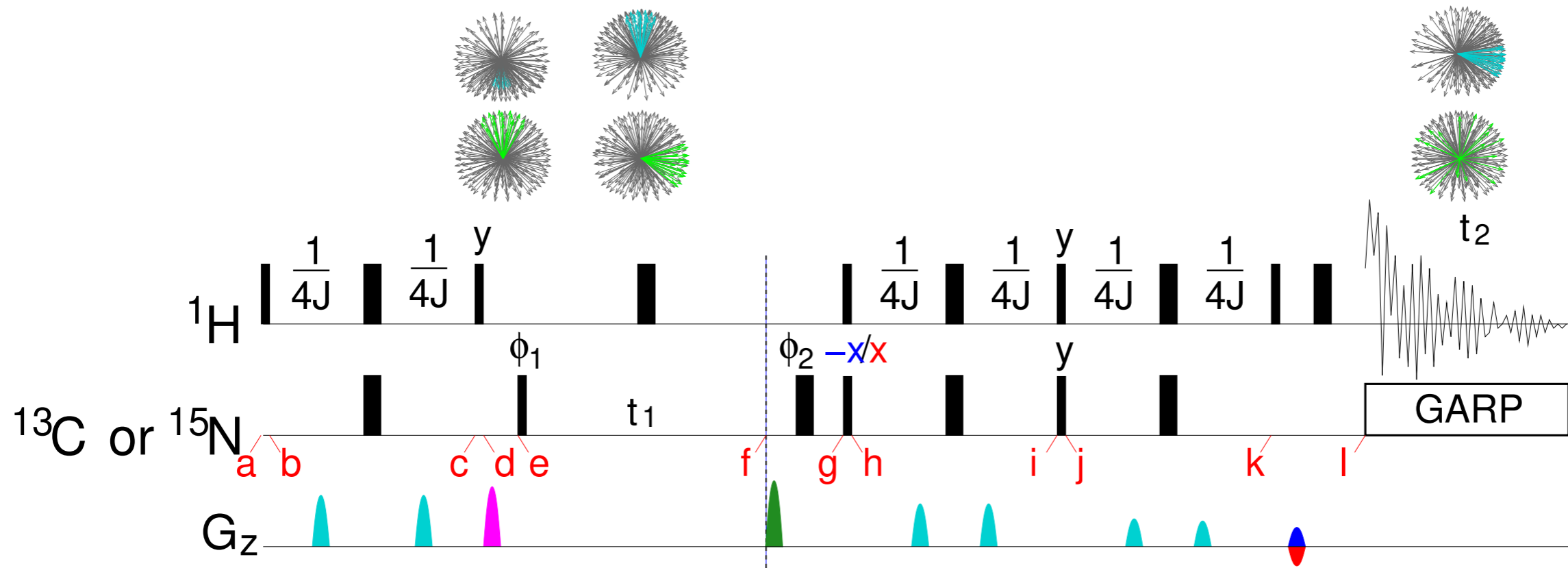
a

b

Gradients in HSQC



Preservation of equivalent pathways



$\phi_1 = x, -x, x, -x, x, -x, x, -x$ and receiver phase $x, -x, -x, x, x, -x, -x, x$ for odd increments of t_1

$\phi_1 = -x, x, -x, x, -x, x, -x, x$ and receiver phase $-x, x, x, -x, -x, x, x, -x$ for even increments of t_1

$\phi_2 = x, x, y, y, -x, -x, -y, -y$