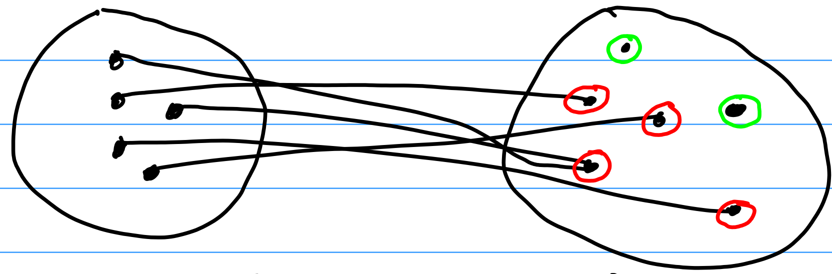
Konzultace 1.12. $\qquad f: M \to P$

Obraz zobrazení

$M \qquad\qquad P$



$\operatorname{im} f = \{ p \in P,\ \exists m \in M \ \ f(m) = p \}$

$\cos: \mathbb{R} \to \mathbb{R}$

$\operatorname{im} \cos = \{ y \in \mathbb{R},\ \exists x \in \mathbb{R} \ \ \cos x = y \}$
$= [-1, 1]$

Jádra definujeme pouze pro lin. zobrazení

$$\varphi: U \to V$$

$$\ker \varphi = \{ u \in U,\ \varphi(u) = \vec{o} \}$$

Vzor množiny v zobrazení

$$f: M \to P \qquad Q \subseteq P$$

Vzor množiny Q: $f^{-1}(Q) = \{ m \in M,\ f(m) \in Q \}$

$$\ker \varphi = \varphi^{-1}(\{\vec{o}\}).$$

$$f: M \to P \qquad g: P \to S \qquad g \circ f: M \to S$$

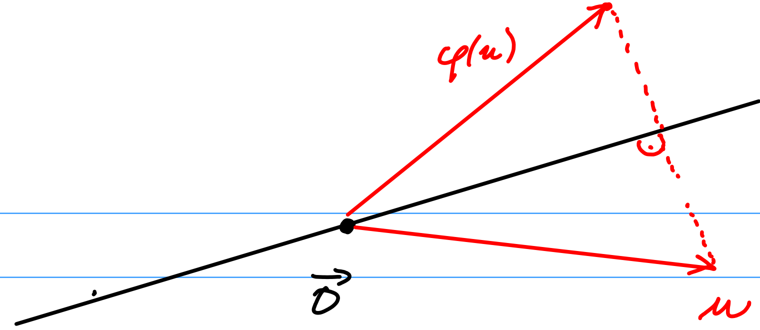
$m \in M$: $(g \circ f)(m) = g(f(m))$.

$$\begin{tikzcd} M \arrow[r, "f"] \arrow[rr, bend right, "g \circ f"'] & P \arrow[r, "g"] & S \end{tikzcd}$$

$f: \mathbb{R}^n \to \mathbb{R}^k \qquad f(x) = Ax$
$g: \mathbb{R}^k \to \mathbb{R}^l \qquad g(y) = By$

$g \circ f: \mathbb{R}^n \to \mathbb{R}^l \qquad (g \circ f)(x) = g(f(x)) = g(Ax) = B(Ax)$
$= \underline{(B \cdot A)\, x}$

Symetrie podle roviny

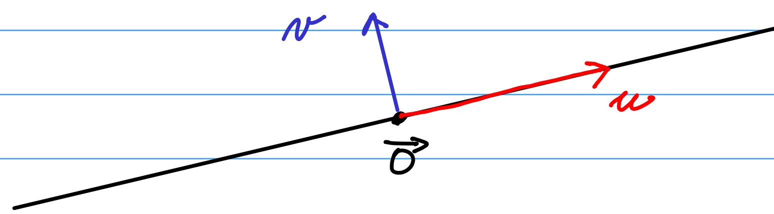


φ je symetrie podle přímky procházející počátkem.

φ je lineární zobrazení

Stačí najít k obrazy nějaké báze, tím je φ určena na všech dalších vektorech.

$\mathbb{R}^2$



$\varphi(u) = u$

$\varphi(v) = -v$

$u \longmapsto u$

$v \longmapsto -v$ z toho určíme matici A 2×2

$$\varphi(x) = Ax = A\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\left(\begin{array}{c|c} u & \varphi(u) \\ v & \varphi(v) \end{array}\right) \xrightarrow{\text{řádk.}} \left(\begin{array}{c|c} cu & c\varphi(u) = \varphi(cu) \\ \underline{v+u} & \varphi(v)+\varphi(u) = \underline{\varphi(v+u)} \end{array}\right)$$

A $\qquad \varphi(x) = Ax$ $\quad | \quad \varphi(e_1) = A\cdot\begin{pmatrix}1\\0\end{pmatrix} = 1.$ sloupec A

$\varphi(e_2) = A\cdot\begin{pmatrix}0\\1\end{pmatrix} = s_2(A)$

$$\left(\begin{array}{c|c} u & \varphi(u) \\ v & \varphi(v) \end{array}\right) \xrightarrow{\text{ERO}} \left(\begin{array}{c|c} e_1 & \varphi(e_1) \\ e_2 & \varphi(e_2) \end{array}\right) \qquad A = \left(\varphi(e_1)^T \; \varphi(e_2)^T\right)$$

E

Symetrie podle roviny $\varphi : \mathbb{R}^3 \to \mathbb{R}^3$

2 vektory v rovině proch. počátkem LN

3. vektor kolmý na rovinu $2x_1 - 3x_2 + x_3 = 0$

kolmý vektor $(2, -3, 1)$.

$\varphi : \mathbb{R}_{50}[x] \longrightarrow Mat_{5\times5}(\mathbb{R})$

$\dim \operatorname{im} \varphi = 10$

$\underline{a_{50}}\, x^{50} + \underline{a_{49}}\, x^{49} + \dots + \underline{a_1} x + \underline{a_0}$ $\qquad \dim \mathbb{R}_{50}[x] = 51$

$$Mat_{5\times5}(\mathbb{R}) = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{15} \\ - & - & - & - \\ b_{51} & & & b_{55} \end{pmatrix} \qquad \dim Mat_{5\times5}(\mathbb{R}) = 25$$

Podprostor v $Mat_{5\times5}(\mathbb{R})$ dimenze 10

$$Q = \left\{ \begin{pmatrix} c_1 & c_2 & c_3 & c_4 & c_5 \\ c_6 & c_7 & c_8 & c_9 & c_{10} \\ & & 0 & & \end{pmatrix} \in Mat_{5\times5}(\mathbb{R}), \right\}$$ podprostor dim 10

Báze $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & & & & \\ & & 0 & & \end{pmatrix}$ $\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ & & 0 & & \end{pmatrix}$ $\dots$ $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ & & 0 & & \end{pmatrix}$

$$\varphi(a_{50}x^{50} + \dots + a_1 x + a_0) = \begin{pmatrix} a_0 & a_1 & \dots & a_4 \\ a_5 & a_6 & \dots & a_9 \\ & & 0 & \end{pmatrix}$$

$\operatorname{im} \varphi$ má dim 10, určí báze je

$$\varphi(p) = \begin{pmatrix} p(1) & p(2) & \dots & p'''(3) \\ & & & \end{pmatrix}$$

$$\bullet\ \varphi(p) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ - & & - & & - \end{pmatrix} \quad \text{pro } \forall p \geq 0$$

$\mathbb{R}_{50}[x] \longrightarrow \mathbb{R}^{51}$ $\qquad$ 1) lineární zobrazení

$a_{50}x^{50} + \dots + a_1 x + a_0 \longmapsto \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{50} \end{pmatrix} \approx$ $\qquad$ 2) bijekce

lin. izomorfismus

$\varepsilon = (1, x, x^2, \dots x^{50})$

$a_{50}x^{50} + \dots + a_1 x + a_0 \longmapsto (a_{50}x^{50} + \dots)_\varepsilon$

$$\mathrm{Mat}_{5\times 5}(\mathbb{R}) \longrightarrow \mathbb{R}^{25}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{15} \\ a_{21} & - & - & - \\ & & & \\ & & & \end{pmatrix} \longmapsto \begin{pmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{15} \\ a_{21} \\ \vdots \end{pmatrix} = (A)_\varepsilon$$

$$\varepsilon = \begin{pmatrix} 1\ 0\ 0\ 0\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0\ 1\ 0 \dots 0 \\ 0 \end{pmatrix} \dots\dots \begin{pmatrix} 0 \\ \quad 1 \end{pmatrix}$$

$e_1\, e_2\, e_3\, e_4$

$\mathbb{R}^4$

Počítání příkladů

$\varphi : U \to V$ $\qquad u_1, \dots u_4$ báze U

$$\mathrm{im}\,\varphi = [\varphi(u_1), \varphi(u_2), \dots \varphi(u_4)]$$

$$[\varphi(e_1)\ \varphi(e_2), \varphi(e_3), \varphi(e_4)]$$

$\varphi \quad \mathbb{R}^4 \longrightarrow \mathbb{R}^3$ $\qquad \dim \mathbb{R}^4 = \dim \ker\varphi + \dim \mathrm{im}\,\varphi$

$u \neq 0 \longrightarrow \vec{0}$ $\qquad 4 = \geq 1 \qquad \leq 3$

$Q = \{ A \in \mathrm{Mat}_{4\times 2}(\mathbb{R}),$ součet čísel ve dvou sloupcích je $0 \}$

$$A \in Q \qquad A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ -a_{11}-a_{21}-a_{31} & -a_{12}-a_{22}-a_{32} \end{pmatrix}$$

$\dim Q = 6$

$$a_{11} \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ -1 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ -1 & 0 \end{pmatrix} \dots\dots + a_{32} \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & -1 \end{pmatrix}$$

$$Q \xrightarrow{\text{lin. bijekce izo}} \mathbb{R}^6 \longleftrightarrow \begin{array}{c} \text{báze } \mathbb{R}^6 \\ e_1 \; e_2 \; \dots \; e_6 \end{array}$$

báze Q

$$\begin{array}{cc} a_{11} & a_{12} \\ a_{21} & \\ a_{31} & \\ \dots & \end{array} \qquad \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \\ a_{12} \\ a_{22} \\ a_{32} \end{pmatrix} \qquad \begin{array}{cc} e_1 & e_2 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & \vdots \\ 0 & \\ 0 & 0 \end{array}$$

Soustava rovnic

$$x_1 = 2a+b, \quad x_2 = a-3b, \quad x_3 = a, \quad x_4 = b$$

Prostor řešení $\qquad\qquad$ $\mathbb{R}^2$ prostor parametrů

$$(2a+b, a-3b, a, b) \xleftarrow{\text{lin. izo.}} \begin{pmatrix} a \\ b \end{pmatrix}$$

$$(2,1,1,0) \longleftarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$(1,-3,0,1) \longleftarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

tvoří báze $\qquad\qquad$ báze