Cross section for momentum transfer

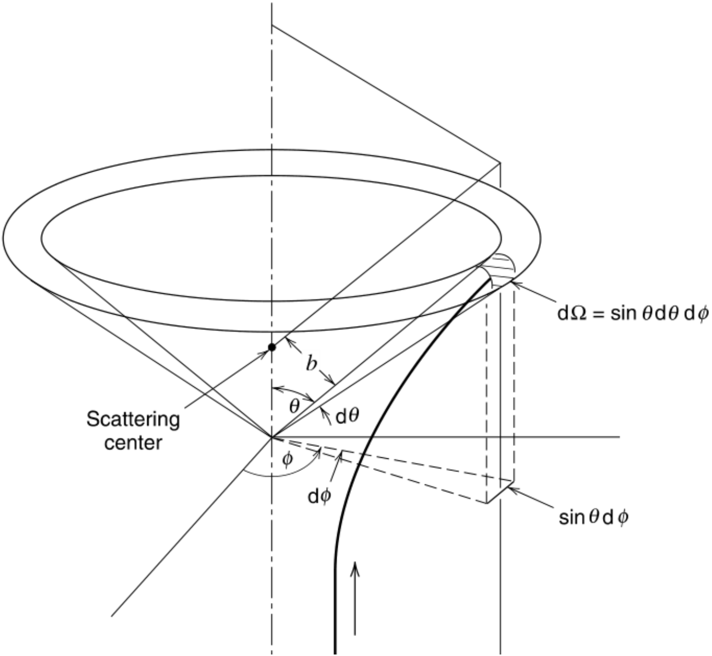
Definition of cross section for momentum transfer

$$\sigma_m = \int_0^{2\pi} \mathrm{d}\varphi \int_0^{\pi} (1 - \cos\theta) I(v, \theta) \sin\theta \mathrm{d}\theta \tag{1}$$

Reminder, the total cross section is

$$\sigma = \int_0^{2\pi} \mathrm{d}\varphi \int_0^{\pi} I(v, \theta) \sin\theta \mathrm{d}\theta \tag{2}$$

Where does the factor $1 - \cos\theta$ come from??



dΩ = sin θ dθ dφ
b
θ
dθ
Scattering
center
φ
dφ
sin θ d φ

Average change of velocity

$$\langle \Delta \vec{v} \rangle = \langle \vec{v}' - \vec{v} \rangle = \frac{1}{\sigma} \int_0^{2\pi} \mathrm{d}\varphi \int_0^{\pi} \Delta \vec{v} I(v, \theta) \sin\theta \mathrm{d}\theta \tag{3}$$

If an electron interacts with a heavy ion, its speed does not change too much:

$$v' \approx v \tag{4}$$

We split the velocity after the collision into a velocity component that is parallel and perpendicular to the velocity before the collision:

$$\vec{v}' = \vec{v}'_{\perp} + \vec{v}'_{\parallel} \tag{5}$$

Also, the angle between the velocities in the scattering angle θ:

$$\vec{v} \cdot \vec{v}' = vv' \cos\theta = v^2 \cos\theta \tag{6}$$

Then:

$$\vec{v}' - \vec{v} = \vec{v}'_{\perp} + \vec{v}'_{\parallel} - \vec{v} = \vec{v}'_{\perp} + \vec{v}(1 - \cos\theta) \tag{7}$$

Average change of velocity

$$\langle \Delta \vec{v} \rangle = \langle \vec{v}'_{\perp} \rangle + \langle \vec{v}(1 - \cos\theta) \rangle = \langle \vec{v}'_{\perp}(\theta, \varphi) \rangle + \vec{v} \langle (1 - \cos\theta) \rangle \quad (8)$$

For azimuthally symmetric potetnials, cross section does not depend on φ $I = I(\theta)$ and the perpendicular velocity will be distrubted symmetrically. For every vector of perpendicular velocity there will one vector with the same magnitude but opposite direction. The velocity integral through φ of the perpendicular velocity will be zero:

$$\langle \vec{v}'_{\perp} \rangle = 0 \quad (9)$$

Also see wikipedia pages for the particular integrals.

Last remarks

$$\langle \Delta \vec{v} \rangle = \vec{v} \langle (1 - \cos\theta) \rangle = \tag{10}$$

$$\frac{\vec{v}}{\sigma} 2\pi \int_0^\pi (1 - \cos\theta) I(v, \theta) \sin\theta \mathrm{d}\theta = \tag{11}$$

$$\vec{v} \frac{\sigma_m}{\sigma} \tag{12}$$

Relation between cross section can be written:

$$\sigma_m = \sigma \langle (1 - \cos\theta) \rangle \tag{13}$$