Example 3-2 Ionization fraction of air

Estimate the degree of ionization of air at 1 atm pressure and room temperature (300 K). Repeat the calculation for a temperature of 8000 K. The main component of air is nitrogen, with an ionization potential of 14.5 eV.

Saha equation

$$\frac{n_i^2}{n_n} = 2.405 \cdot 10^{21}\, T^{3/2} \mathrm{e}^{-\frac{U}{k_\mathrm{B} T}} \tag{1}$$

Common unit conversions in plasma physics

- $1\text{ eV} \approx 1.602 \cdot 10^{-19}\text{ J}$
- $1\text{ eV} = \frac{e}{k}\text{ K} \approx 11604\text{ K}$ [1]

8000 K of temperature is only about 0.6894 eV of energy.

[1] This just a unit conversion. A particle with temperature T will, generally, have a different mean energy (based on the particle's degrees of freedom) -> **Use this with caution!**

Ideal gas law

The gas consists of neutral and charged particles. Assume charged particles are positive ions and electrons. Assume all species have the same temperature T.

Then the ideal gas law is:

$$p = p_e + p_i + p_n = \frac{kT}{V}(N_e + N_i + N_n) = kT(n_e + n_i + n_n) \quad (2)$$

To maintain overall neutrality:

$$n_e + n_i = 2n_i \quad (3)$$

Combine every equation. . .

$$\frac{n_i^2}{n_n} = 2.405 \cdot 10^{21}\, T^{3/2} \mathrm{e}^{-\frac{U}{k_\mathrm{B} T}} \equiv A \tag{4}$$

$$n_n = \frac{p}{kT} - 2n_i \equiv b - 2n_i \tag{5}$$

$$\tag{6}$$

$$n_i^2 + 2An_i - Ab = 0 \tag{7}$$

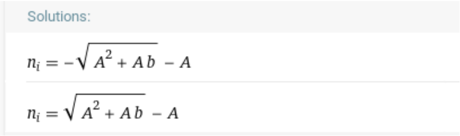

Figure 1: wolframalpha.com can solve this

In numbers..

For $T = 300$ K:

$$b = 2.5 \cdot 10^{25} \tag{8}$$

$$A = 3.2 \cdot 10^{-219} \tag{9}$$

$$n_i = 2.8 \cdot 10^{-97} \tag{10}$$

For $T = 8000$ K:

$$b = 9.2 \cdot 10^{23} \tag{11}$$

$$A = 1.3 \cdot 10^{18} \tag{12}$$

$$n_i = 1.1 \cdot 10^{21} \tag{13}$$

Degree of ionization

For $T = 300$ K:

$$\frac{n_i}{n_n + n_i} = 1.2 \cdot 10^{-122} \tag{14}$$

For $T = 8000$ K:

$$\frac{n_i}{n_n + n_i} = 0.0012 \tag{15}$$