

Gauss' law

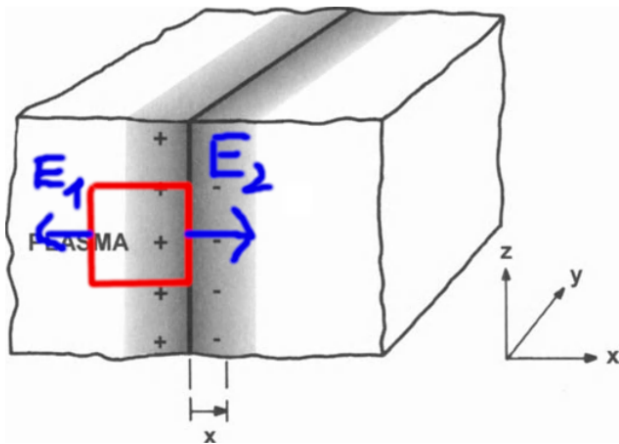
$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad (1)$$

$$\text{LHS : } \int \nabla \cdot \vec{E} \, dV = \oint_{\partial V} \vec{E} \cdot d\vec{S} \quad (2)$$

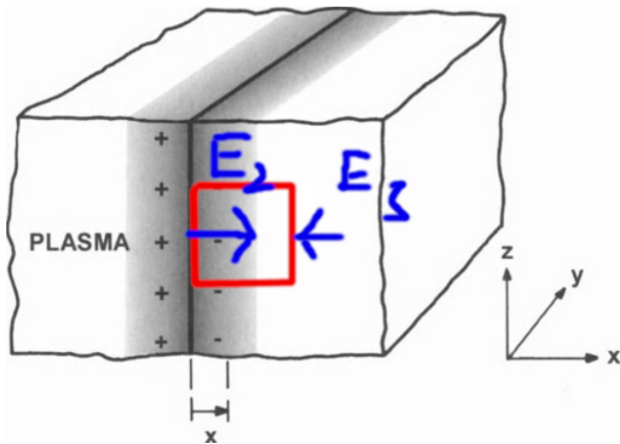
$$\text{RHS : } \int \frac{\rho}{\epsilon_0} \, dV = \frac{Q}{\epsilon_0} \quad (3)$$

Parallel and constant vectors \vec{E} and \vec{S}

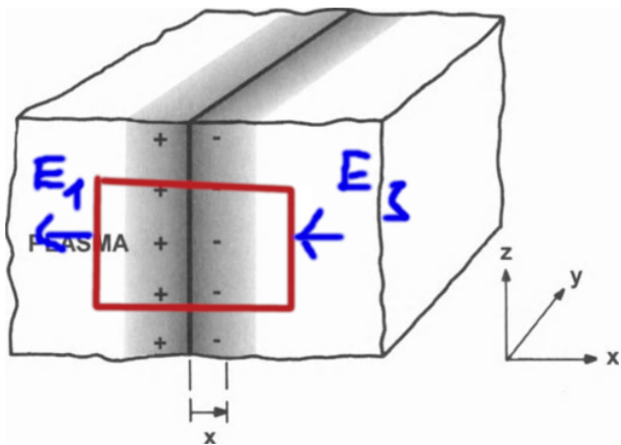
$$\sum_{i \in \partial V} E_i S_i = \frac{Q}{\epsilon_0} \quad (4)$$



$$E_1 + E_2 = \frac{enSx}{S\epsilon_0} \quad (5)$$

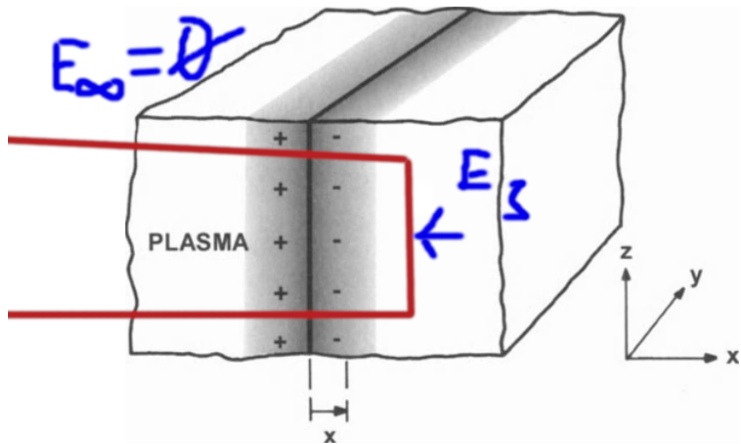


$$-E_2 - E_3 = -\frac{enSx}{S\epsilon_0} \quad (6)$$

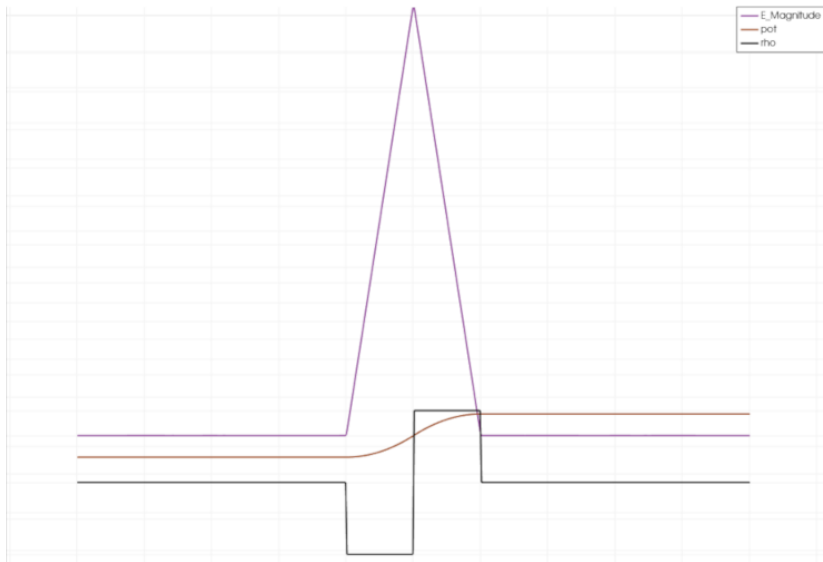


$$E_1 - E_3 = 0$$

(7)



$$E_{\text{interface}} = \frac{nex}{\epsilon_0} \quad (8)$$



Equation of motion

Lorentz force on electron;

$$m\ddot{x} = -eE \quad (9)$$

$$\ddot{x} = -\frac{ne^2x}{m\epsilon_0} \quad (10)$$

Solution

Characteristic polynomial:

$$\lambda^2 = -\frac{ne^2}{m\epsilon_0} \quad (11)$$

$$\lambda = i\sqrt{\frac{ne^2}{m\epsilon_0}} \quad (12)$$

General solution:

$$x(t) = Ae^{i\lambda t} + Be^{-i\lambda t} \simeq \sin\left(\sqrt{\frac{ne^2}{m\epsilon_0}}t\right) \quad (13)$$