4. příklad z 11. dú   Inverze pomocí alg. doplňků k matici

$$\left(\begin{array}{cccccc|cccccc} 1 & & & & & b & 1 & 0 & 0 & & & 0 \\ & 1 & & 0 & & b & 0 & 1 & 0 & & & 0 \\ & & 1 & & & b & & & & & & \\ 0 & & & \ddots & 1 & 0\; b & & & & & & \\ & & & & 1 & b & 0 & 0 & 0 & & 1 & 0 \\ & & & & & 1 & 0 & 0 & 0 & & 0 & 1 \end{array}\right) \sim \left(\begin{array}{c|cccc} & 1 & 0 & & -b \\ E & 0 & 1 & & -b \\ & & & \ddots & \\ & 0 & & 1 & -b \\ & & & & 1 \end{array}\right)$$

$$B^{-1} = \frac{1}{\det B}\left(\tilde{b}_{ij}\right)^T$$

$$\tilde{b}_{ij} = (-1)^{i+j} \det \left(\quad \right)$$ (matrix with row i and column j crossed out)

$$\det B = 1$$

$$B = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 & b \\ 0 & 1 & 0 & \cdots & 0 & b \\ 0 & 0 & 1 & & 0 & b \\ & & & & & \\ 0 & 0 & 0 & & 1 & b \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\tilde{b}_{22} = (-1)^{2+2} \det \begin{pmatrix} 1 & & & b \\ & 1 & & b \\ & & \ddots & b \\ & & & 1 \end{pmatrix} = 1$$

samé 0

$$\tilde{b}_{2,n-2} = (-1)^{2+n-2} \det \begin{pmatrix} 1 & 0 & 0 & & 0 & b \\ 0 & 0 & 1 & & 0 & b \\ & & & & 1 & b \\ & & & & & 1 \end{pmatrix} = 0$$

$\tilde{b}_{ij}$

$$\tilde{b}_{n-1,3} = (-1)^{n-1+3} \det \begin{pmatrix} & & & 0 & \\ & & & \vdots & \\ & & & 0 & \\ 0 & 0 & 1 & 0 & b \\ 0 & 0 & & 0 & 1 \end{pmatrix} = 0$$

samé 0

$$\tilde{b}_{n,3} = (-1)^{n+3} \det$$

$n-1 \quad n-1-3$

$4-3$

$$= (-1)^{n+3} \det \begin{pmatrix} 1 & 0 & 0 & & \\ 0 & 1 & 0 & & \\ & & 0 & \cdots & b \\ & & 1 & & b \\ 0 & 0 & 0 & \cdots & 1\,b \end{pmatrix}$$

$(-1)^{2n-1}\, b$

$$= (-1)^{n+3} (-1)^{n-4} \det \begin{pmatrix} 1 & 0 & 0 & & \\ 0 & 1 & 0 & & \\ & & 1 & & \\ & 0 & & \ddots & \\ & & & & 1\,b \\ & & & & b \end{pmatrix} = (-1)^{2n-1}\, b$$

$$= \underline{\underline{-b}}$$

Jinak: ~~Stupl. rozvoj podle posl. řádku~~

$\cancel{(-1)^{n-1+n-2}}$

$$(\tilde{b}_{ij}) = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ -b & -b & \cdots -b & 1 \end{pmatrix} \qquad B^{-1} = (\tilde{b}_{ij})^T = \begin{pmatrix} 1 & & -b \\ & \ddots & -b \\ & & -b \\ & & 1 \end{pmatrix}$$

Co lze čekat v teoretické části:

(1) Napište předpis lin. zobrazení

$$\varphi : \mathbb{R}_{10}[x] \longrightarrow Mat_{3\times 3}(\mathbb{R})$$

$$\dim \ker \varphi = 5, \quad \varphi(x^{10} - 3x^2 + 5) = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

(2) Dokažte, že pro vektory $u_1, u_2, u_3 \in U$ platí

$$[u_1, u_2, u_2 + u_3] = [2u_1 - u_2, u_3, u_2 + 2u_3]$$

$\subseteq$

$\supseteq$