$\varphi : U \to V$

1) $V = \mathbb{R}^3$ = trojice reálných čísel

$\varphi(u) = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ nebo $(1, 2, 3)$

$e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$\varepsilon = (e_1, e_2, e_3)$

$$\left(\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}\right)_\varepsilon = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$\varphi : \mathbb{R}_2[x] \to Mat_{2\times 2}(\mathbb{R})$

2) $\varphi(3x^2 - 8x - 3)$

γ báze $\mathbb{R}_2[x]$

δ báze $Mat_{2\times 2}(\mathbb{R})$

$(\varphi)_{\delta,\gamma}$ $\qquad \varphi(3x^2 - 8x - 3) = \begin{pmatrix} & \end{pmatrix}$

$$(\varphi(3x^2 - 8x - 3))_\delta = \begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

$\varphi : \mathbb{R}_2[x] \to Mat_{2\times 2}(\mathbb{R})$

$\varphi(\text{polynom}) = \text{matice}$

$(\varphi)_{\delta,\gamma} \, (\text{polynom})_\gamma = (\varphi(p))_\delta$ souřadnice matice v bázi δ

sloupec

matice

sloupec

$p \in \mathbb{R}_2[x]$

$\varphi : \mathbb{R}_2[x] \to Mat_{2\times 2}(\mathbb{R})$

$$\varphi(p) = \begin{pmatrix} p'(0), & p(1) \\ p''(1), & p(0) \end{pmatrix}$$

$(\varphi)_{\delta,\chi}$ $\quad (p)_\chi$

matice

$$\begin{array}{cc} & 3 \\ 4 & \square \end{array} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \bullet \\ \bullet \\ \bullet \\ \bullet \end{pmatrix} = \left(\begin{pmatrix} p'(0) & p(1) \\ p''(1) & p(0) \end{pmatrix}\right)_\delta$$

- $\varphi \; : \; \mathbb{R}_2[x] \to Mat_{2\times 2}(\mathbb{R})$
- $(\varphi)_{\delta,\chi} \quad \mathbb{R}^3 \to \mathbb{R}^4$

  $x \mapsto (\varphi)_{\delta,\chi} \cdot x$

$U \xrightarrow{\varphi} V$

$\alpha \qquad \beta, \delta \qquad (\varphi)_{\beta,\alpha}$ známe

$\chi \qquad 2$

$3 \begin{pmatrix} \\ \\ \end{pmatrix}$

$$\begin{tikzcd} U \arrow[r, "id_U"] \arrow[rrr, bend left, "\varphi"] & U \arrow[r, "\varphi"] & V \arrow[r, "id_V"] & V \\ \chi & \alpha & \beta & \delta \end{tikzcd}$$

$$(\varphi)_{\delta,\chi} = (id)_{\delta\beta} \cdot (\varphi)_{\beta,\alpha} \cdot (id)_{\alpha,\chi}$$

Přednáška 8

DÚ č. 8

Zjistěte, zda dané zobrazení φ je lineární.

$$\varphi : U \to V$$

1) $\underline{\forall u_1, u_2}$ $\quad \varphi(u_1 + u_2) = \varphi(u_1) + \varphi(u_2)$

2) $\forall a \in K \ \forall u \in U \quad \varphi(au) = a\,\varphi(u)$.

$$\varphi : \mathbb{R}_2[x] \longrightarrow Mat_{2\times 2}(\mathbb{R})$$

$$\varphi(p) = \begin{pmatrix} p'(0) & p(1) \\ p''(2) & p(0) \end{pmatrix}$$

Je lineární: Byl i tento důkaz

$p(x) = x^2 + 1 \qquad q(x) = x + 2$

$$\varphi(x^2+1) = \begin{pmatrix} 0 & 2 \\ 2 & 1 \end{pmatrix} \qquad \varphi(x+2) = \begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix}$$

$$\varphi(x^2+x+3) = \begin{pmatrix} 1 & 5 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 2 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix} = \varphi(x^2+1) + \varphi(x+2)$$

Zobrazení φ je lineární.

Důkaz φ splňuje pravidla pro všechny dvojice polynomů

$$\varphi(p+q) = \begin{pmatrix} (p+q)'(0) & (p+q)(1) \\ (p+q)''(2) & (p+q)(0) \end{pmatrix} = \begin{pmatrix} (p'+q')(0) & p(1)+q(1) \\ (p''+q'')(2) & p(0)+q(0) \end{pmatrix}$$

$$= \begin{pmatrix} p'(0)+q'(0) & p(1)+q(1) \\ p''(2)+q''(2) & p(0)+q(0) \end{pmatrix}$$

$$= \begin{pmatrix} p'(0) & p(1) \\ p''(2) & p(0) \end{pmatrix} + \begin{pmatrix} q'(0) & q(1) \\ q''(2) & q(0) \end{pmatrix} = \varphi(p) + \varphi(q)$$

Chceme dk, že $\quad (1) \wedge (2)$

$\varphi \quad U \to V$ není lineární

Stačí dk $\quad \neg(1) \vee \neg(2)$

nebo

$\neg(1) \qquad \exists (u_1, u_2) \in U \times U$

$$\varphi(u_1 + u_2) \neq \varphi(u_1) + \varphi(u_2)$$

$\neg(2) \qquad \exists a \in K \ \exists u \in U$

$$\varphi(au) \neq a\varphi(u)$$

$$\varphi : \mathbb{R}_2[x] \to \mathbb{R}$$
$$\varphi(p) = p(1) \cdot p(2)$$

Stačí dk $\neg(1) \qquad p(x) = x^2, \ q(x) = x+1$

$$\varphi(p+q) = \varphi(x^2+x+1) = 3 \cdot 7 = 21$$
$$\varphi(x^2) = 1 \cdot 4$$
$$\varphi(x+1) = 2 \cdot 3$$
$$\varphi(x^2) + \varphi(x+1) = 4+6 = 10 \neq \varphi(x^2+x+1) = 21$$

není potřeba se zabývat vlastností (2).

$=$
$$\varphi(p+q) = (p+q)(1) \cdot (p+q)(2) = (p(1) + q(1))(p(2) + q(2))$$
$$= \underline{p(1)p(2)} + \underbrace{p(1)q(2) + q(1)p(2)}_{\neq 0} + \underline{q(1)q(2)}$$

$$\varphi(p) + \varphi(q) = \underline{p(1)p(2)} + q(1)q(2)$$

$p(1)q(2) + q(1)p(2) \neq 0$

$\varphi: \mathbb{R}_{2020}[x] \longrightarrow Mat_{3\times 3}(\mathbb{R})$

lineární, $\dim \ker \varphi = 2015$

$\dim 2021$

Předpis $\varphi: \mathbb{R}_{2020}[x] \longrightarrow \begin{pmatrix} a_0 & a_1 & a_2 \\ a_3 & a_4 & a_5 \\ 0 & 0 & 0 \end{pmatrix}$

$Mat_{3\times 3}(\mathbb{R})$

Není to předpis

odsud vezme

$$\varphi(a_0 + a_1 x + \dots + a_{2020} x^{2020}) = \begin{pmatrix} a_0 & a_1 & a_2 \\ a_3 & a_4 & a_5 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\varphi(\qquad) = \begin{pmatrix} \sum_{i=0}^{5} a_i & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\ker \varphi = \left\{ \sum_{i=0}^{2020} a_i x^i,\ \varphi(p) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right\}$$

2015

$$\sum_{i=0}^{5} a_i = 0 \qquad a_0, a_1 \dots a_5\ a_6 \dots a_{2020}$$

libovolné libovolné

$$a_0 = -(a_1 + a_2 + a_3 + a_4 + a_5)$$

$\dim \ker \varphi = 20\cancel{15}\,20$

$$-(a_1 + a_2 + \dots + a_5)\cdot 1 + a_1 x + \dots + a_6 x^6 + \dots + a_{2020} x^{2020}$$

báze jádra

$$x^{2020},\ x^{2019},\ \dots\ x^{6},\qquad x^5-1,\ x^4-1,\ x^3-1,\ x^2-1,\ x-1$$

má ~~..~~ 2020 prvků

nikoliv 2015!

$$\begin{pmatrix} \sum_{i=0}^{5} a_i & a_6 & a_7 \\ a_8 & a_9 & a_{10} \\ 0 & 0 & 0 \end{pmatrix}.$$

2021 není

6 lin. kombinac

$A = (a_{ij})$ permutace n prvkové množiny $\{1, 2, \dots n\}$

všechna možná pořadí těchto čísel

3142

My chápeme permutace ~~...~~ jako bijektivní zobrazení

n	1	2	3	4
$\sigma(n)$	3	1	4	2

$\sigma : \{1,2,3,4\} \to \{1,2,3,4\}$

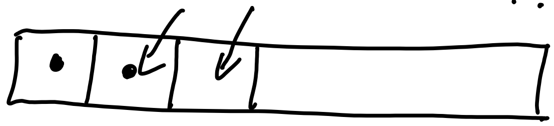
Každé permutaci lze přiřadit tzv. znaménko permutace

$$\operatorname{sign} \sigma = \begin{matrix} 1 \\ -1 \end{matrix}$$

$$\det A = \sum_{\sigma \in S_n} \operatorname{sgn} \sigma \; a_{1\sigma(1)}\, a_{2\sigma(2)} \cdots a_{n\sigma(n)}$$

n počet permutací je $n! = n\cdot(n-1)\cdots 2\cdot 1$

n čísel $\quad n-1 \quad n-2$



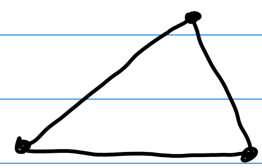
$4! = 24$

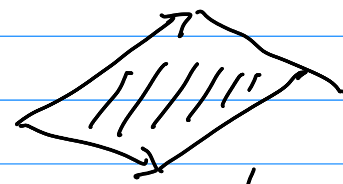
$5! = 120$

$10! > 10^6$

$\left|\det\begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \end{pmatrix}\right|$ = obsah ~~rovnoběžnostěnu~~ rovnoběžníku

$x = (x_1, x_2)$

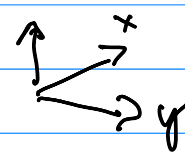




$y = (y_1, y_2)$

konec 1. semestru

$\left|\det\begin{pmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{pmatrix}\right|$ = objem rovnoběžnostěnu



Počítání n-úhel → 2. semestr