GB471
Stabilita a dynamika přírodních systémů

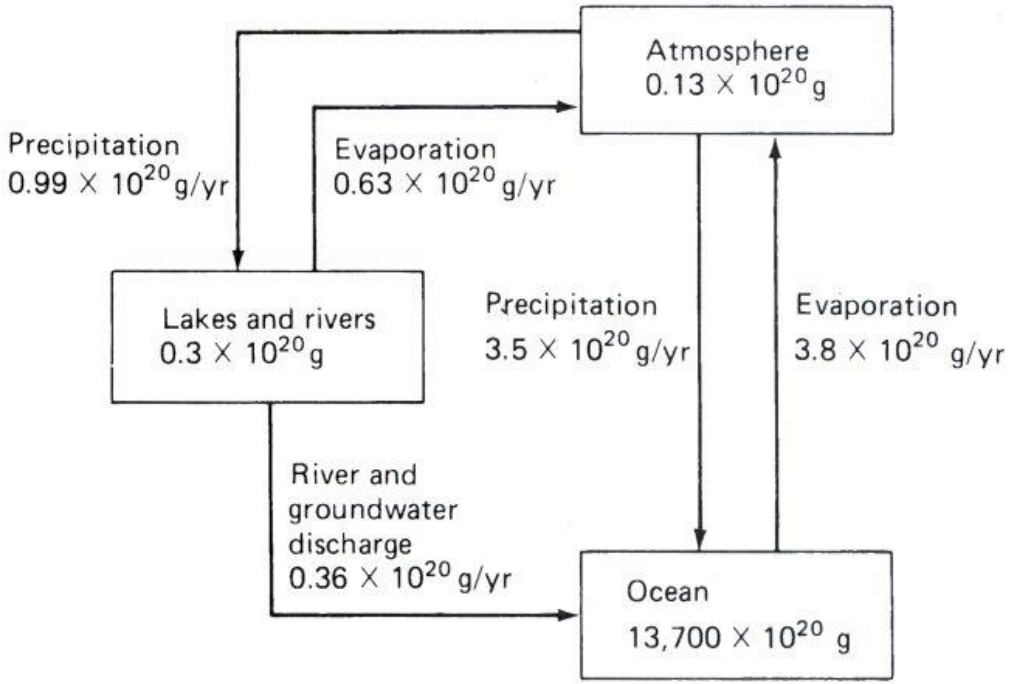
Josef Zeman

2020

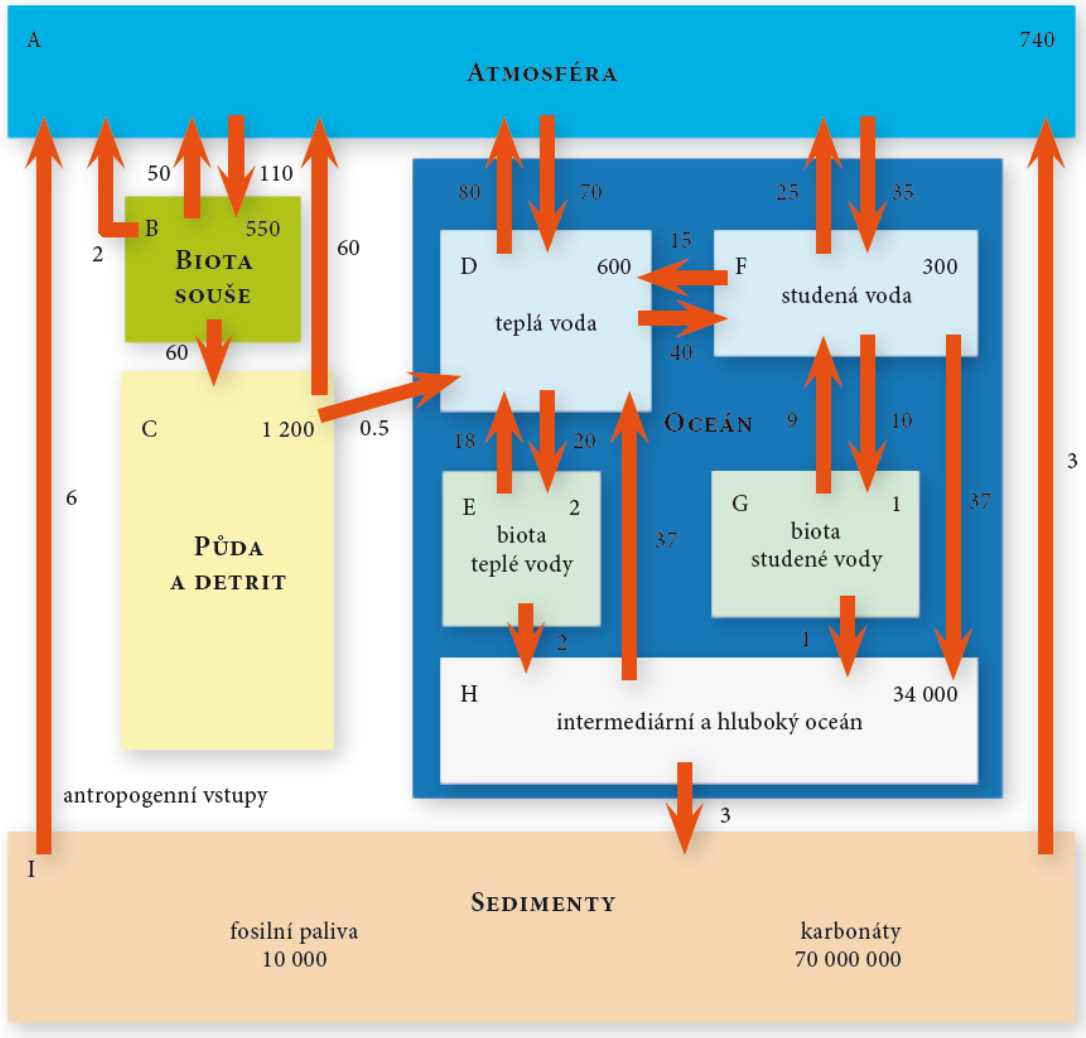
3 Dynamika

Koncepce rezervoárů

Zemský systém

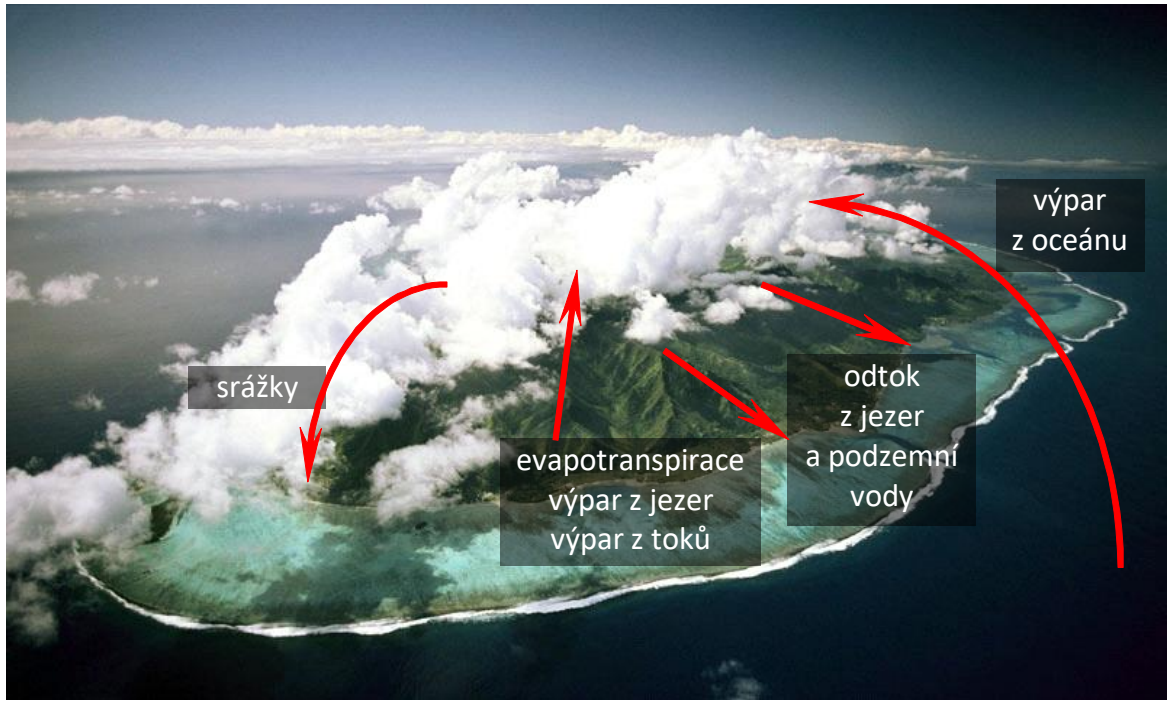


Hydrogeologický systém

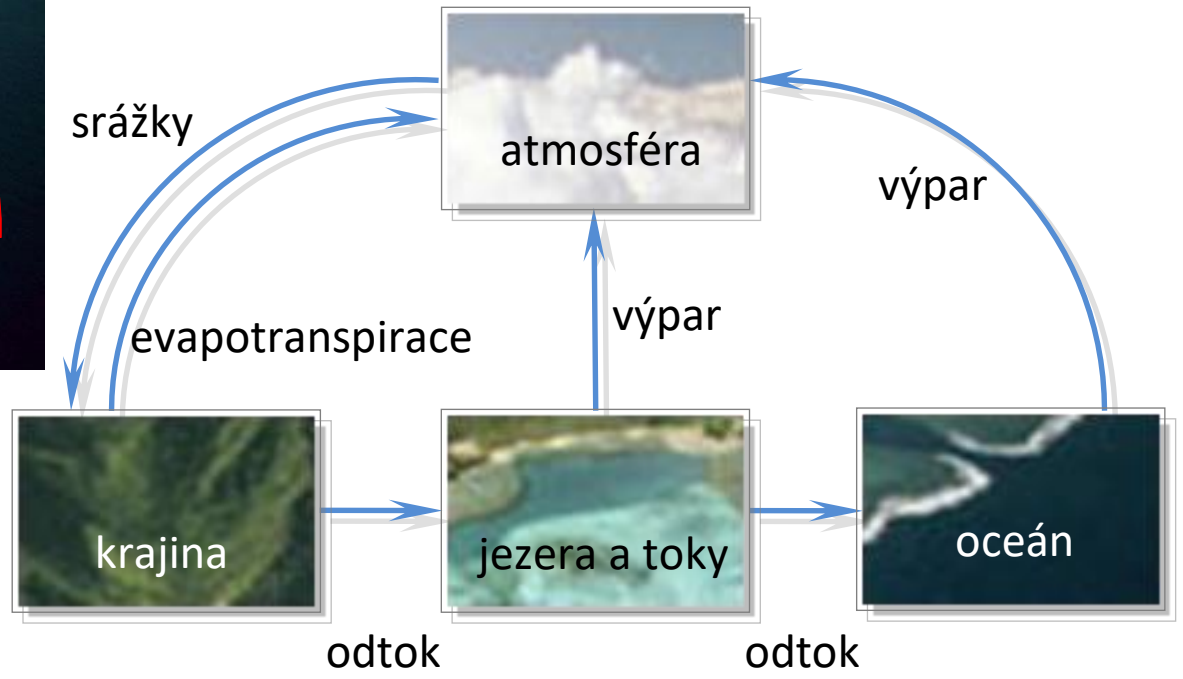


Koncepce rezervoárů

Reálný systém

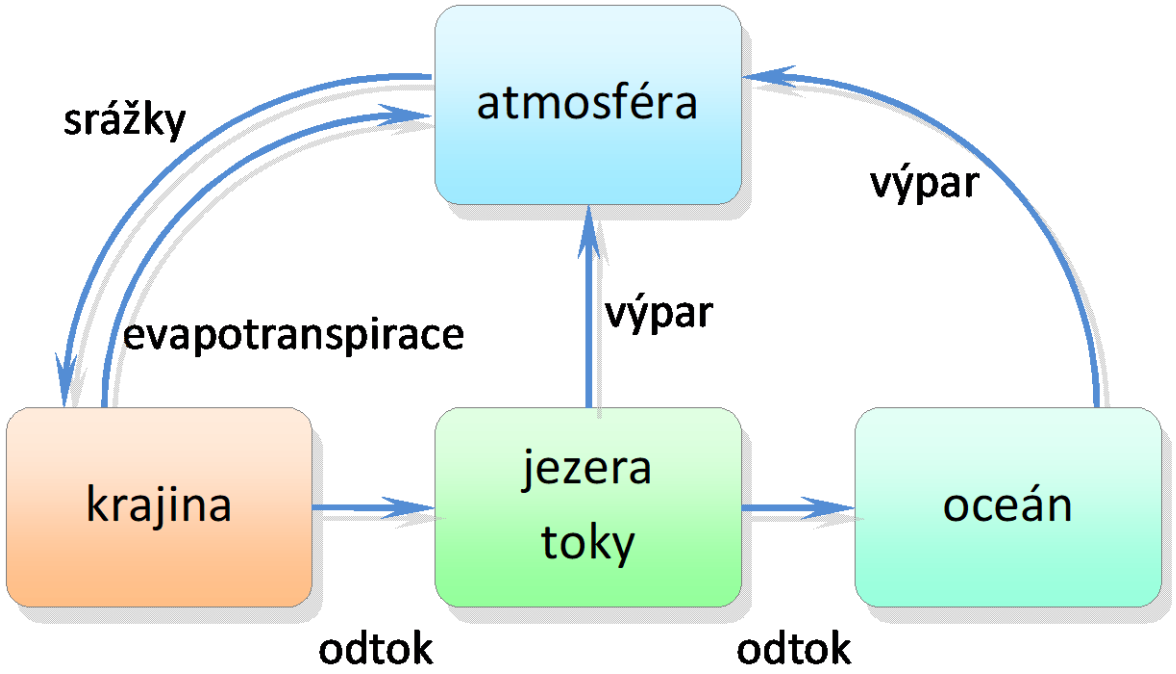


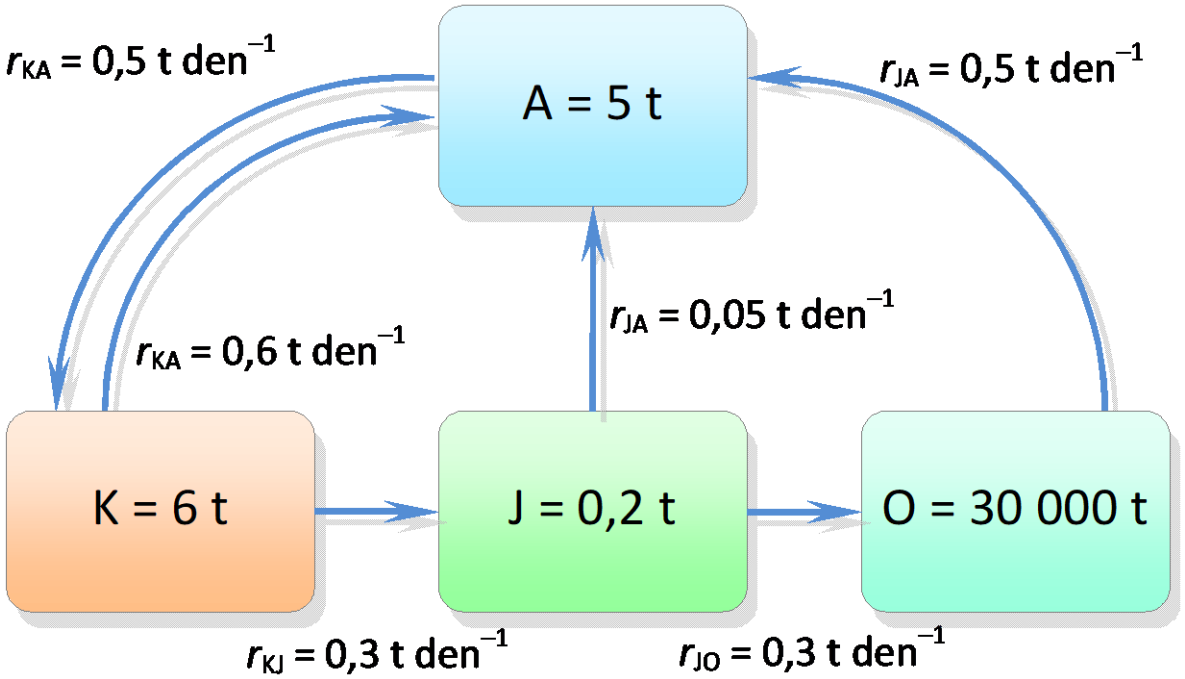
Rezervoárový model



Koncepce rezervoárů

Rezervoárový model





Koncepce rezervoárů

Hranice – fyzická, ale také pouze virtuální

$$\mathrm{CO_2\,(aq) + H_2O \underset{r_{-1}}{\overset{r_{+1}}{\rightleftharpoons}} H_2CO_3 \underset{r_{-2}}{\overset{r_{+2}}{\rightleftharpoons}} H^+ + HCO_3^- \underset{r_{-3}}{\overset{r_{+3}}{\rightleftharpoons}} H^+ + CO_3^{2-}}$$

rychlost $r_{\mathrm{Y}} = kX$

$r_{\mathrm{jo}} = k_{\mathrm{jo}} m_{\mathrm{j}}$

toky lineární $r_{+1} = k_{+1} a_{\mathrm{CO_2(aq)}} a_{\mathrm{H_2O}}$

toky nelineární $r_{\mathrm{Y}} = kX^2$

výsledný tok $r_{\mathrm{výsledný}} = \sum_i r_i - \sum_j r_j$

časová změna obsahu rezervoáru $\dfrac{dm_{\mathrm{C}}}{dt} = \sum_i r_i - \sum_j r_j$

Jednorezervoárový systém

Časový vývoj X

$$r_{vstup} = A$$

$$r_{výstup} = kX$$

$$r_{výsledný} = r_{vstup} - r_{výstup}$$

$$\frac{dX}{dt} = r_{vstup} - r_{výstup}$$

$$\frac{dX}{dt} = A - kX$$

$$\frac{dX}{A - kX} = dt$$

$$\int_{X_0}^{X} \frac{dX}{A - kX} = \int_0^t dt$$

$$\ln(A - kX) - \ln(A - kX_0) = -k(t - 0)$$

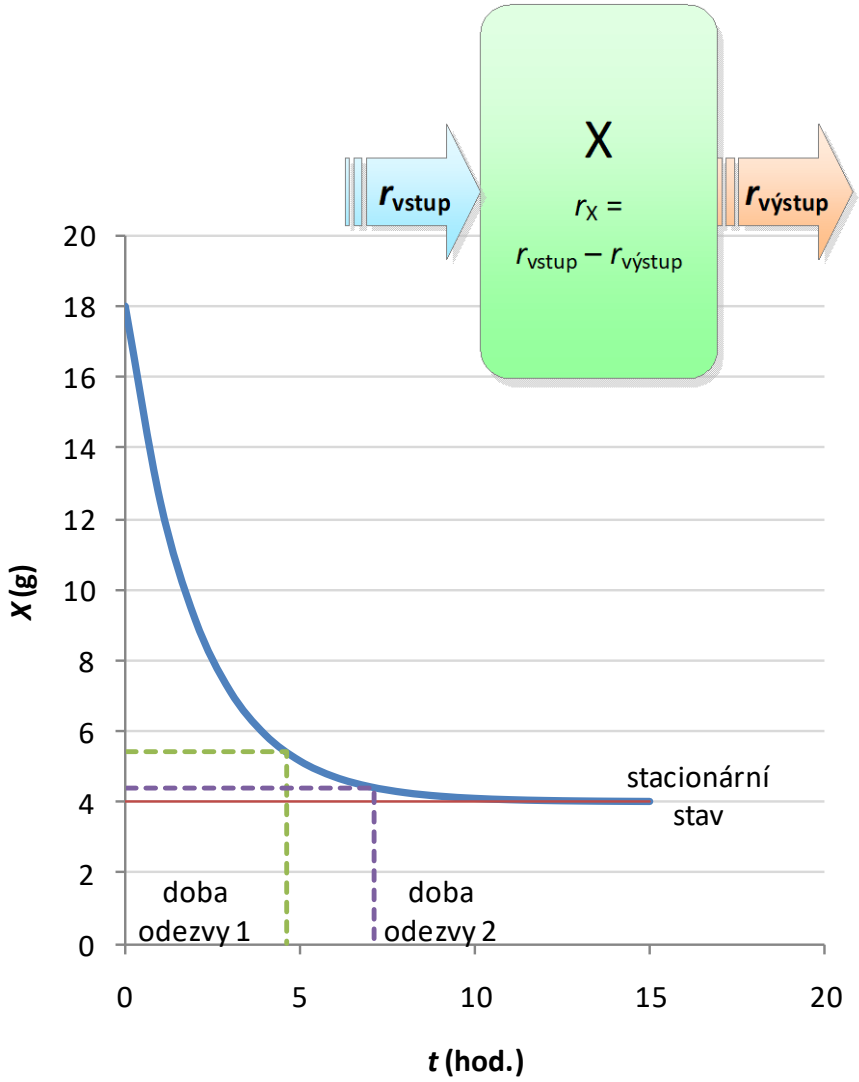
$$\ln \frac{A - kX}{A - kX_0} = -kt$$

$$\frac{A - kX}{A - kX_0} = e^{-kt}$$

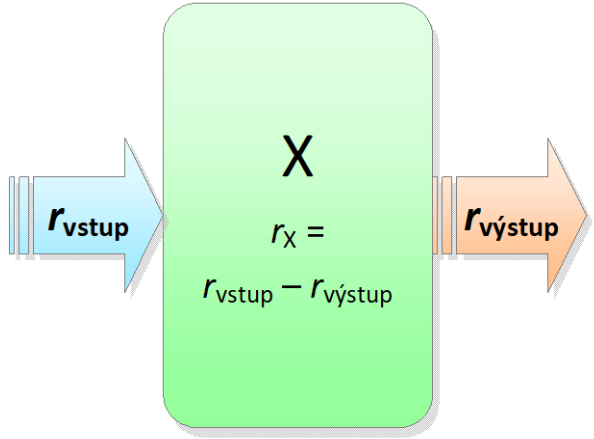
$$A - kX = (A - kX_0)e^{-kt}$$

$$-kX = (A - kX_0)e^{-kt} - A$$

$$X = \frac{A}{k} + \left(X_0 - \frac{A}{k}\right)e^{-kt}$$



Jednorezervoárový systém



Stacionární stav

$$r_{vstup} = r_{výstup}$$

$$r_{výsledný} = \frac{dX}{dt} = A - kX_S = 0$$

$$X_S = \frac{A}{k}$$

$$X = \frac{A}{k} + \left(X_0 - \frac{A}{k} \right) e^{-kt}$$

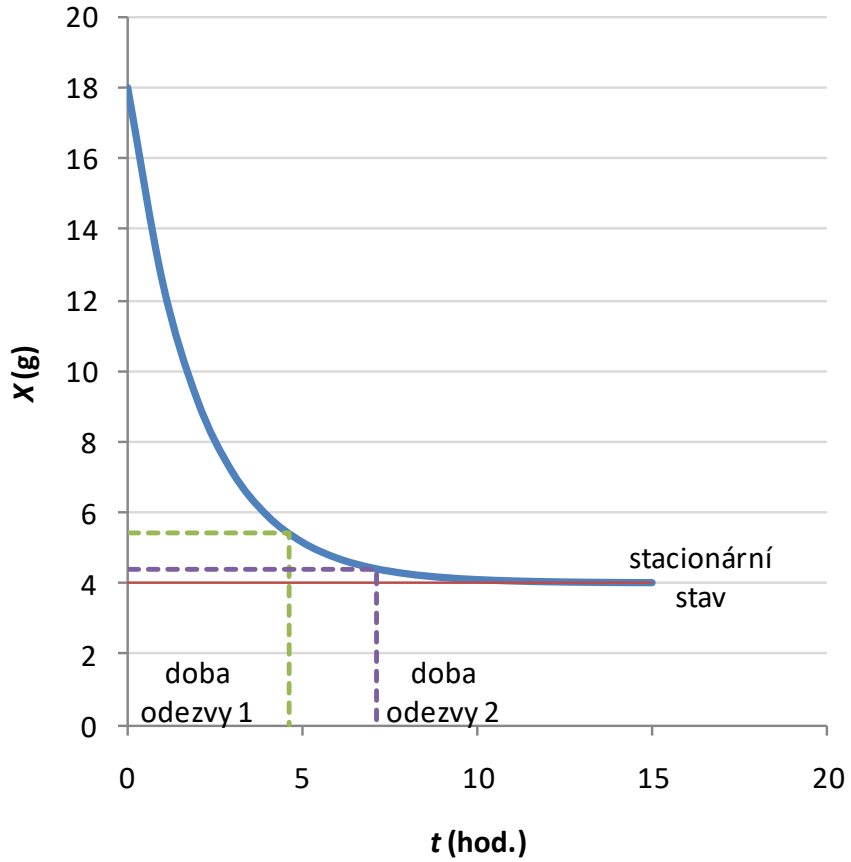
$$X = X_S + \left(X_0 - X_S \right) e^{-kt}$$

Doba zdržení

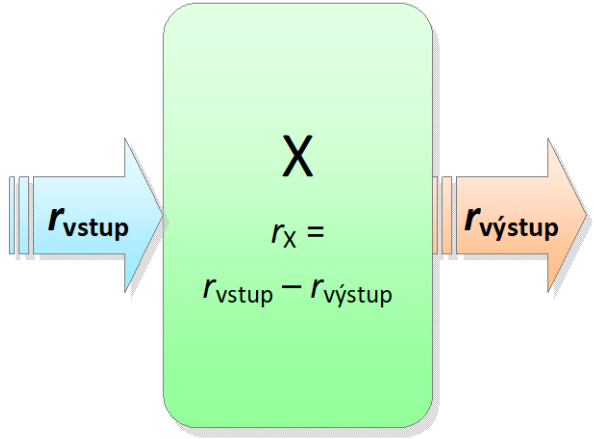
$$t_{zdržení} = \frac{X_S}{r_{vstup}} = \frac{X_S}{r_{výstup}}$$

$$t_{zdržení} = \frac{X_S}{A} = \frac{\frac{A}{k}}{A} = \frac{1}{k}$$

$$t_{zdržení} = \frac{X_S}{kX_S} = \frac{1}{k}$$



Jednorezervoárový systém



Doba odezvy (response time)

absolutní relativní

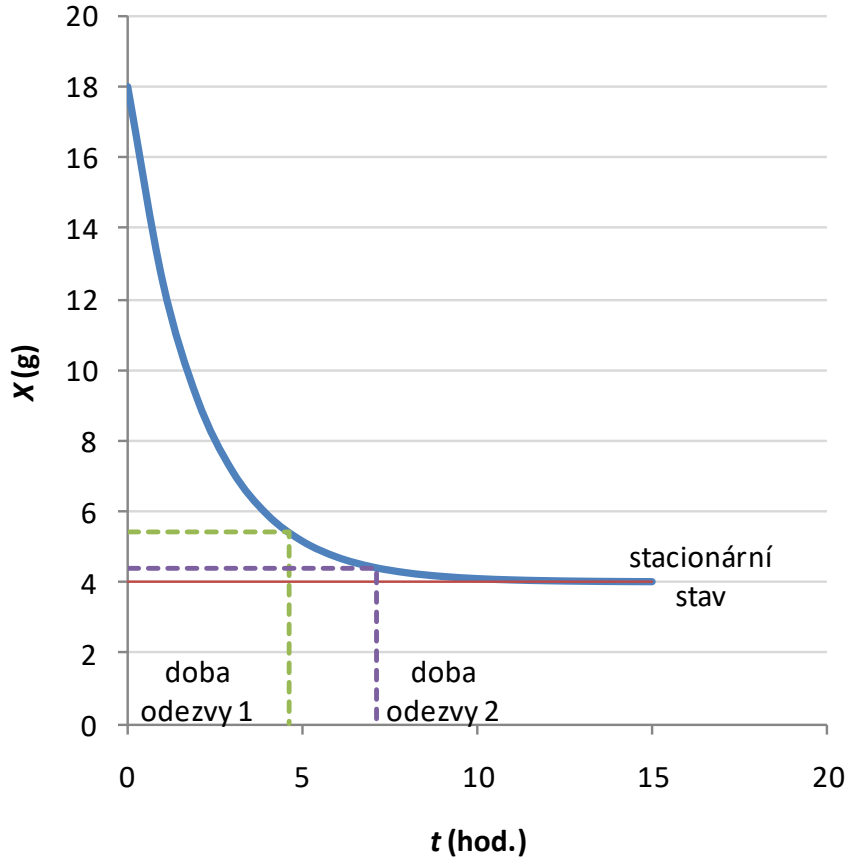
$$X = \frac{A}{k} + \left(X_0 - \frac{A}{k} \right) e^{-kt}$$

$$\left(X_0 - \frac{A}{k} \right) e^{-kt} = \left(X_0 - X_S \right) e^{-kt}$$

$$e^{-kt_{odezvy}} = 0{,}1$$

$$-kt_{odezvy} = \ln 0{,}1$$

$$t_{odezvy} = -\frac{\ln 0{,}1}{k} = \frac{2{,}303}{k}$$



Jednorezervoárový systém

Doba odezvy (response time)

relativní

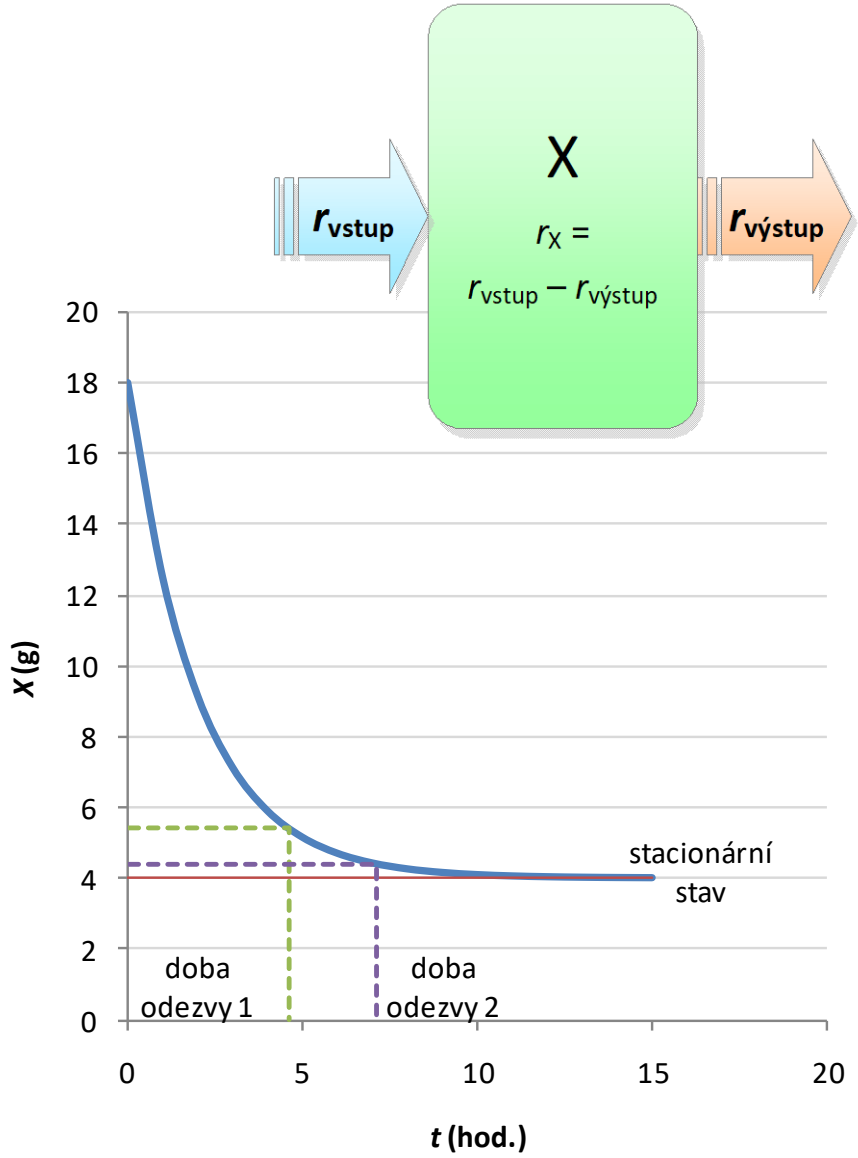
$$X = \frac{A}{k} + \left(X_0 - \frac{A}{k} \right) e^{-kt}$$

$$\Delta X_{rel} = \frac{velikost\ odchylky}{stacionární\ stav} = \frac{(X_0 - X_S)e^{-kt}}{X_S} = \left(\frac{X_0}{X_S} - 1 \right) e^{-kt}$$

$$0{,}1 = \left(\frac{X_0}{X_S} - 1 \right) e^{-kt_{odezvy}} \qquad \frac{0{,}1X_S}{X_0 - X_S} = e^{-kt_{odezvy}}$$

$$0{,}1 = \left(\frac{X_0 - X_S}{X_S} \right) e^{-kt_{odezvy}} \qquad \ln \frac{0{,}1X_S}{X_0 - X_S} = -kt_{odezvy}$$

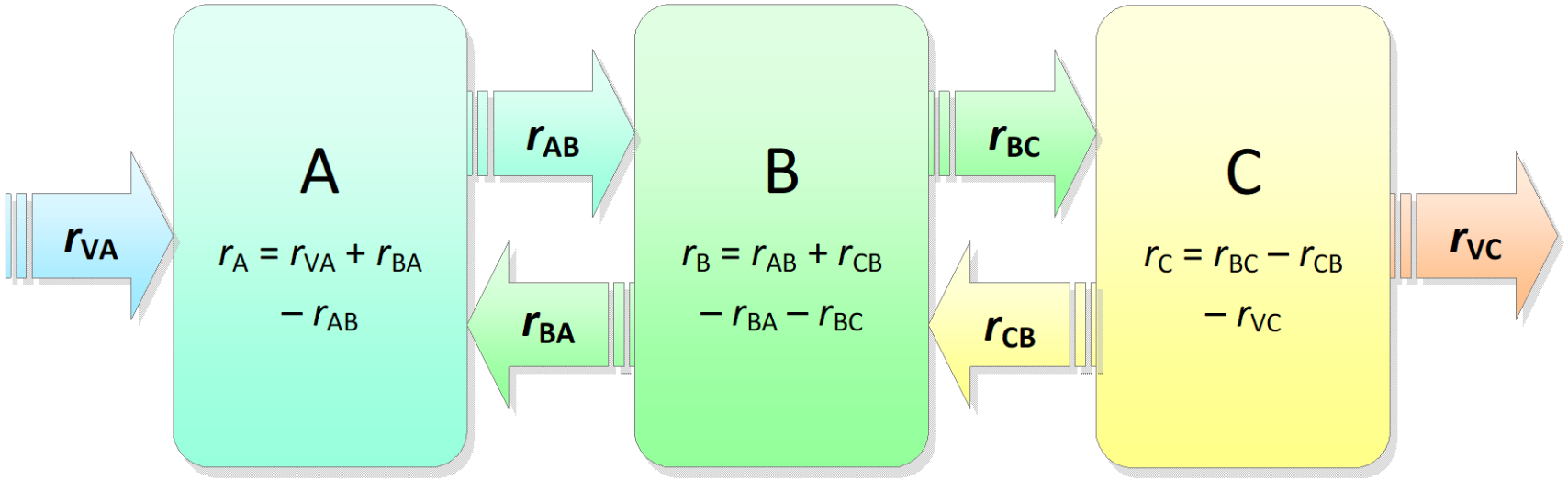
$$t_{odezvy} = \frac{1}{k} \ln \frac{X_0 - X_S}{0{,}1X_S}$$



Třírezervoárový SYSTÉM

Vývoj koncentrací (obsahů)

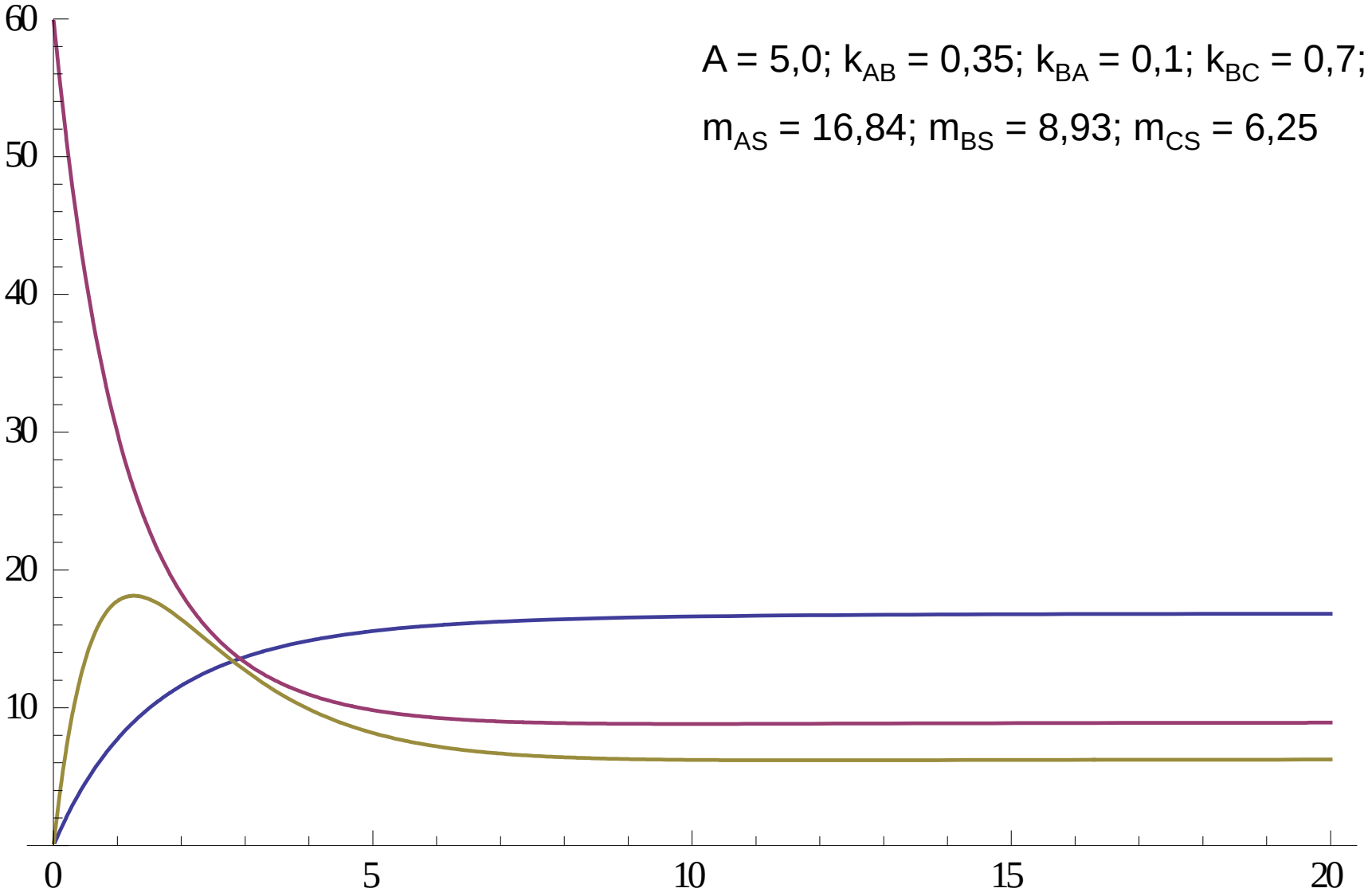
$$\rightarrow CO_2\,(aq) + H_2O \underset{r_{-1}}{\overset{r_{+1}}{\rightleftharpoons}} H_2CO_3 \underset{r_{-2}}{\overset{r_{+2}}{\rightleftharpoons}} H^+ + HCO_3^- \underset{r_{-3}}{\overset{r_{+3}}{\rightleftharpoons}} H^+ + CO_3^{2-} \rightarrow$$



$$r_{VA} = A \qquad r_{BA} = k_{BA} m_B \qquad r_{CB} = k_{CB} m_C$$

$$r_{AB} = k_{AB} m_A \qquad r_{BC} = k_{BC} m_B \qquad r_{VC} = k_{VC} m_C$$

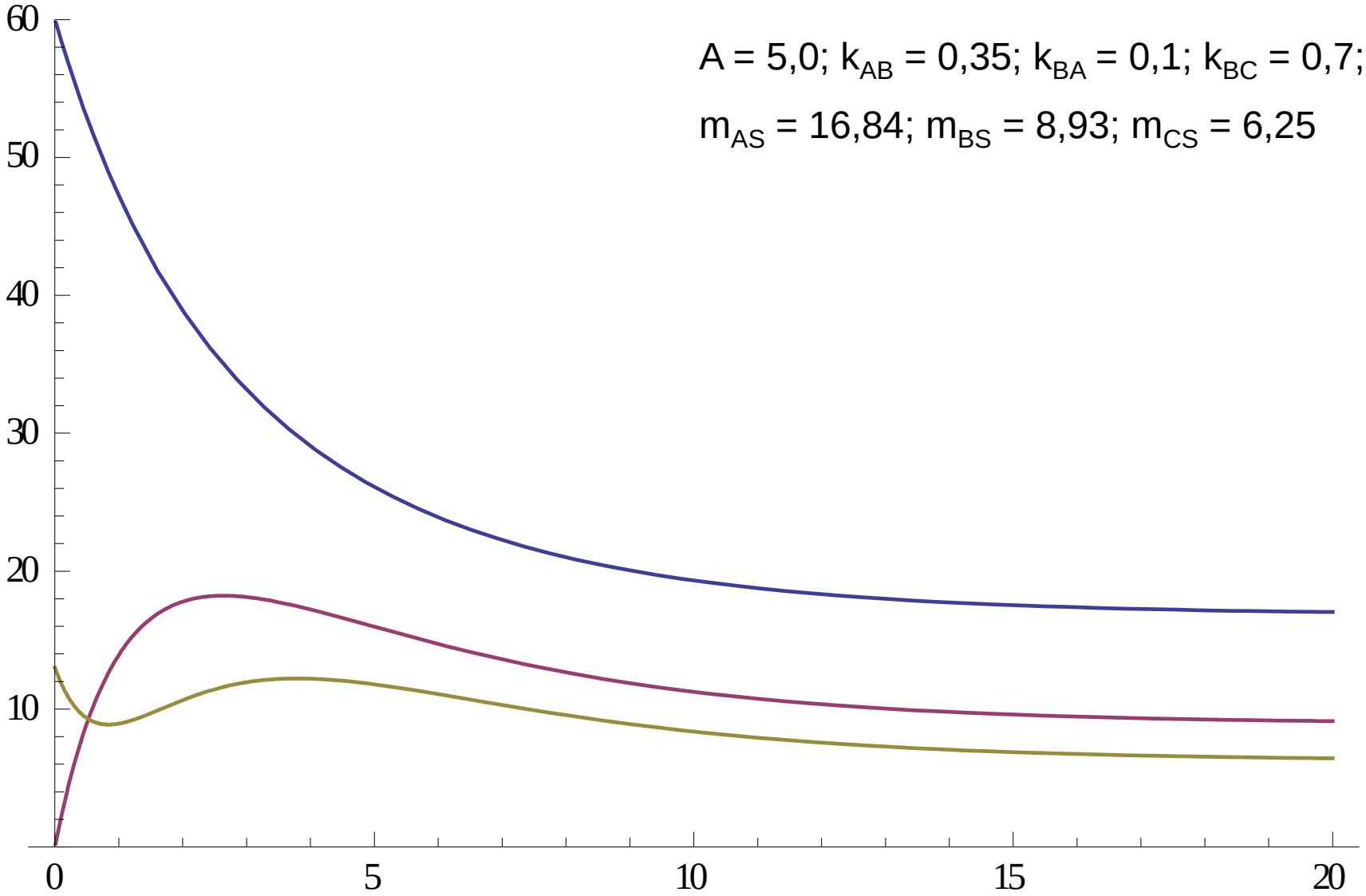
Třírezervoárový systém



$A = 5{,}0$; $k_{AB} = 0{,}35$; $k_{BA} = 0{,}1$; $k_{BC} = 0{,}7$; $k_{CB} = 0{,}2$; $k_{VC} = 0{,}8$

$m_{AS} = 16{,}84$; $m_{BS} = 8{,}93$; $m_{CS} = 6{,}25$ $\quad$ $m_{A0} = 0$; $m_{B0} = 60$; $m_{C0} = 0$

Třírezervoárový systém



$A = 5{,}0$; $k_{AB} = 0{,}35$; $k_{BA} = 0{,}1$; $k_{BC} = 0{,}7$; $k_{CB} = 0{,}2$; $k_{VC} = 0{,}8$

$m_{AS} = 16{,}84$; $m_{BS} = 8{,}93$; $m_{CS} = 6{,}25$ $\quad$ $m_{A0} = 0$; $m_{B0} = 60$; $m_{C0} = 0$

Modelový příklad

viz Dynamika cyklu Ostrov