C7790 Introduction to Molecular Modelling
TSM Modelling Molecular Structures

Lesson 13
Potential Energy Surface I

PS/2020 Distant Form of Teaching: Rev1

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Context

macroworld | microworld

states

(thermodynamic properties, G, T,…)

phenomenological thermodynamics

equilibrium (equilibrium constant)
kinetics (rate constant)

↕

free energy
(Gibbs/Helmholtz)

↕

partition function

statistical thermodynamics

microstates

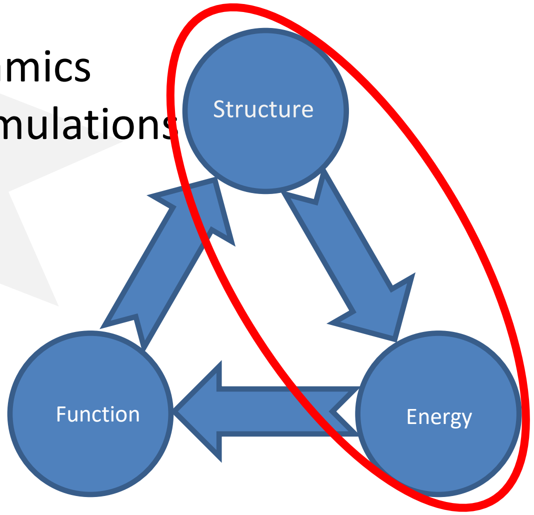
(mechanical properties, E)

Description levels (model chemistry):

- quantum mechanics
  - semiempirical methods
  - ab initio methods
  - post-HF methods
  - DFT methods
- molecular mechanics
- coarse-grained mechanics

Simulations:

- molecular dynamics
- Monte Carlo simulations
- docking
- …



microstate ≠ microworld

Revision

$$\hat{H}\phi(\mathbf{x},t) = i\hbar\frac{\partial\phi(\mathbf{x},t)}{\partial t}$$

time dependent Schrödinger equation

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time dependent Schrödinger equation

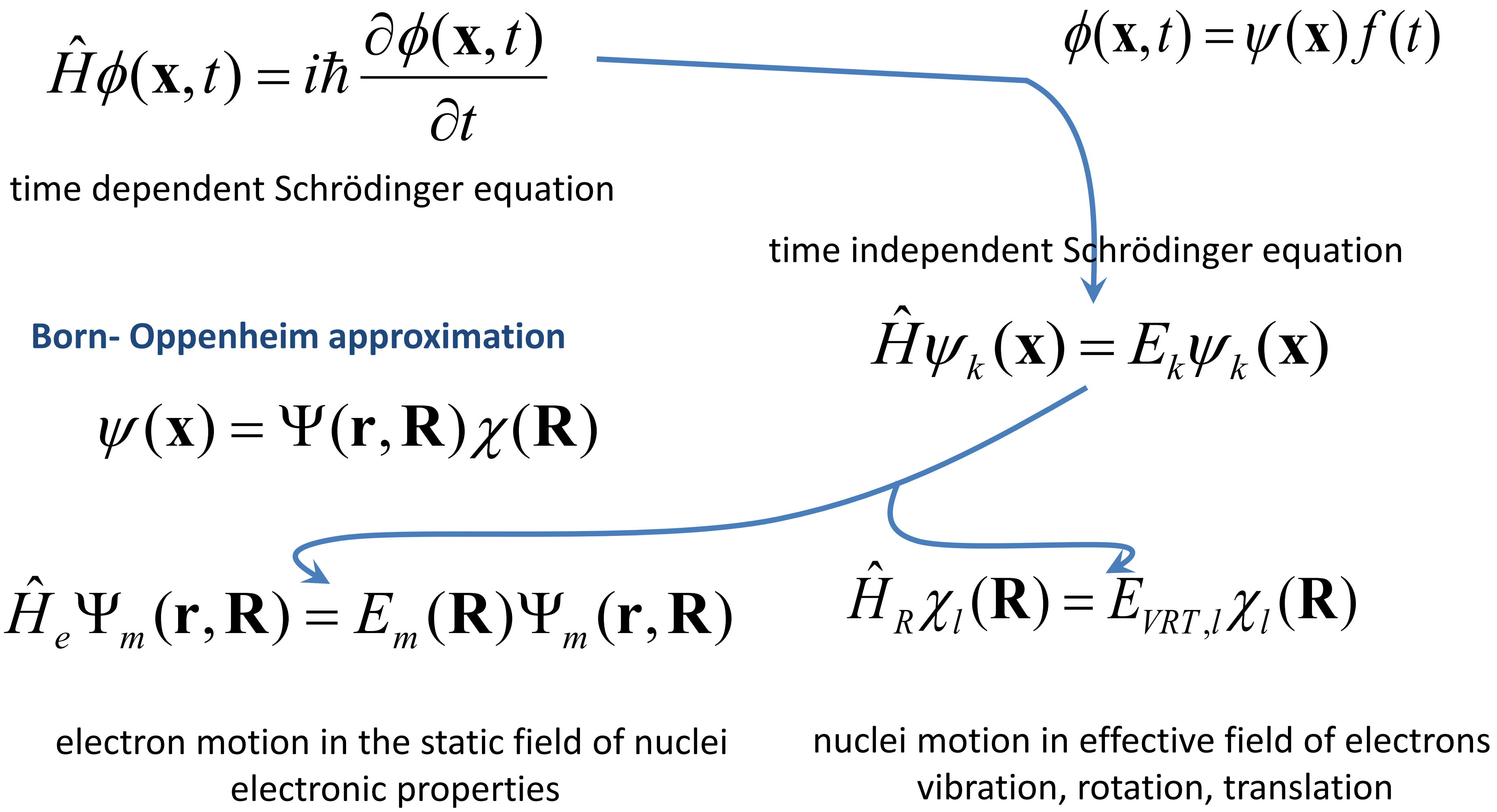
$$\phi(\mathbf{x},t) = \psi(\mathbf{x})f(t)$$

time independent Schrödinger equation

$$\hat{H}\psi_k(\mathbf{x}) = E_k\psi_k(\mathbf{x})$$

system can exist in several quantum states, each state is described by wavefunction ψ_k and has energy E_k

Revision



$$\hat{H}\phi(\mathbf{x},t) = i\hbar\frac{\partial\phi(\mathbf{x},t)}{\partial t}$$

time dependent Schrödinger equation

$$\phi(\mathbf{x},t) = \psi(\mathbf{x})f(t)$$

time independent Schrödinger equation

$$\hat{H}\psi_k(\mathbf{x}) = E_k\psi_k(\mathbf{x})$$

Born- Oppenheim approximation

$$\psi(\mathbf{x}) = \Psi(\mathbf{r},\mathbf{R})\chi(\mathbf{R})$$

$$\hat{H}_e\Psi_m(\mathbf{r},\mathbf{R}) = E_m(\mathbf{R})\Psi_m(\mathbf{r},\mathbf{R})$$

electron motion in the static field of nuclei
electronic properties

$$\hat{H}_R\chi_l(\mathbf{R}) = E_{VRT,l}\chi_l(\mathbf{R})$$

nuclei motion in effective field of electrons
vibration, rotation, translation

Hydrogen molecule

HW: What is the dissociation energy of H_2, D_2, and T_2?

$$H_2 \xrightarrow[\Delta G_r = ?]{} H^{\cdot} + H^{\cdot}$$

$$\begin{tikzcd}
\Delta G_r = ? \arrow[d, bend left, "\text{approximation}"] \\
\Delta E_r = ?
\end{tikzcd}$$

approximation
do not consider thermal effects

$$\hat{H}_e \Psi_m(\mathbf{r},\mathbf{R}) = E_m(\mathbf{R})\Psi_m(\mathbf{r},\mathbf{R})$$

Energy is a function of nuclei positions.
The function and its projections to lower dimensional configurational spaces are called potential energy surface.

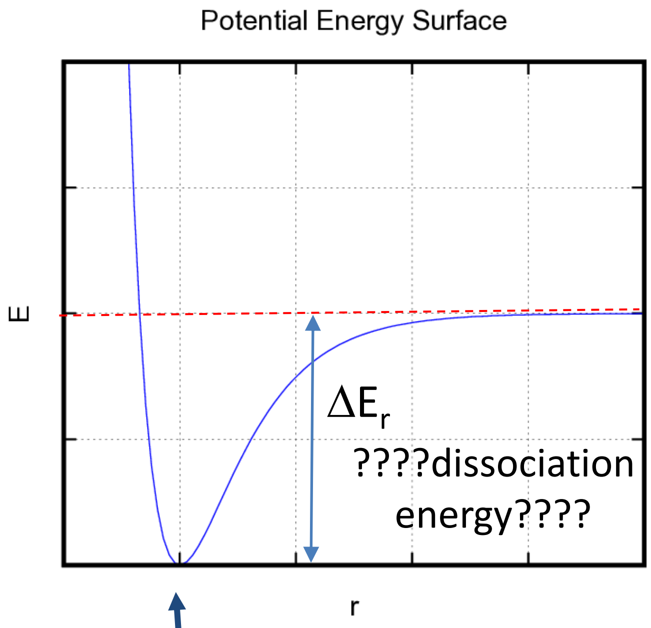
What is the potential energy surface for H_2, D_2, and T_2?
Do they differ?

H_2 - Potential Energy Surface

$$\hat{H}_e \Psi_m(\mathbf{r},\mathbf{R}) = \boxed{E_m(\mathbf{R})} \Psi_m(\mathbf{r},\mathbf{R})$$

dissociated state

$2H^{\cdot}$



bound state H_2

What is the energy of dissociated state? How is it related to reference states?

Two suitable reference states:

- **standard QM reference state** (infinite separation of electrons and nuclei, no kinetic energy) - negative energy
- **dissociated state** is considered as a reference with zero energy

REMEMBER:

- The **reference state** represents well defined state with well defined energy, usually zero.
- Its choice is arbitrary, but it must be consistent for all compounds and their states.

Recall Hamiltonian of chemical system

Hamiltonian of chemical system consisting of *N* nuclei of mass *M* and charge *Z* and *n* electrons is given by:

kinetic energy operator **(ELECTRONS ONLY)** potential energy

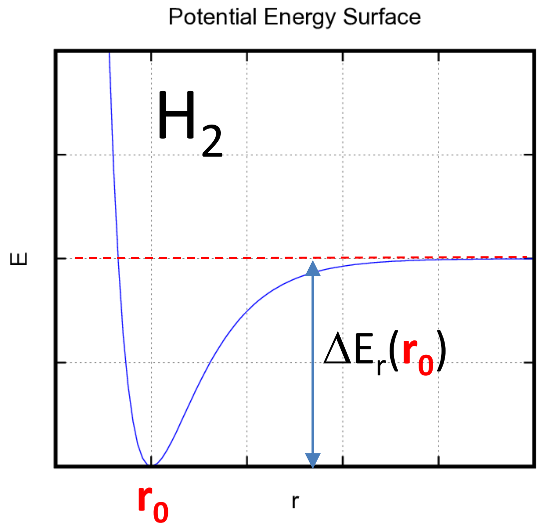
$$\hat{H} = \overbrace{-\frac{1}{2m}\underbrace{\sum_{i=1}^{n}}_{\text{electrons}}\nabla_i^2}^{\text{kinetic energy operator}} \overbrace{+\underbrace{\sum_{i=1}^{N}\sum_{j>i}^{N}\frac{Z_iZ_j}{r_{ij}}}_{\text{nucleus-nucleus}} - \underbrace{\sum_{i=1}^{N}\sum_{j=1}^{n}\frac{Z_i}{r_{ij}}}_{\text{electron-nucleus}} + \underbrace{\sum_{i=1}^{n}\sum_{j>i}^{n}\frac{1}{r_{ij}}}_{\text{electron-electron}}}^{\text{potential energy}}$$

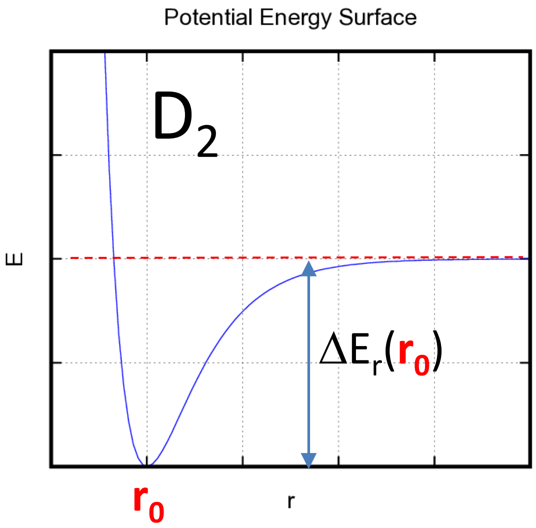
Schrödinger equation:

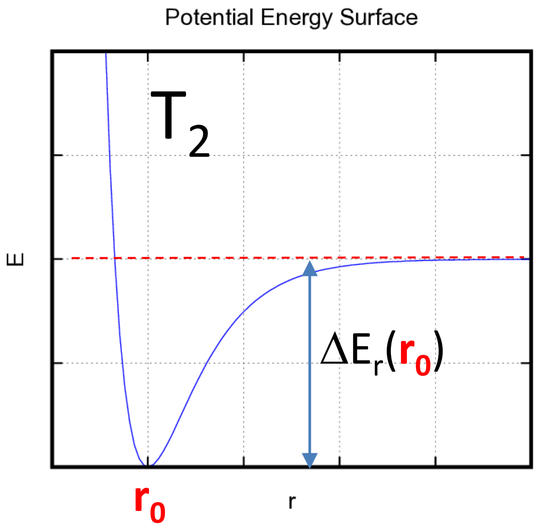
$$\boxed{\hat{H}\psi_k(\mathbf{r},\mathbf{R}) = E_k(\mathbf{R})\psi_k(\mathbf{r},\mathbf{R})}$$

Nuclei motion (nuclei mass) is not considered in the BO approximation.

H_2, D_2, T_2 - Potential Energy Surface



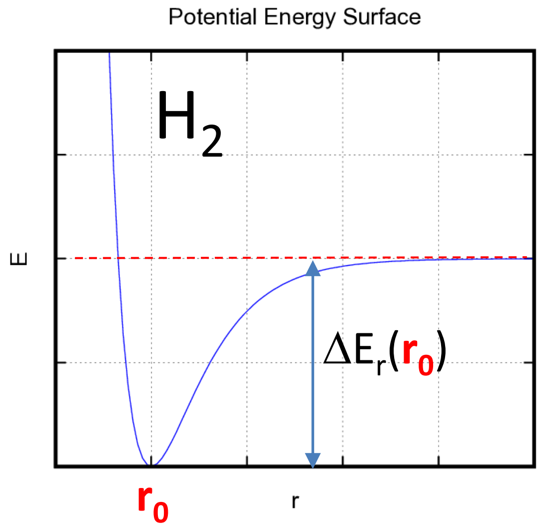


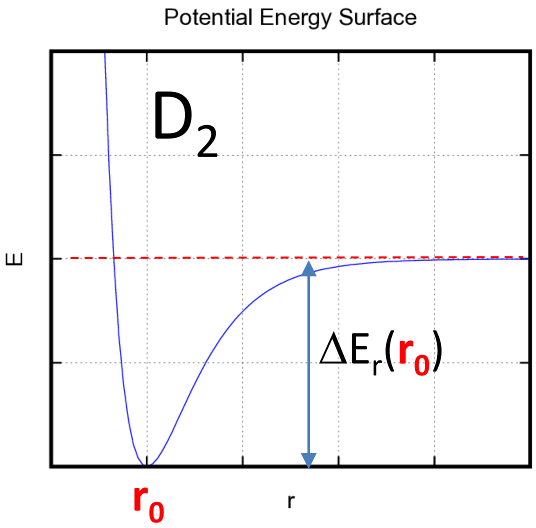


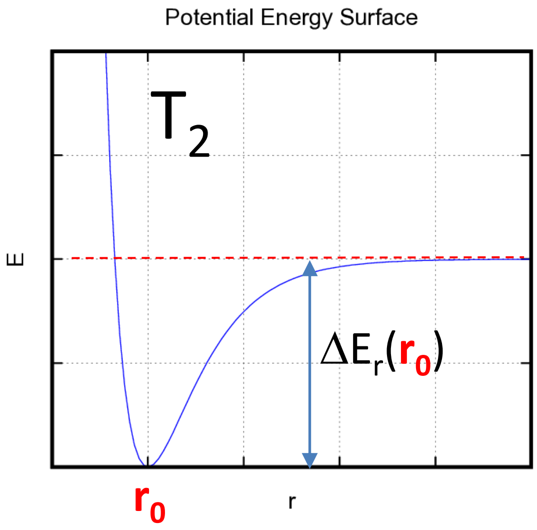
Potential energy surfaces are the same!
r_0 is the same as well.

Reason:
All three systems are chemically identical (two electrons and two +1 charged nuclei).

H_2, D_2, T_2 - Potential Energy Surface







What about vibrations? Do they contribute and how?

Consider harmonic oscillator (approximation):

characteristic frequency

$$E_V = \left(v + \frac{1}{2}\right) h v$$

non-zero energy even in the ground vibration state!!!

v = 0, 1, 2, …

H_2, D_2, T_2 - Vibrations

Harmonic oscillator:

characteristic frequency

$$E_V = \left(v + \frac{1}{2}\right) h\upsilon \qquad \upsilon = \frac{1}{2\pi}\sqrt{\frac{K}{\mu}}$$

v = 0, 1, 2, …

force constant; does it differ?

reduced mass (clearly this differs among H_2, D_2, and T_2)

H_2, D_2, T_2 - Vibrations

Harmonic oscillator:

characteristic frequency

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What about the force constant?

Harmonic potential:

tangent

curvature

$$V(r) = \frac{1}{2}K(r - r_0)^2 \longrightarrow \frac{\partial V(r)}{\partial r} = K(r - r_0) \longrightarrow \frac{\partial^2 V(r)}{\partial r^2} = K$$

first derivative with respect to r

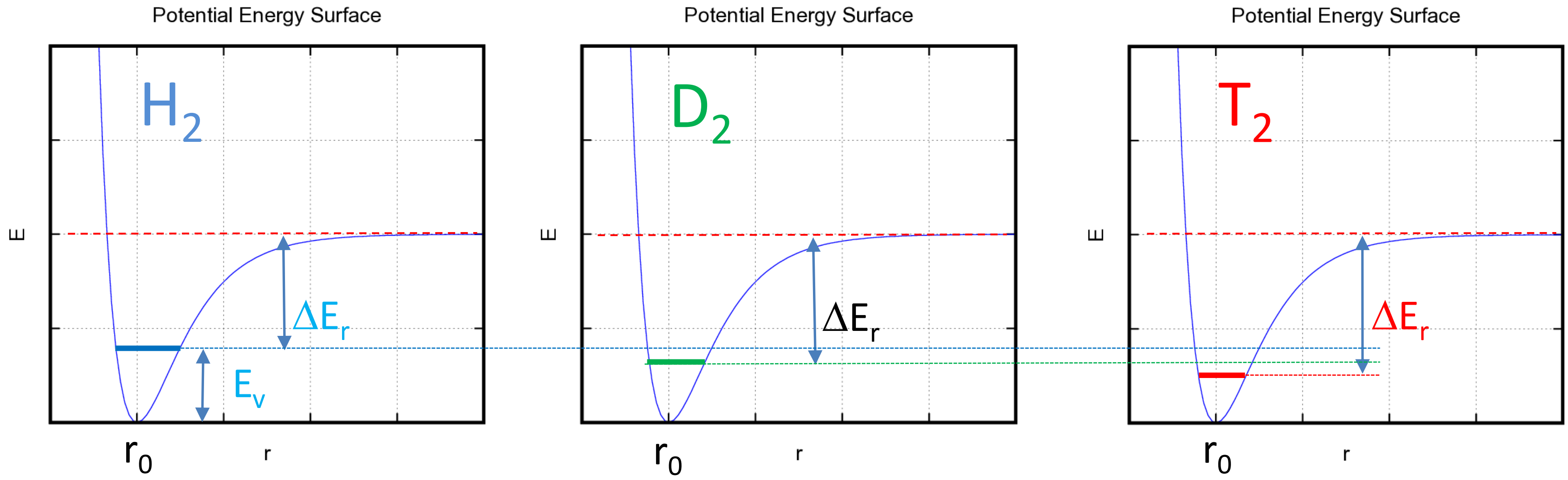
second derivative with respect to r

The force constant can be determined from the PES curvature at equilibrium distance (r_0) in harmonic approximation.

⇨

All three systems have the same PES and thus the same K as well .

H_2, D_2, T_2 - PES + Vibrations



!! not in scale !!

$|\Delta E_r| < |\Delta E_r| < |\Delta E_r|$

$$E_V = \left(\nu + \frac{1}{2}\right) h\nu$$

$$\nu = \frac{1}{2\pi}\sqrt{\frac{K}{\mu}}$$

$r_0 = r_0 = r_0$

$r_e \sim r_e \sim r_e$ observable equilibrium bond lengths impact of anharmonicity and QM character of vibrations

bigger mass -> smaller frequency -> lower energy

Revision

$$\hat{H}_e \Psi_m(\mathbf{r},\mathbf{R}) = E_m(\mathbf{R})\Psi_m(\mathbf{r},\mathbf{R})$$

electron motion in the static field of nuclei
electronic properties

$$\hat{H}_R \chi_l(\mathbf{R}) = E_{VRT,l}\chi_l(\mathbf{R})$$

nuclei motion in effective field of electrons
vibration, rotation, translation

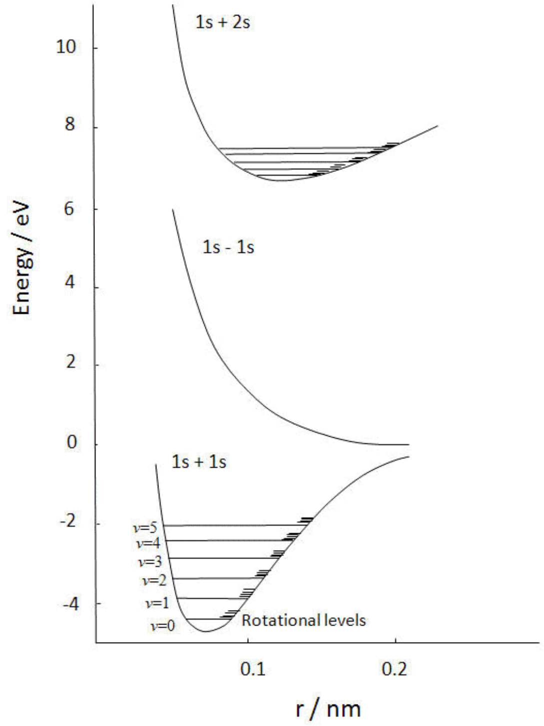
electronic energy part

vibration, rotation, translation energy part

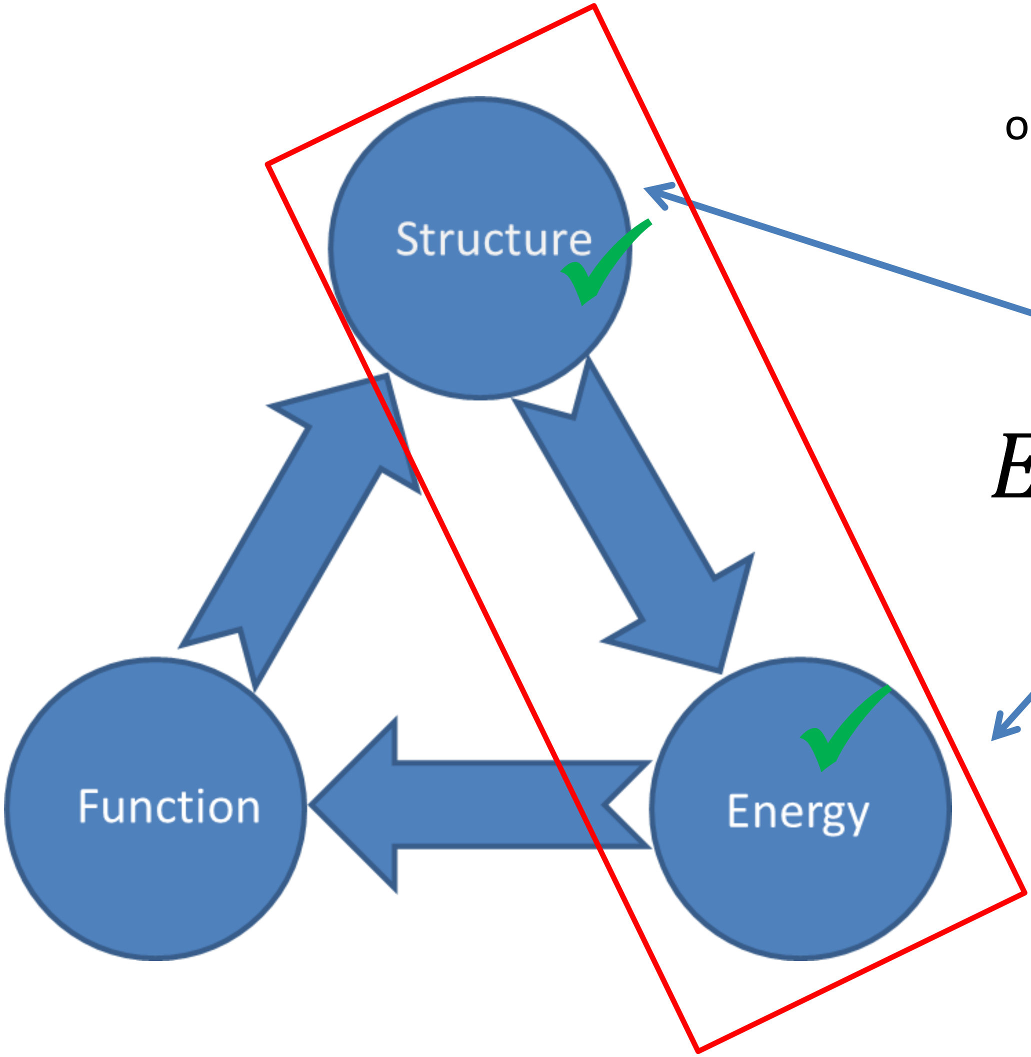
$$E_k = E_m(R_{opt,m}) + E_{VRT,l}$$

total energy of the state

optimal geometry, at which E_m is minimal



Structure *vs* system state



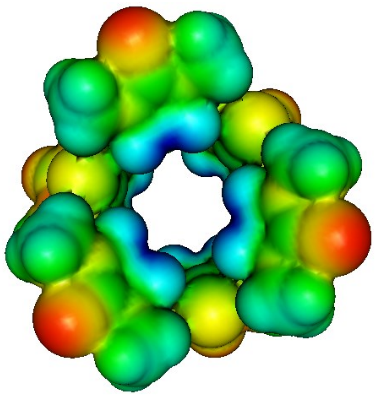
only part of the quantum state description

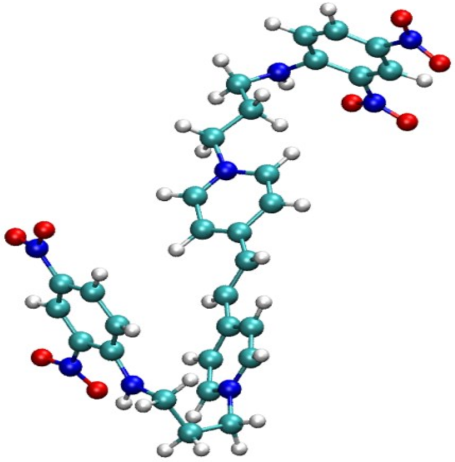
$$E = E(r_o) + E_{VRT}$$

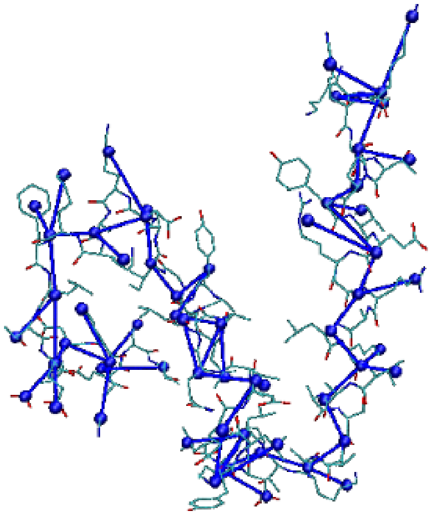
!!!!!!

E_{VRT} is nonzero even at 0 K because of vibrational (and translational) energy.

Method overview (model chemistry)







QM (Quantum mechanics) | **MM (Molecular mechanics)** | ***CGM (Coarse-grained* mechanics)**

$$\boxed{E(\mathbf{R})} \xrightarrow{\text{approximations}} \boxed{E(\mathbf{R})} \xrightarrow{\text{approximations}} \boxed{E(\mathbf{R})}$$

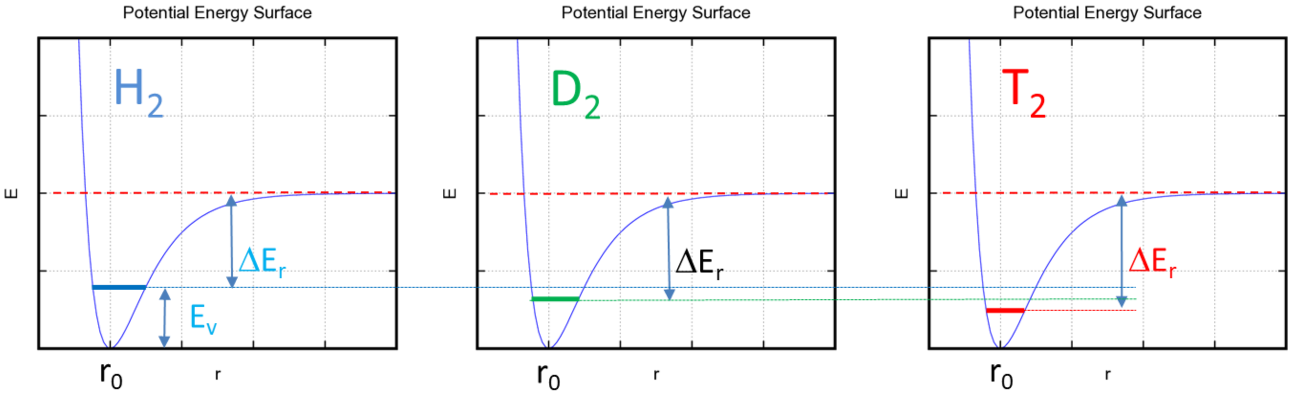
R - position of atom nuclei

R - position of atoms

R - position of beads

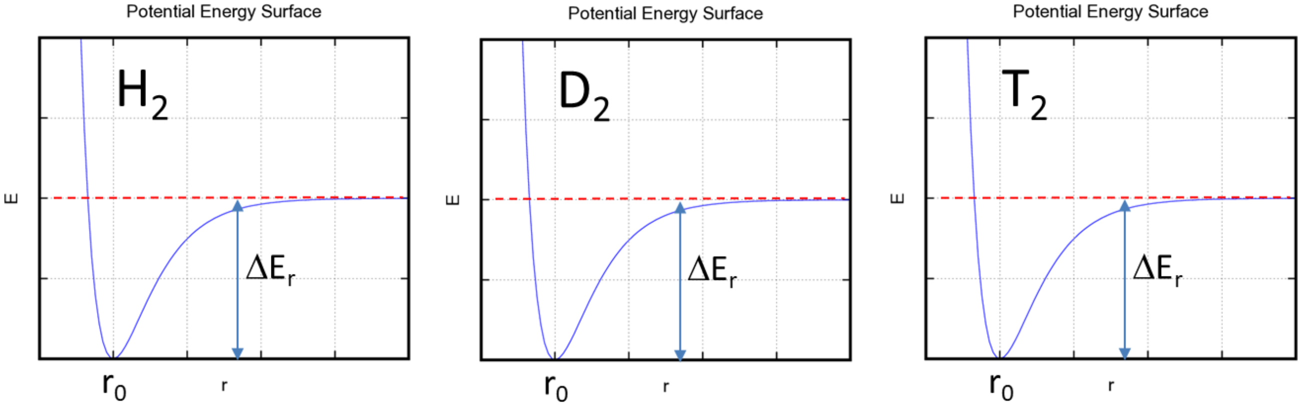
Quantum vs Classical description

- **Fully QM**
- QM, MM + **QM harmonic approximation**, or similar
- QM, MM + **path integral molecular dynamics**



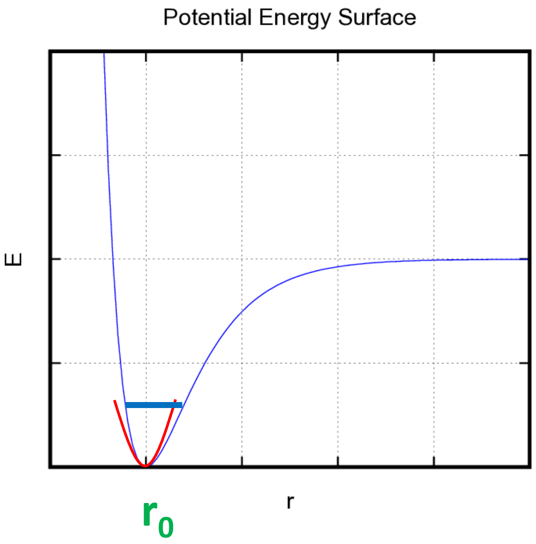
thermal energy not shown in graphs is $1/2k_BT$ (equipartition principle) in all cases (fully quantum/classical)

- QM, MM, CG + classical nuclei/atom motions, molecular dynamics (MD)



NO DIFFERENCE

Summary



To characterize a quantum state:

$$E = E(r_o) + E_{VRT}$$

a) we need to find **a potential energy minimum**

b) we can further evaluate vibrations from **the PES curvature at the minimum**

(too difficult to calculate, thus it is usually neglected)

- PES cannot describe mass effect of nuclei; it only describes electronic effects.
- Isotope effects can be measured experimentally
  - Primary Isotope Effect (kinetics)
  - Secondary Isotope Effect (kinetics)
  - It can be even tasted by your tongue, see:

Ben Abu, N.; Mason, P. E.; Klein, H.; Dubovski, N.; Ben Shoshan-Galeczki, Y.; Malach, E.; Pražienková, V.; Maletínská, L.; Tempra, C.; Chamorro, V. C.; Cvačka, J.; Behrens, M.; Niv, M. Y.; Jungwirth, P. Sweet Taste of Heavy Water. *Communications Biology* **2021**, *4* (1), 1–10. https://doi.org/10.1038/s42003-021-01964-y.