Cvicenia 6.1, 6.5a) a 6.4 z Bittencourta

6.1

$$f(v) = K_0 \qquad \text{if} \qquad |v_i| \leq v_0 \qquad \text{else} \qquad f(v) = 0 \tag{1}$$

Z prikladu 5.1 sme ziskali K_0:

$$K_0 = \frac{n_0}{8v_0^3} \tag{2}$$

a)

Teplota je definova ako:

$$T = \frac{1}{N_{\mathrm{sv}} k} \langle m c^2 \rangle, \tag{3}$$

ked N_{sv} je pocet stupnov volnosti a $c^2 = \vec{c} \cdot \vec{c}$ je kvadrat nahodne rychlosti: $\vec{c} = \vec{v} - \langle \vec{v} \rangle$.

$\langle \vec{c} \rangle$ je z definicie vzdy nula, ale $\langle c^2 \rangle$ nemusi (iba pre $0K$).

a)

Zo symetrie rozdelovacej funkcie mozeme vidiet:

$$\langle \vec{v} \rangle = 0, \tag{4}$$

rychlosti su rovnako pravdepodobne ako ich opacne $(-v)$ hodnoty.

$$\langle mc^2 \rangle = \frac{1}{n_0} \int m(\vec{v} - \vec{0})^2 f(v)\, \mathrm{d}^3 v = \tag{5}$$

$$\frac{m}{n_0} \int_{-v_0}^{v_0} \int_{-v_0}^{v_0} \int_{-v_0}^{v_0} K_0 \left(v_x^2 + v_y^2 + v_z^2 \right) \, \mathrm{d}v_x \mathrm{d}v_y \mathrm{d}v_z = \tag{6}$$

$$\frac{8m}{n_0} K_0 v_0^5 = m v_0^2 \tag{7}$$

$$T = \frac{m v_0^2}{3k}, \tag{8}$$

pre 3 stupne volnosti.

b)

$$P_{ij} = nm\langle c_i c_j \rangle \qquad i,j \in \{x,y,z\} \tag{9}$$

$$P_{ij} = m \int f(v) v_i v_j \mathrm{d}^3 v = \tag{10}$$

$$m \int_{-v_0}^{v_0} \int_{-v_0}^{v_0} \int_{-v_0}^{v_0} K_0 v_i v_j \mathrm{d}v_i \mathrm{d}v_j \mathrm{d}v_k = \tag{11}$$

$$2mv_0 K_0 \int_{-v_0}^{v_0} \int_{-v_0}^{v_0} v_i v_j \mathrm{d}v_i \mathrm{d}v_j , \tag{12}$$

kde sme vyintegrovali rychlost, ktora nie je i ani j.

b)

Su dve moznosti:

- $i \neq j$:

$$2mv_0K_0 \int_{-v_0}^{v_0} \int_{-v_0}^{v_0} v_i v_j \mathrm{d}v_i \mathrm{d}v_j = \tag{13}$$

$$2mv_0K_0 \int_{-v_0}^{v_0} \frac{1}{2} \left(v_0^2 - v_0^2 \right) v_j \mathrm{d}v_j = 0 \tag{14}$$

- $i = j$

$$2mv_0K_0 \int_{-v_0}^{v_0} \int_{-v_0}^{v_0} v_i^2 \mathrm{d}v_i \mathrm{d}v_j = \tag{15}$$

$$4mv_0^2K_0 \int_{-v_0}^{v_0} v_i^2 \mathrm{d}v_i = 4mv_0^2K_0 \frac{2}{3} v_0^3 = \frac{1}{3} v_0^2 mn_0 \tag{16}$$

c)

Vektor toku tepla:

$$\vec{q} = \frac{1}{2} mn \langle c^2 \vec{c} \rangle = \frac{1}{2} mn \langle \left(v_x^2 + v_y^2 + v_z^2 \right) \cdot (v_x, v_y, v_z) \rangle, \qquad (17)$$

co vyrobi integraly typu:

$$\langle v_i^3 + v_i v_j^2 + v_i v_k^2 \rangle, \qquad (18)$$

kde $i = \{x, y, z\}$, $i \neq j \neq k$.

Vsetky tieto funkcie su ale neparne a kvoli integracnym medziam:

$$\langle v_i^3 \rangle = 0 \qquad \langle v_i v_j^2 \rangle = n_0 \langle v_i \rangle \langle v_j^2 \rangle = 0, \qquad (19)$$

6.5a)

Zo zadania:

$$\vec{w}_\alpha = \vec{u}_\alpha - \vec{u}_0 \tag{20}$$

$$\vec{c}_{\alpha 0} = \vec{v} - \vec{u}_0 = \vec{v} + \vec{w}_\alpha - \vec{u}_\alpha = \vec{c}_\alpha + \vec{w}_\alpha \tag{21}$$

Originalny tlak:

$$\mathbf{P}_\alpha = n_\alpha m_\alpha \langle \vec{c}_\alpha \vec{c}_\alpha \rangle \tag{22}$$

Alternativny tlak:

$$\mathbf{P}_{\alpha 0} = n_\alpha m_\alpha \langle \vec{c}_{\alpha 0} \vec{c}_{\alpha 0} \rangle \tag{23}$$

6.5a)

$$\langle \vec{w}_\alpha \rangle = \vec{w}_\alpha \tag{24}$$

$$\langle \vec{c}_{\alpha 0} \vec{c}_{\alpha 0} \rangle = \langle (\vec{c}_\alpha + \vec{w}_\alpha)(\vec{c}_\alpha + \vec{w}_\alpha) \rangle = \tag{25}$$

$$\langle \vec{c}_\alpha \vec{c}_\alpha \rangle + \langle \vec{c}_\alpha \vec{w}_\alpha \rangle + \langle \vec{w}_\alpha \vec{c}_\alpha \rangle + \langle \vec{w}_\alpha \vec{w}_\alpha \rangle = \tag{26}$$

$$\langle \vec{c}_\alpha \vec{c}_\alpha \rangle + 2 \langle \vec{c}_\alpha \vec{w}_\alpha \rangle + \langle \vec{w}_\alpha \vec{w}_\alpha \rangle = \tag{27}$$

$$\langle \vec{c}_\alpha \vec{c}_\alpha \rangle + 2 \vec{w}_\alpha \langle \vec{c}_\alpha \rangle + \vec{w}_\alpha \vec{w}_\alpha = \langle \vec{c}_\alpha \vec{c}_\alpha \rangle + \vec{w}_\alpha \vec{w}_\alpha \tag{28}$$

6.5a)

$$\mathbf{P}_{\alpha 0} = n_\alpha m_\alpha \langle \vec{c}_{\alpha 0} \vec{c}_{\alpha 0} \rangle = n_\alpha m_\alpha \langle \vec{c}_\alpha \vec{c}_\alpha \rangle + n_\alpha m_\alpha \vec{w}_\alpha \vec{w}_\alpha = \qquad (29)$$

$$\mathbf{P}_\alpha + n_\alpha m_\alpha \vec{w}_\alpha \vec{w}_\alpha \qquad (30)$$

Skalarny tlak je: $p = \frac{1}{3}\mathrm{Tr}\mathbf{P}$:

$$p_{\alpha 0} = p_\alpha + \frac{1}{3} n_\alpha m_\alpha w_\alpha^2 \qquad (31)$$

6.4

Jednoduchsi problem: tlakovy tenzor (2. radu) ma 3^2 elementov ale len 6 je nezavislych

$$\begin{pmatrix} c_x c_x & c_x c_y & c_x c_z \\ c_y c_x & c_y c_y & c_y c_z \\ c_z c_x & c_z c_y & c_z c_z \end{pmatrix} \tag{32}$$

$c_x c_y = c_y c_x$, atd.

$$\begin{pmatrix} \color{blue}{c_x c_x} & \color{blue}{c_x c_y} & \color{blue}{c_x c_z} \\ c_y c_x & \color{blue}{c_y c_y} & \color{blue}{c_y c_z} \\ c_z c_x & c_z c_y & \color{blue}{c_z c_z} \end{pmatrix} \tag{33}$$

6.4

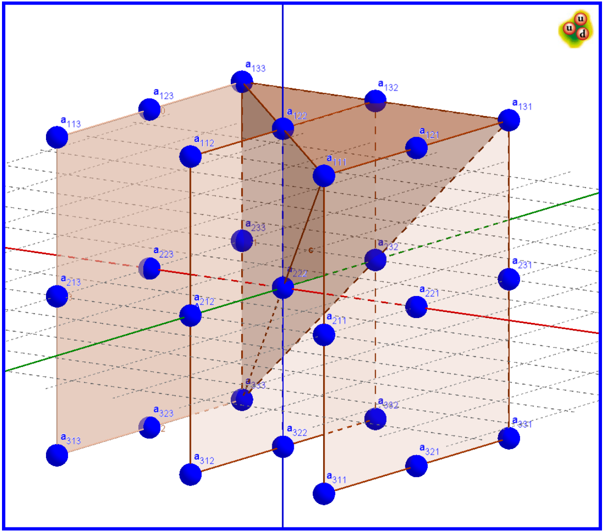
Podobne, triada ma 3^3 elementov. Nezavisle elementy dostaneme kominaciami s opakovanim. Vyberame 3 indexy na 3 miesta. Indexy, x, y, z, sa mozu opakovat:

$$\binom{n+k-1}{k} \tag{34}$$

$$\binom{3+2-1}{2} = 6 \tag{35}$$

$$\binom{3+3-1}{3} = 10 \tag{36}$$

6.4



Zdroj: https://i.imgur.com/HK0dOSZ.png