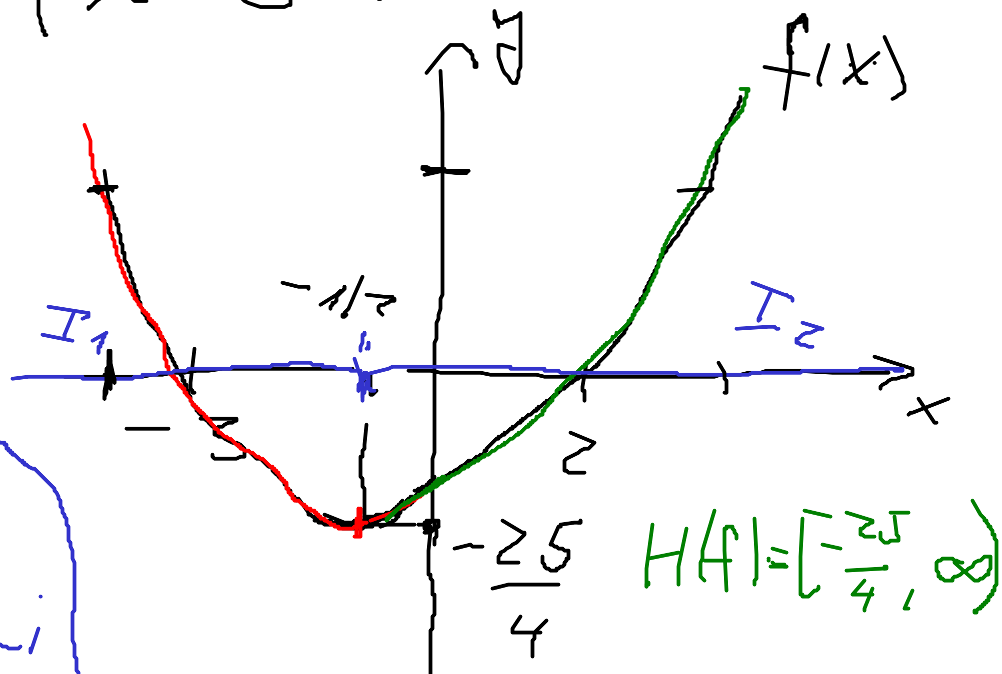
10.1

① $f(x) = x^2 + x - 6 =$
$= (x+3)(x-2)$

$f(-\frac{1}{2}) = \frac{1}{4} - \frac{1}{2} - 6$
$= -\frac{1}{4} - 6 = \frac{-25}{4}$



- $I_1 = (-\infty, -\frac{1}{2}]$
  f(x) klesající
- $I_2 = [-\frac{1}{2}, \infty)$
  f(x) roste

$h(y) := f^{-1}(y)$
inverzní funkce

① $I_1 = (-\infty, -\frac{1}{2}]$

$h(y) = -\frac{1}{2} - \frac{1}{2}\sqrt{4y+25}$

$D(h) = [-\frac{25}{4}, \infty)$

$H(h) = (-\infty, -\frac{1}{2}]$

$y = x^2 + x - 6$

$0 = x^2 + x - y - 6$

$x_{1,2} = \frac{-1 \pm \sqrt{1 + 4(y+6)}}{2}$

$= -\frac{1}{2} \pm \frac{1}{2}\sqrt{4y+25}$

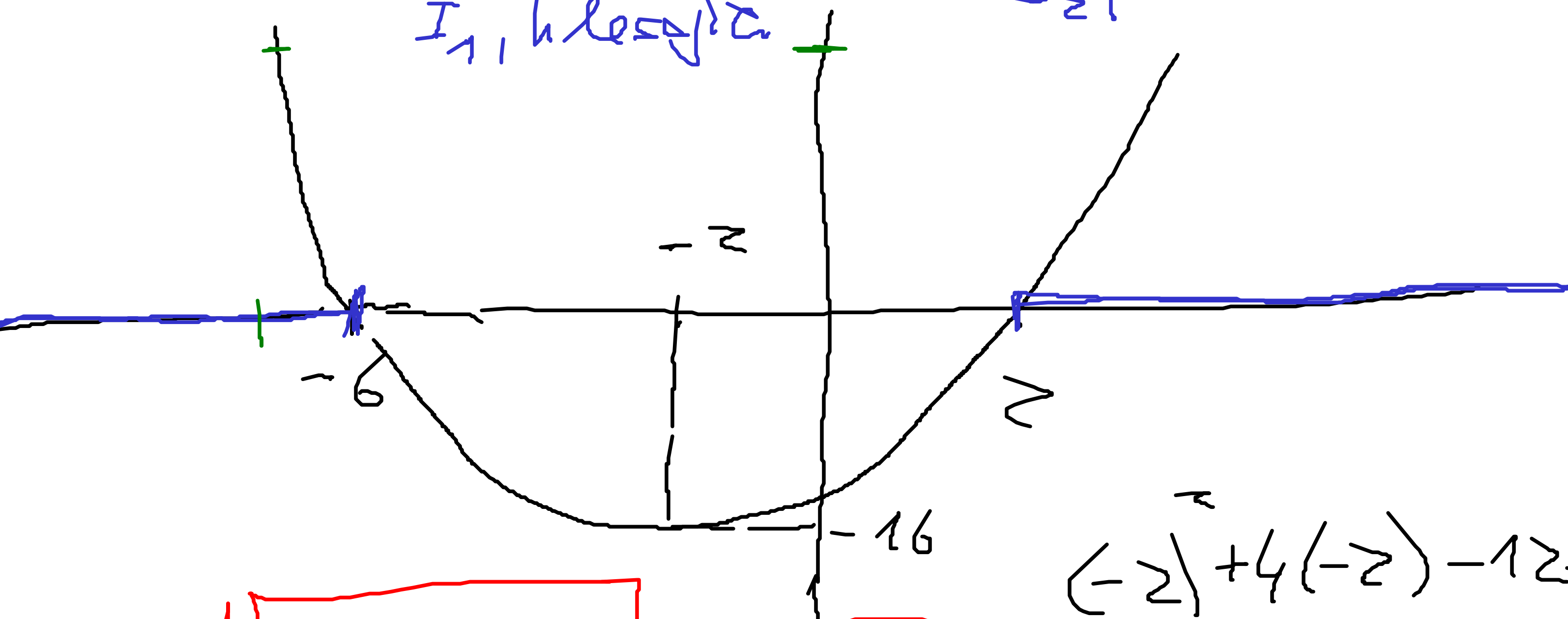
② I_2 podobně

2

$f(x)=\sqrt{x^2+4x-12}$

$x^2+4x-12=(x+6)(x-2)$

$D(f)=\underbrace{(-\infty,-6]}_{I_1,\ \text{klesající}}\cup\underbrace{[2,\infty)}_{I_2,\ \text{rostoucí}}$ $\quad$ $\underline{H(f)=[0,\infty)}$



$(-2)^2+4(-2)-12=$
$=4-8-12$
$=-16$

$y=\sqrt{x^2+4x-12}$, $y\geq 0$

$y^2=x^2+4x-12$

$0=x^2+4x-y^2-12$

$$x_{1,2}=\frac{-4\pm\sqrt{16+4(y^2+12)}}{2}=$$

$$=-2\pm\sqrt{4+y^2+12}=-2\pm\sqrt{y^2+16}$$

$h(y)=f^{-1}(y)$ inverzní funkce

$I_1 = (-\infty, -6]$:

$h(y) = -2 - \sqrt{y^2+16}$

$D(h) = [0, \infty)$

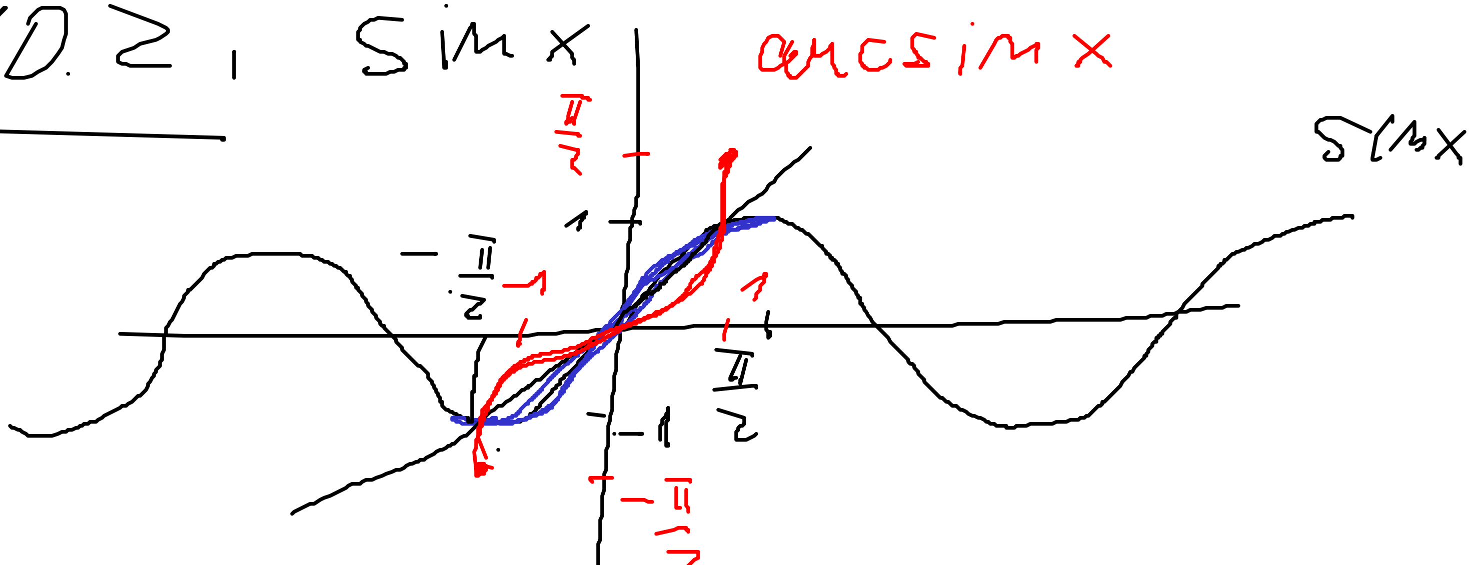
$H(h) = (-\infty, -6]$

$I_2 = [2, \infty)$: $h(y) = -2 + \sqrt{y^2+16}$

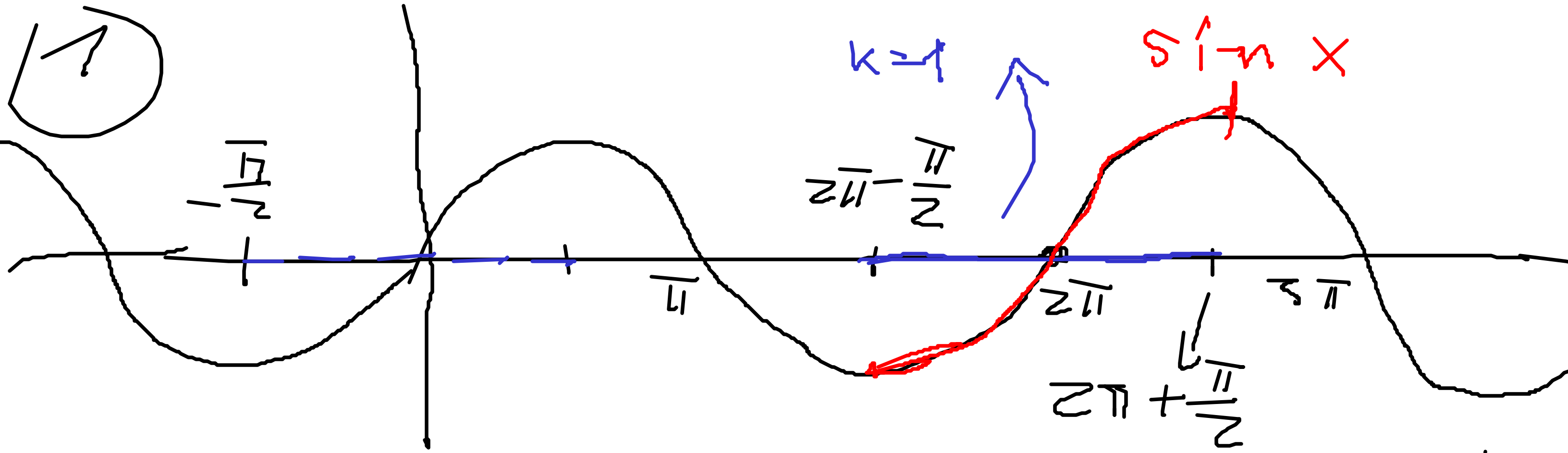
$D(h) = [0, \infty)$

$H(h) = [2, \infty)$

10.2.



arcsin je inverzní funkce k funkci sin x na interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$



Pomocí funkce arcsin napište inverzní funkci k funkci $\sin x$ na intervalu $[2k\pi - \frac{\pi}{2}, 2k\pi + \frac{\pi}{2}]$

$y = \sin x$ pro $\forall x$

$y = \sin(\underbrace{x - 2k\pi}_{\in [-\frac{\pi}{2}, \frac{\pi}{2}]})$

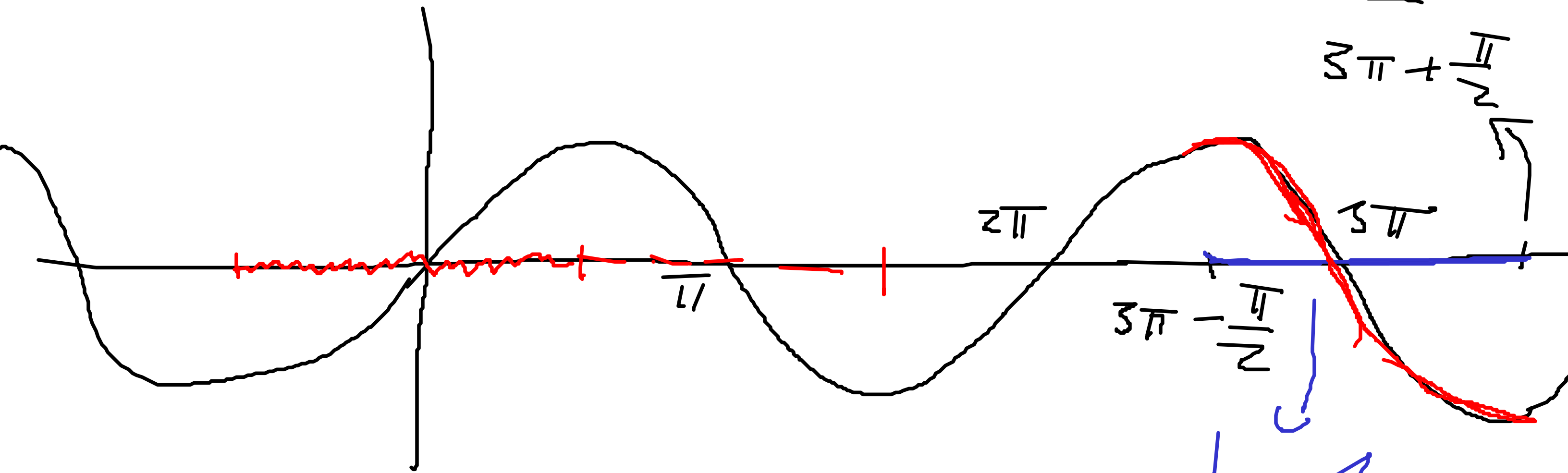
$\arcsin y = x - 2k\pi$

$x = \arcsin y + 2k\pi$

$\boxed{h(y) = \arcsin y + 2k\pi}$

$H(h) = [-\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi]$

(2) $x \in \left[(2k+1)\pi - \frac{\pi}{2}, (2k+1)\pi + \frac{\pi}{2}\right]$



$y = \sin x \neq \sin(x - (2k+1)\pi)$

$y = \sin(\underbrace{x - 2k\pi}_{\in\left[\pi - \frac{\pi}{2}, \pi + \frac{\pi}{2}\right] = \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]}) = -\sin(x - 2k\pi - \pi)$

Zmiana: $\sin x = \sin(\pi - x)$

$= -\sin(x - \pi)$

$$\boxed{\sin x = -\sin(x - \pi)}$$

$y = -\sin(\underbrace{x - (2k+1)\pi}_{\in\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]})$

$-y = \sin(x - (2k+1)\pi)$

$\underbrace{\arcsin(-y)}_{\text{lichá funkce}} = x - (2k+1)\pi$

$-\arcsin y = x - (2k+1)\pi$

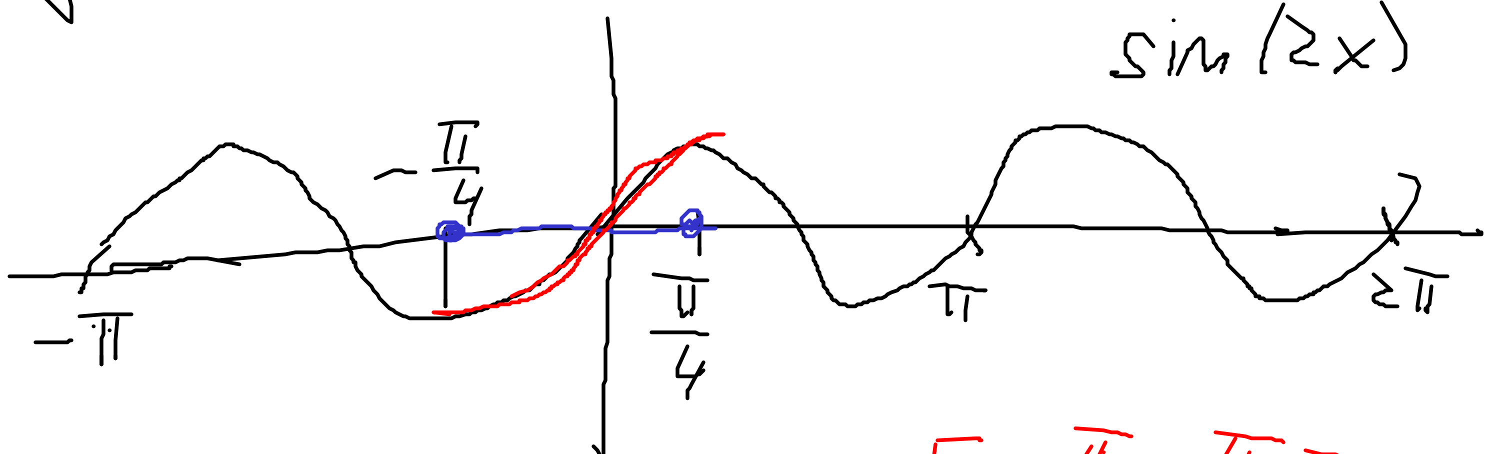
$\boxed{x = -\arcsin y + (2k+1)\pi = h(y)}$

10.3: Interval monotonie obsahující 0

(1) $f(x) = \sin x \cdot \cos x$ + inverze

$\sin 2x = 2\sin x \cos$

$y = \frac{1}{2}\sin(2x)$



Interval je $I = \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ rostoucí

$\hookrightarrow 2y = \sin(2x)$

$\arcsin(2y) = 2x$

$$\boxed{x = \frac{1}{2}\arcsin(2y) = h(y)}$$

2) $f(x) = \sin x + \cos x$

$$\boxed{\begin{aligned} &\cos x = \sin\left(\frac{\pi}{2} - x\right) \\ &\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2} \end{aligned}}$$

$y = \sin x + \cos x =$

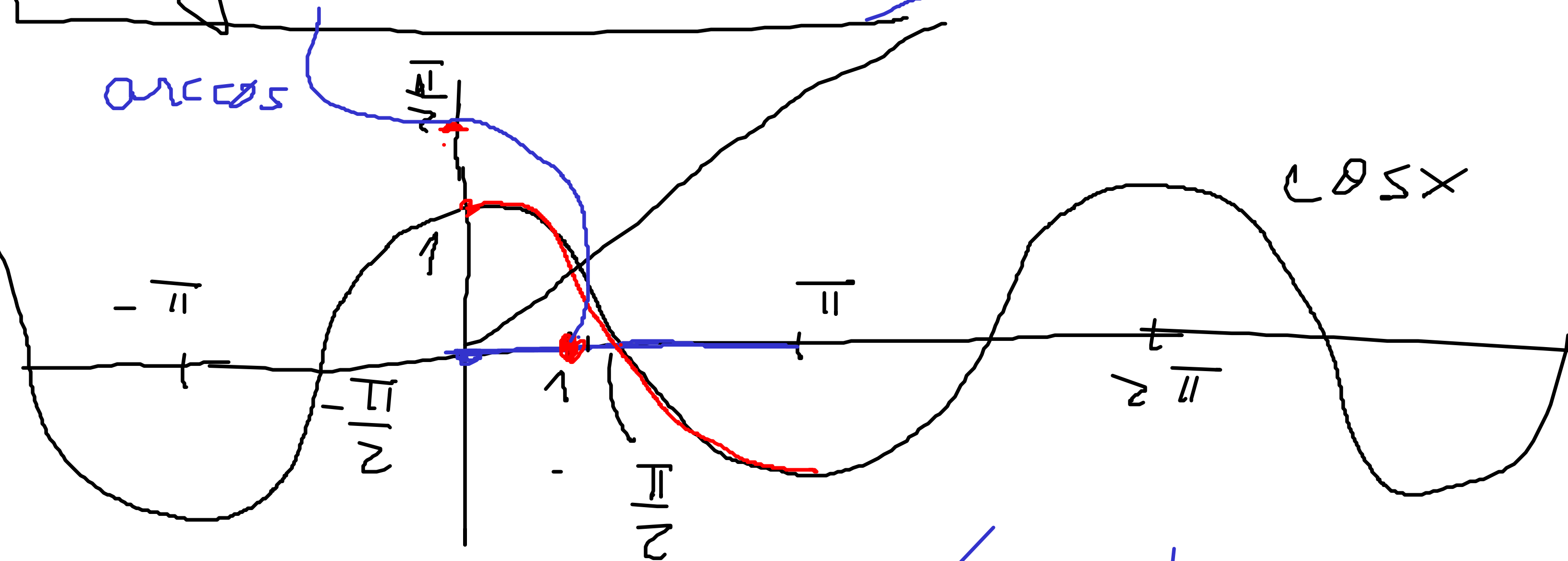
$= \sin x + \sin\left(\frac{\pi}{2} - x\right)$

$= 2\sin\frac{x + \left(\frac{\pi}{2} - x\right)}{2}\cos\frac{x - \left(\frac{\pi}{2} - x\right)}{2}$

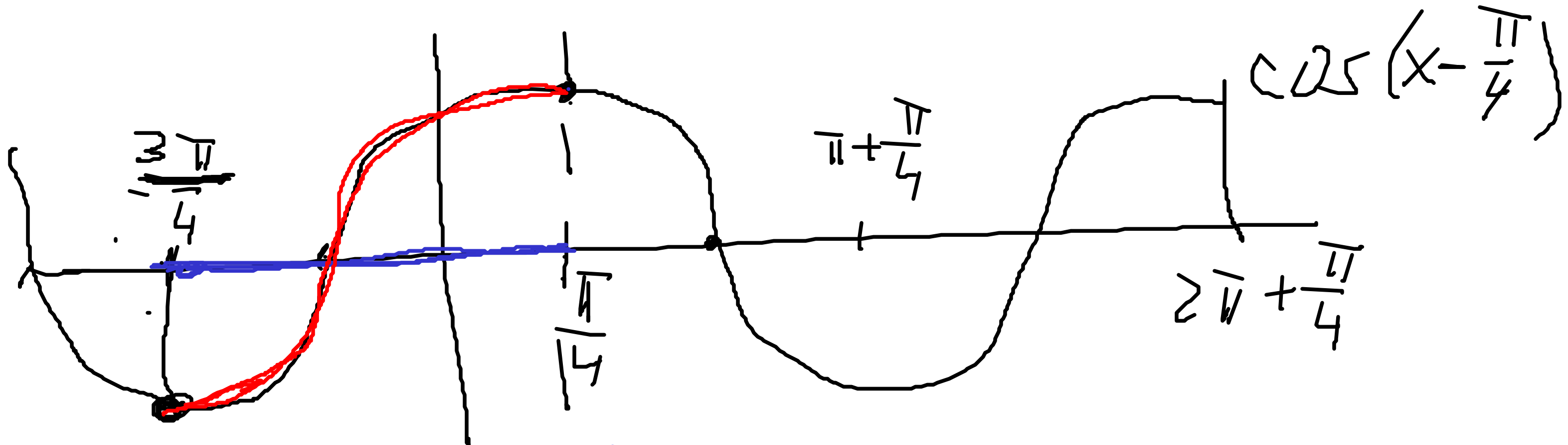
$= 2\underbrace{\sin\frac{\pi}{4}}_{\frac{\sqrt{2}}{2}}\cos\frac{2x - \frac{\pi}{2}}{2}$

$= \sqrt{2}\cos\left(x - \frac{\pi}{4}\right)$

$$\boxed{\frac{1}{\sqrt{2}}y = \cos\left(x-\frac{\pi}{4}\right)} = \cos\left(-x+\frac{\pi}{4}\right)$$



arccos je inverzní funkce k funkci $\cos x$ pro $x \in [0, \pi]$



$$I = \left[-\frac{3\pi}{4}, \frac{\pi}{4}\right]$$

$$\frac{1}{\sqrt{2}}y = \cos\left(\underbrace{-x+\frac{\pi}{4}}_{\in[0,\pi]}\right)$$

$$\arccos\left(\frac{y}{\sqrt{2}}\right) = -x+\frac{\pi}{2}$$

$$\boxed{x = -\arccos\left(\frac{y}{\sqrt{2}}\right) + \frac{\pi}{2} = h(y)}$$

2. vnitrosemestrální test

② Kvadratická soustava

$$ax^2 + x + a = 0$$

$$x^2 + ax + a = 0 \quad / \cdot (-a)$$

Určete, pro které reálné hodnoty parametru a má soustava společné reálné řešení.

$$x + a - a(ax + a) = 0$$

$$x + a - a^2 x - a^2 = 0$$

$$(1 - a^2)x + a - a^2 = 0$$

$$(1+a)(1-a)x + a(1-a) = 0$$

: $a = 1$ řešení

• $a \neq 1 \Rightarrow \underbrace{(a+1)}\,x + a = 0$

$a = -1$ není možné

$$x = \frac{-a}{a+1} \qquad a \neq 1$$

$$x^2 + x + 1 = 0$$

$D = 1 - 4 < 0$

nemá reálné řešení

$\left.\begin{array}{l} x^2+x+1=0 \\ D = 1-4<0 \\ \text{nemá reálné řešení} \end{array}\right\} a = 1$ nejde

$$x^2 + ax + a = 0$$

$$\left(-\frac{a}{a+1}\right)^2 + a\left(-\frac{a}{a+1}\right) + a = 0 \quad / \cdot \frac{1}{a}$$

• $\boxed{a = 0 \rightsquigarrow x = 0 \text{ ok}}$

• $a \neq 0 \Rightarrow \dfrac{a}{(a+1)^2} - \dfrac{a}{a+1} + 1 = 0 \quad / \cdot (a+1)^2$

$$a - a(a+1) + (a+1)^2 = 0$$

$$a - a^2 - a + a^2 + 2a + 1 = 0$$

$$\boxed{a = -\frac{1}{2}}$$

$$\left.\begin{array}{l} ax^2 + x + a = 0 \\ x^2 + ax + a = 0 \end{array}\right\} a = -\frac{1}{2}$$

$$-\frac{1}{2}x^2 + x - \frac{1}{2} = 0 \quad / \cdot (-2)$$

$$x^2 - \frac{1}{2}x - \frac{1}{2} = 0 \quad / \cdot 2$$

$$x^2 - 2x + 1 = 0 = (x-1)^2$$

$$2x^2 - x - 1 = 0$$

$x = 1$

řešení

obou

rovnic

Závěr

$a = 0 \longrightarrow x = 0$

$a = -\frac{1}{2} \longrightarrow x = 1$