# Plasma diagnostics and simulations

## Phase-resolved schlieren photography

## Goals of the experiment

In this experiment you will assemble and align a schlieren setup from laboratory components and use it to visualize development of repetitive plasma discharge. Upon completion of this experiment, you should:

- Have a fundamental understanding of the theory of schlieren and shadowgraph visualization techniques

- Have a working knowledge of schlieren and shadowgraph setups including placement and alignment of optics and practical issues involved with the individual components involved: mirrors, light source, cutoff, focusing optics, and camera. Given components, you should be able to set up your own schlieren

- Understand what types of phenomena are good candidates for schlieren and shadowgraph visualization

## Basic principles

This experiment deals with schlieren and shadowgraph visualization techniques, which translate phase speed differences in light, invisible to the eye, into changes in intensity which we perceive as regions of light and dark. A brief discussion of these techniques is presented here; for a more complete description the reader is referred to the famous book by Settles [**?**].

Change in phase speed as light passes through a transparent medium is called refraction.

The refractive index $n = c_0/c$ of a medium describes the change in phase speed where c is the speed of light in the medium and $c_0$ is the speed of light in a vacuum. For gases, the refractive index is linearly dependent on the gas density according to the Gladstone-Dale relation:

$$n = K\rho + 1, \tag{1}$$

where $K$ is the Gladstone-Dale constant and $\rho$ is the gas density. K is dependent on gas composition, temperature, and the wavelength of the light passing through the medium.

In gases $K$ is typically between $0.1\times10^{-3}$ and $1.5\times10^{-3}$ $(\text{kg/m}^3)^{-1}$. $K\rho$ is therefore much smaller than 1, in fact the refractive index varies only in the third or fourth decimal place. So very sensitive optics is required to detect changes in density in gases.

On the other hand the refractive indices of transparent liquids and solids are quite higher than of gases, so much less optical sensitivity is required to observe refractions in liquids or solids than gases. Actually 'schlieren' (from German; singular "Schliere", meaning "streak") was originaly a method how to look for optical inhomogeneities in transparent material not necessarily visible to the human eye (eg. defects in lenses).

There is a dispute whereas to write 'schlieren' or 'Schlieren' with initial capital letter or not. Logically it is not a name so it should be written without capitals on the other hand in German language all nouns start with capital letter and this word have been adopted from German so... use what you want just be consistent.

Gas flows with changing density are collectively called compressible flows, and compressibility can be produced by temperature variations and/or appreciable Mach number such that $M^2$ is not substantially smaller than 1. Mach number is a dimensionless quantity representing the ratio of flow velocity past a boundary to the local speed of sound:

$$M = \frac{u}{c} \tag{2}$$

Where $M$ is the Mach number, $u$ is the local flow velocity with respect to the boundaries and $c$ is the speed of sound in the medium.

A light ray passing orthogonally through a change in refractive index will experience a change in phase velocity but will continue traveling in the same direction. If the light ray intersects the change in refractive index obliquely, however, it will bend towards the region with greater n. For a light ray travelling in the z-direction intersecting a region of varying refractive index the ray curvature is given by

$$\frac{\partial^2 x}{\partial z^2} = \frac{1}{n}\frac{\partial n}{\partial x} \tag{3}$$

Here x is used to represent a direction orthogonal to the ray propagation. Integrating once, the angular ray deflection is

$$\theta = \frac{1}{n} \int \frac{\partial n}{\partial x} \partial z \tag{4}$$

From Equation 4 we see that ray deflection is dependent not only on the refractive index of the medium but mainly on the gradients in n orthogonal to the ray propagation direction. Because of the integral in z, deflection is also quite sensitive to the extent of the region of varying density in the z-direction. These deflections are what are made visible by **schlieren** and **shadowgraph** techniques.

**Schlieren** visualization and the **shadowgraphy** are similar techiques. The primary difference is that while shadowgraphs are sensitive to changes in the second derivative in density, schlieren systems detect changes to the first derivative in density.

# General implementation of schlieren or shadowgraph

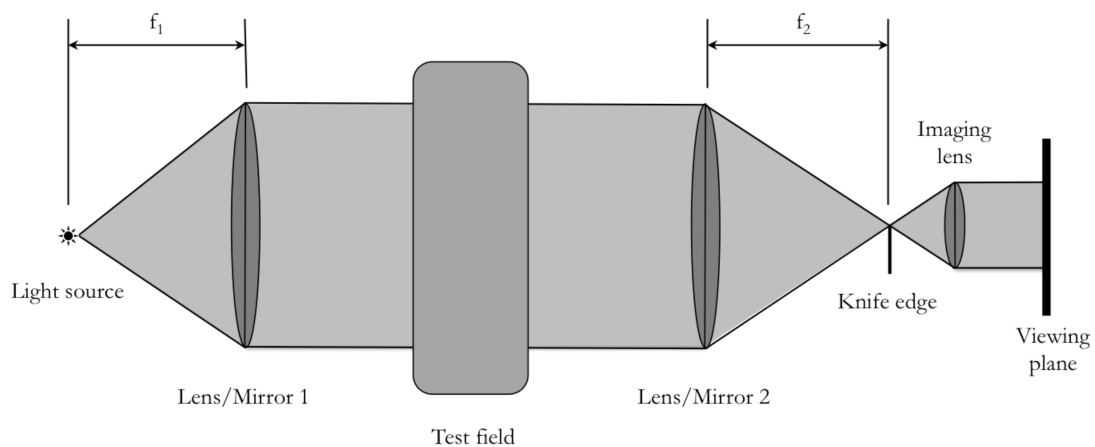Figure 1 shows a schematic of a typical schlieren setup.



Figure 1: Schematic of a typical schlieren setup.

Light from a small source is collimated (made parallel) by a focusing mirror or lens and passes through a test field with varying refractive index. The light source must be small to avoid blur in

the image, which is given by $ld/f_1$ where $l$ is the distance of the observation plane from the optical disturbance and depends on the physical size of the camera detector, $f_1$ is the focal length of the first focusing mirror or lens, and $d$ is the size of the light source. The source cannot be too small, however, because very small light sources suffer from a lack of sharpness due to diffraction. The parallel light is then refocused by a second lens or mirror and imaged by a camera or screen. Third lens/mirror may or may not be added, they serve only for purposes of changing the dimensions of resulting image - to fit the image on a screen or camera chip.

If the second derivative of density is not constant, i.e. $\frac{\partial \rho^3}{\partial y^3} \neq 0$, the region where this occurs will appear dark, or as a shadow (thus shadowgraphy). Figure 2 illustrates the creation of shadows from environments with different derivatives.
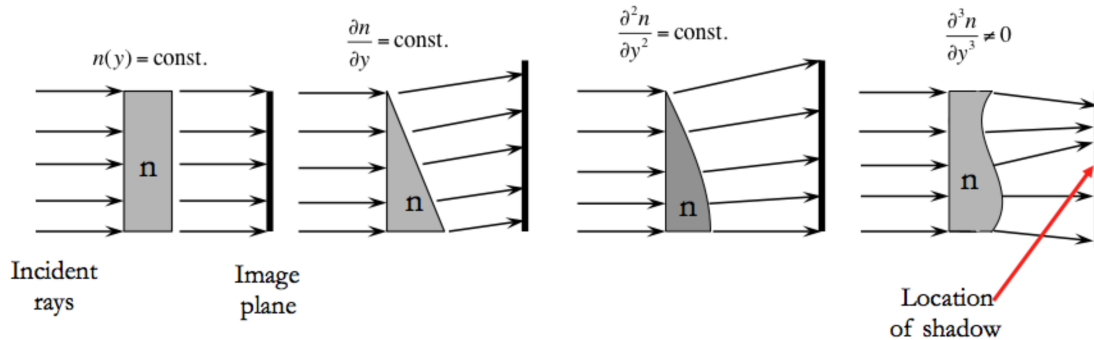


Figure 2: Schematic showing the refraction of light rays by density fields with density and its first, second, and third derivatives constant. The two-dimensional description is easily extended to three dimensions.

Now we add the knife edge at the focal point of the second lens or mirror. Undeflected light rays are affected uniformly by the knife edge and the amount of light blocked by the knife edge is commonly referred to as "cutoff." The overall intensity of the image is then reduced with the increase of the cutoff. As light passes through a density field with a non-constant first derivative, the light rays are deflected as shown in 2. If light rays are deflected towards the knife edge, the part of the image where those light rays originate from will be darkened more than a part of the image with constant density. Conversely, if light rays are deflected away from the knife edge, that part of the image will appear brighter than unaffected regions of the image. Thus schlieren setup are only sensitive to density gradients normal to the knife edge.

The primary advantage of schlieren visualization over shadowgraph visualization is that of sensitivity. The sensitivity of a shadowgraph setup depends primarily upon optical path length, which is typically difficult and expensive to modify. The sensitivity of a schlieren setup, however, mainly depends on the amount of cutoff used. Higher cutoff leads to decreased brightness in the image while the intensity of deflected rays stays same.

In practice, many variations of the basic schlieren setup described here are used. By far the most common is the Z-type schlieren setup, an almost universal standard see fig. 3. Focusing mirrors provide a much larger field of view for the same price as focusing lenses, so the setup is bent into a "Z" shape to utilize the mirrors. Besides the cost advantage, this also saves space in the laboratory.

A Z-type schlieren setup does come at a price, however. The nonlinear alignment of the optics create two effects that distort the image, coma and astigmatism. Coma can be eliminated by careful arrangement and alignment of the focusing mirrors. Astigmatism cannot be fully eliminated, but by using a slit light source aligned with the knife edge it can be minimized until it is no longer visible. Other variations typically involve the knife edge modification, such as using a circular or double knife edge to eliminate the reversal in brightness of conventional schlieren images. Circular knife edges have the added benefit of being sensitive to density gradients in all
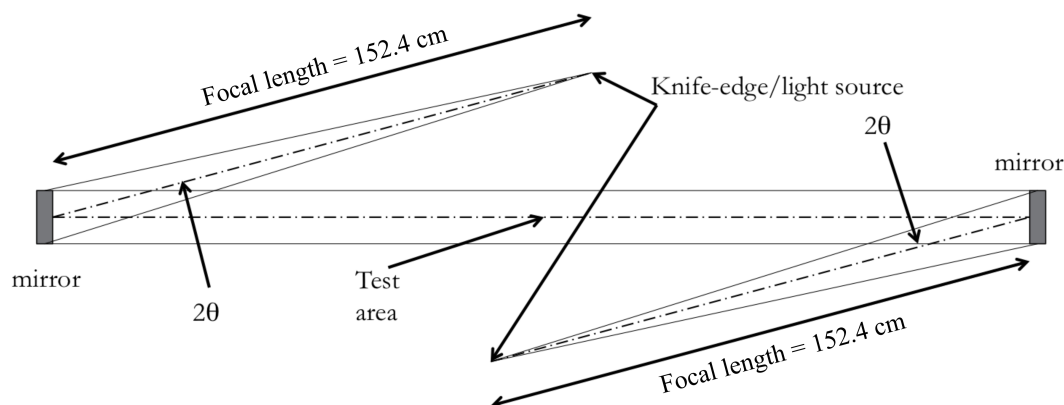
Figure 3: Recommended layout for the schlieren system.

directions. A further modifications are for example to use color strip filters instead of a knife edge, which results in colored schlieren images with the color corresponding to the strength of the change in density gradient.

## Laboratory implementation of schlieren setup

The first task is to assemble and align the schlieren setup. This may seem tedious and might take some time to attain good alignment and focus, but a well-aligned setup will make all the difference. The primary components for the schlieren setup in this experiment are two spherical focusing mirrors. These mirrors have a focal length of 60 inches (152.4 cm). Figure 3 shows a sketch of a recommended layout for the optics. A functional overall length for the system is based on the mirrors and the space available on the optical table. This length between mirrors does not need to be exact as we assume there is parallel optical beam. A recommended entrance/exit angle for the beams is somewhere between $2\theta = 10^o$ and $2\theta = 20^o$ , as shown in the figure. This also does not need to be exact, but **it is crucial that the system is symmetric**.

**Tips and tricks**

- It is best to measure some angle between 10 and 20 degrees relative to the holes on the optical table for this line to make the alignment of other optics easier.

- Remember that the mirrors will need to be raised off the table in order to make their centerline coincide with the position of plasma. If you find this too late you have to start again from scratch.

- Recommended assembly order - mirrors, light source, screen/camera, knife edge as last.

- Key instrument for alignment: a clean sheet of white paper. To make alignment easier, draw a 6"-diameter circle on the paper to designate the size of the test beam should be **same** along the whole inter-mirror space. If not, your light source is too far or too close.

- Assemble your own light source if necessary, we will tell you how.

**Do not be afraid to take everything apart and start from scratch if you cannot get a fully-illuminated, well-focused image at the camera.** Your images will only be as good as the alignment of your system. Once the system is properly aligned and focused, slowly bring the knife-edge into the beam as you watch the image on the screen or camera. When the knife-edge is perfectly aligned with the focus, the brightness of the image will change uniformly

as the knife-edge is moved in and out of the beam. If the image darkens on the same side as the knife-edge, the knife-edge is too close to the second focusing mirror. If the opposite occurs, it is too far away. Adjust the position of the knife-edge until the image darkens uniformly as the knife-edge is moved into the light beam. **Remember that a shadowgraph image will be produced with zero cutoff!**

At this point your schlieren system should be aligned! There are multiple ways to test it, but among the most popular are rubbing one's hands together and placing a hand in the test area and observing the thermal plume from the hand, breathing into the test region and viewing the gradients created by exhaling, or blowing into the test region with a compressed gas duster. Flames from matches or cigarette lighters work well too. At this point you may also want to practice recording single images and short movies with the camera while adjusting the frame rate and resolution.

## Application on plasma

You will get simple repetitive spark discharge at your disposal - Marx generator that is commonly used to ignite gas burner. Your goal is to use ICCD camera triggered by the discharge pulse in order to measure time development of the spark using schlieren and shadowgraph. Try to reason what variables can be obtained from such images and how to exctract them (eg. estimate the speed of the shock-wave propagation).

## References

[1] Settles, Gary S. Schlieren and shadowgraph techniques: visualizing phenomena in transparent media. Springer Science & Business Media, 2012.