• $f(x)$ je periodická s periodou $\ell \in \mathbb{R}_+$,

jestliže $\forall x \in \mathbb{R}: f(x+\ell) = f(x)$

• nejmenší perioda funkce $f(x)$ je nejmenší $\ell \in \mathbb{R}_+$ z předchozí definice.

<u>111</u> 1 $f(x) = \sin x + \cos x$

$\hookrightarrow$ má periodu 2π

$$f(x) = \sin x + \sin\left(x + \frac{\pi}{2}\right)$$

$$= 2\sin\frac{x + \left(x+\frac{\pi}{2}\right)}{2} \cos\frac{x - \left(x+\frac{\pi}{2}\right)}{2}$$

$$= 2\sin\left(x+\frac{\pi}{4}\right)\underbrace{\cos\left(-\frac{\pi}{4}\right)}_{\frac{\sqrt{2}}{2}}$$

$$= \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$$

$\Rightarrow$ nejmenší perioda je 2π

$$\boxed{\sin x + \cos x = \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)}$$

(2) $f(x) = \sin(3x)$

$\Rightarrow$ nejmenší perioda $\frac{2\pi}{3}$

Nechť je nejmenší perioda ℓ

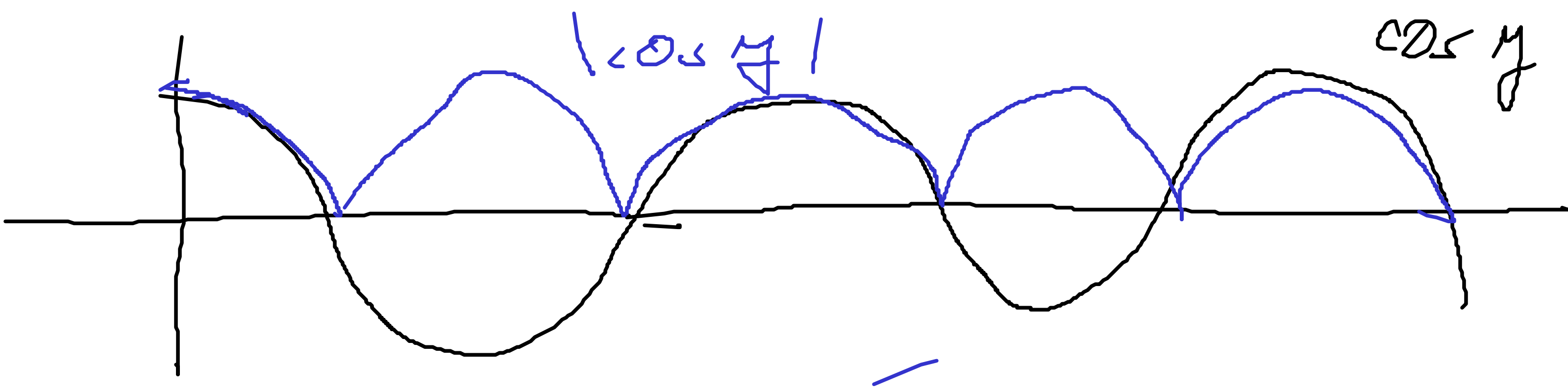
$f(x+\ell) = \sin(3(x+\ell)) = \sin(3x) = f(x)$

$\forall x \qquad \sin(3x+3\ell) = \sin 3x$

$\Rightarrow 3\ell = 2k\pi$ pro nějaké $k \in \mathbb{Z}$

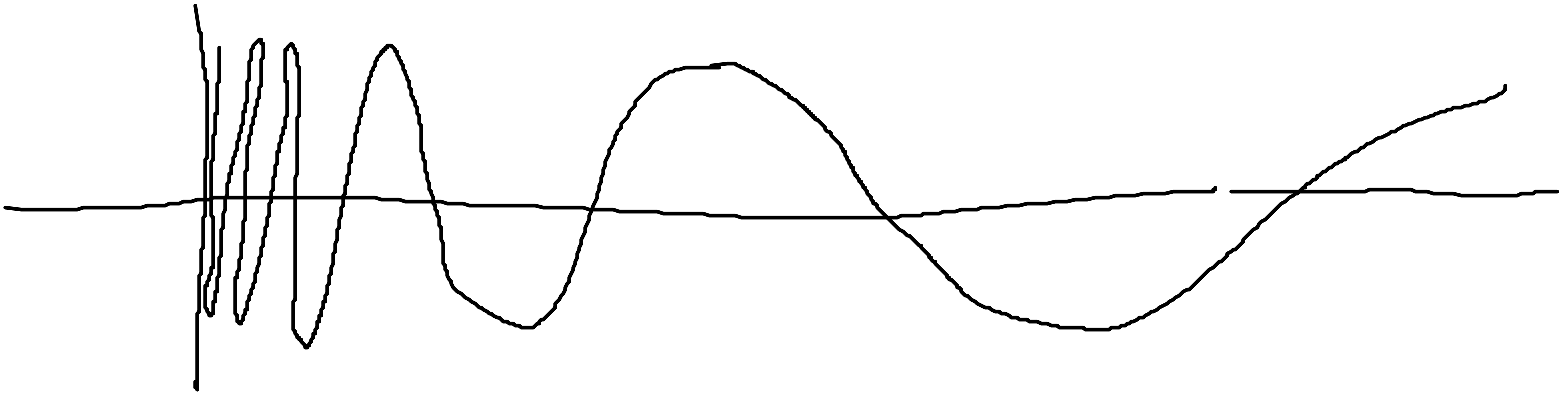
Nejmenší $\rightsquigarrow k=1 \rightsquigarrow \ell = \frac{2\pi}{3}$

(3) $f(x) = |\underbrace{\cos(2x)}_{\text{má nejm. periodu } \pi}|$



Závěr: $f(x)$ má nejm. periodu $\frac{\pi}{2}$

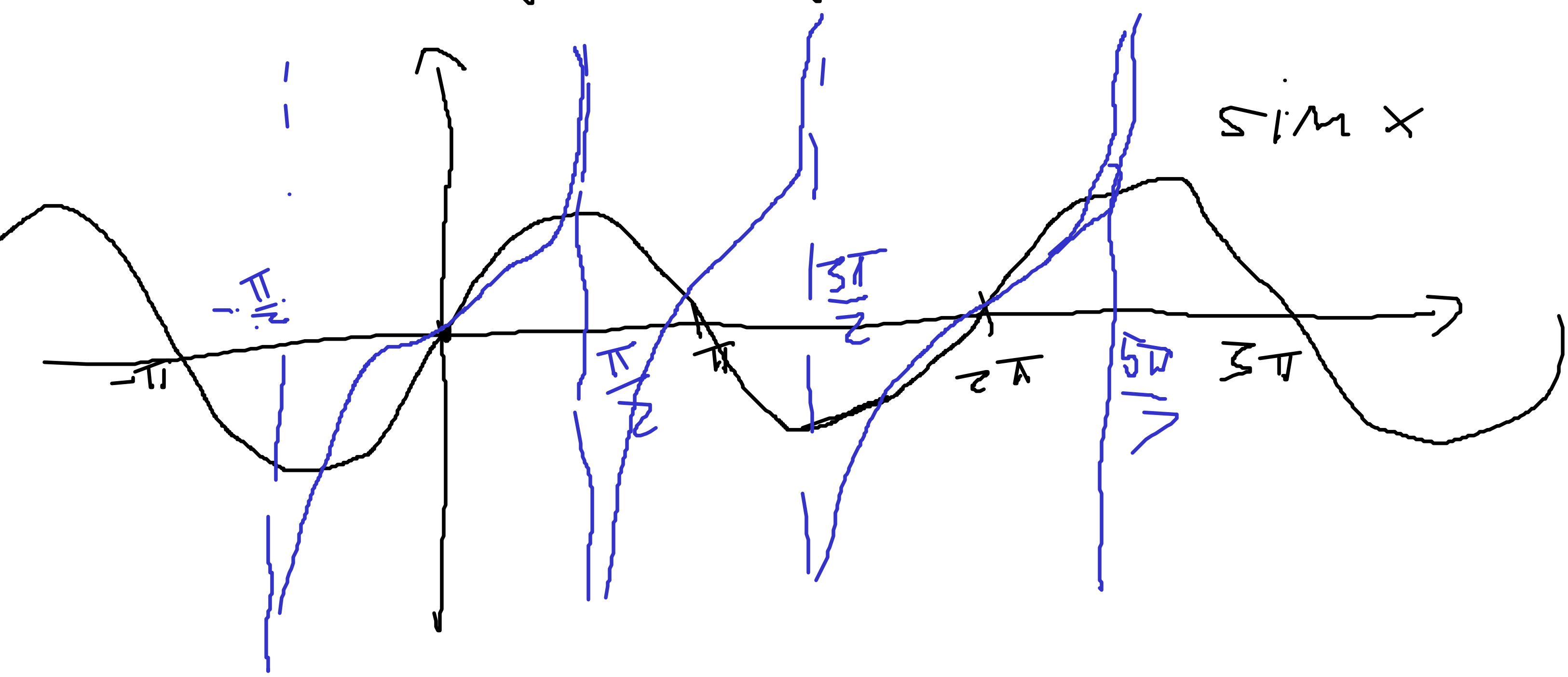
(4) $f(x) = \sin\frac{1}{x}$ **není periodická**



(6) $f(x) = \sin x + \operatorname{tg} x$

$\rightsquigarrow$ má periodu 2π ?

$\rightsquigarrow$ je nejmenší ?



Z asymptot vidíme, že nejmenší perioda musí být $\geq \pi$ $\dashrightarrow$ nejm. per. je 2π

11.2

① $f(x) = x \sin x$

sudá $\quad f(-x) = (-x)\sin(-x)$
$= (-x)(-\sin x)$

② $f(x) = \underbrace{x}_{\text{lichá}} \underbrace{\cos 2x}_{\text{sudá}}$

$f(x)$ lichá

⑦ $f(x) = |\sin x + \cos x|$
$= |\sqrt{2}\sin(x + \frac{\pi}{4})|$

není sudá ani lichá

11.3: ① funkce $f(x)$ s periodou 3π a oborem hodnot $[1,2]$

- perioda 3π
  $\sin(ax)$ má periodu $\frac{2\pi}{a}$
  $\rightsquigarrow \frac{2\pi}{a} = 3\pi \rightarrow a = \frac{2}{3}$

$\sin\left(\frac{2}{3}x\right)$ má periodu 3π

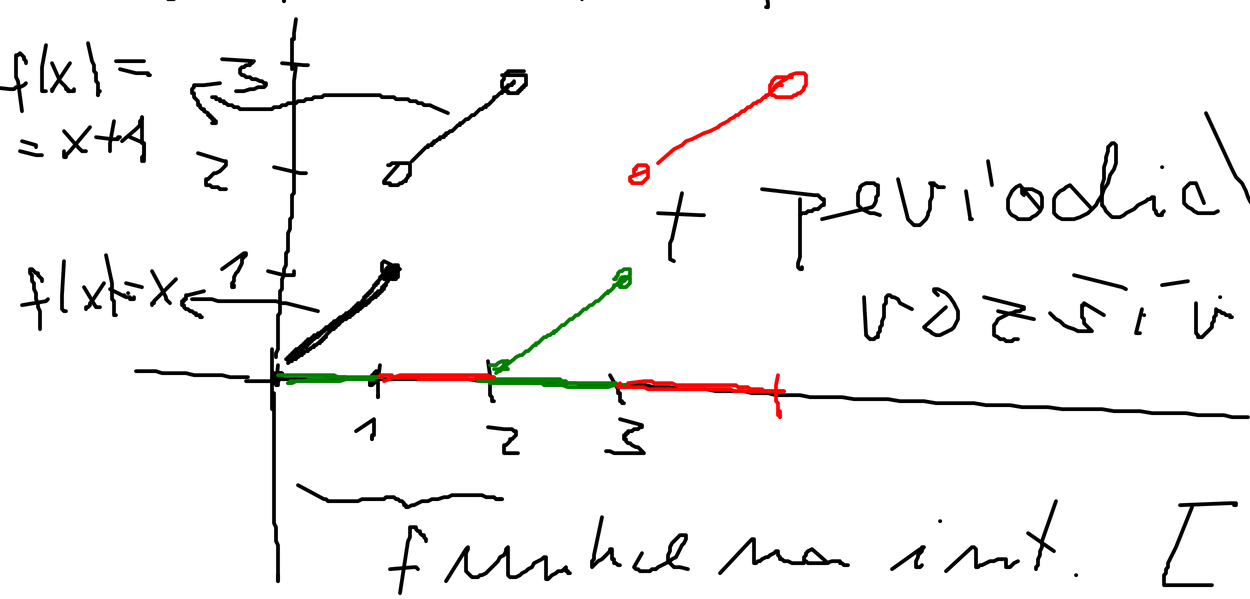
$$f(x) = \frac{1}{2}\sin\left(\frac{2}{3}x\right) + \frac{5}{2}$$

(2) perioda 1, obor hodnot $\mathbb{R}$

$f(x) = \operatorname{tg}(\pi x)$

$\operatorname{tg}(ax)$ má periodu $\frac{\pi}{a} = 1 \Rightarrow a = \pi$

(3) perioda 2, obor hodnot $[0,1] \cup (2,3)$, rostoucí na $(0,2)$



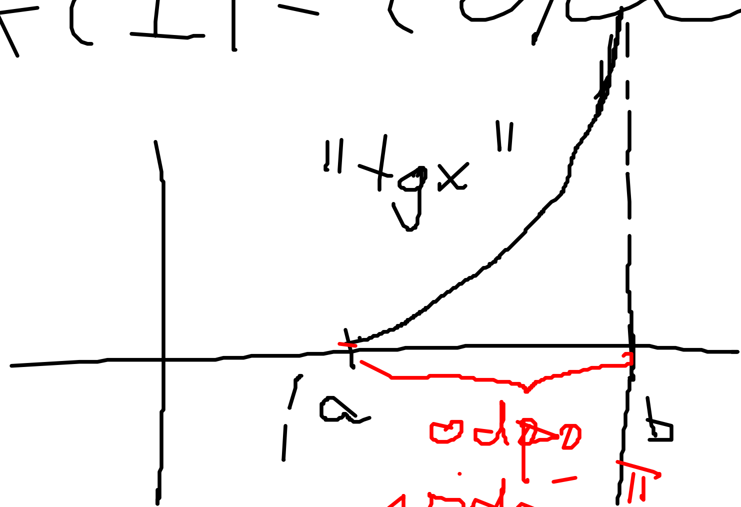
+ periodické rozšíření

funkce na int. $[0,2)$

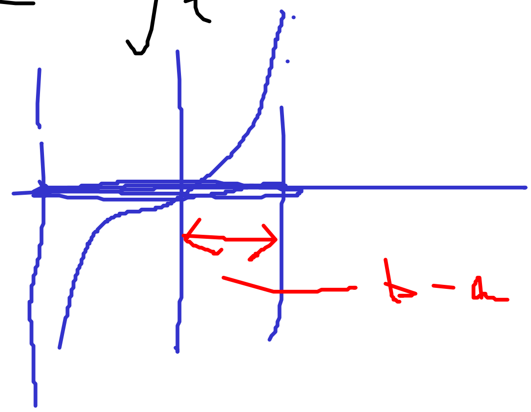
$$f(x) = \begin{cases} x - 2k & x \in [2k, 2k+1] \\ x - 2k + 1 & x \in (2k+1, 2k+2) \end{cases}$$

11.4 (4) $I=(a,b)$, $a,b\in\mathbb{R}$, $a<b$

obor hodnot na I je
$f(I)=(0,\infty)$



odpovídá $\frac{\pi}{2}$ $\Rightarrow$ chceme periodu $2(b-a)$



$f(x)= \dots$

$tg(cx)$ má periodu

$$\frac{\pi}{c}=2(b-a) \Rightarrow c=\frac{\pi}{2(b-a)}$$

$$f(x)=tg\left(\frac{\pi}{2(b-a)}x+r\right)$$

$$f(a)=0 \rightsquigarrow tg\left(\underbrace{\frac{\pi}{2(b-a)}\cdot a+r}_{=0}\right)=0$$

$$\Rightarrow r=-\frac{\pi a}{2(b-a)}$$

Závěr: $f(x) = \mathrm{tg}\left(\frac{\pi}{2(b-a)}x - \frac{\pi a}{2(b-a)}\right)$

11.6

① $\sin x = \sin 2x$

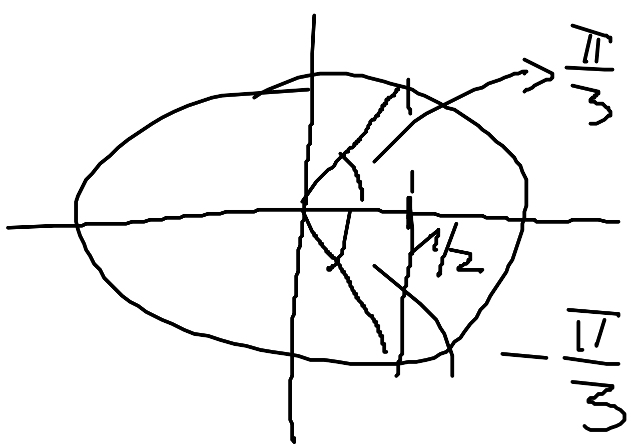
$\sin x = 2\sin x \cos x$

$0 = \underbrace{\sin x}_{=0} \underbrace{(2\cos x - 1)}_{=0}$

$x = k\pi$

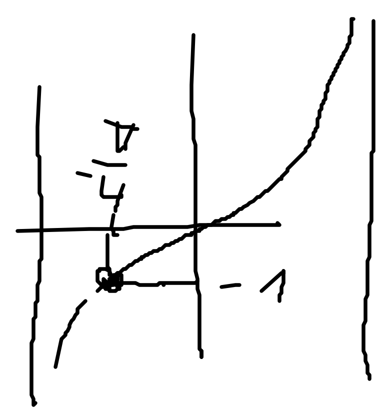
$\cos x = \frac{1}{2}$

$x = \pm\frac{\pi}{3} + 2k\pi$



② $\sin 3x + \cos 3x = 0 \quad / \frac{1}{\cos 3x}$

$\mathrm{tg}(3x) = -1$



$3x = -\frac{\pi}{4} + k\pi$

$x = -\frac{\pi}{12} + \frac{k}{3}\pi, \; k \in \mathbb{Z}$

Pozn.: $\cos(3x) = 0 \Rightarrow \sin(3x) \neq 0$

$\Rightarrow$ není v žádném řešení

(3) $\sin(2x) = \cos(3x)$

$$\sin(2x) - \cos(3x) = 0$$

$$\sin(2x) - \sin\left(3x + \frac{\pi}{2}\right) = 0$$

$$2\sin\frac{2x - \left(3x + \frac{\pi}{2}\right)}{2}\cos\frac{2x + \left(3x + \frac{\pi}{2}\right)}{2} = 0$$

$$\underbrace{\sin\left(-\frac{x}{2} - \frac{\pi}{4}\right)}_{=0}\ \underbrace{\cos\left(\frac{5x}{2} + \frac{\pi}{4}\right)}_{=0} = 0$$

$$-\frac{x}{2} - \frac{\pi}{4} = k\pi$$

$$-x = 2\left(\frac{\pi}{4} + k\pi\right)$$

$$x = -\frac{\pi}{2} - 2k\pi$$

$$k \in \mathbb{Z}$$

$$\frac{5x}{2} + \frac{\pi}{4} = \frac{\pi}{2} + k\pi$$

$$5x = 2\left(\frac{\pi}{4} + k\pi\right)$$

$$x = \frac{\pi}{10} + \frac{k\pi}{5}$$

$$k \in \mathbb{Z}$$

11.7 / 2

$$2\sin^2 x + 7\cos x - 5 = 0$$

$$2(1-\cos^2 x) + 7\cos x - 5 = 0$$

$$\boxed{y = \cos x} \Rightarrow$$

$$2(1-y^2) + 7y - 5 = 0$$

$$-2y^2 + 7y - 3 = 0$$

$$2y^2 - 7y + 3 = 0$$

$$y_{1,2} = \frac{7 \pm \sqrt{49 - 4\cdot 2\cdot 3}}{4}$$

$$= \frac{7 \pm \sqrt{49-24}}{4} = \frac{7 \pm 5}{4} = \begin{cases} 3 \\ \frac{1}{2} \end{cases}$$

- $y = 3 = \cos x \rightsquigarrow$ nejde
- $y = \frac{1}{2} = \cos x \rightsquigarrow x = \pm\frac{\pi}{3} + 2k\pi$

$$k \in \mathbb{Z}$$

Řešení v intervalu $[0, 2\pi)$

- $\frac{\pi}{3} + 2k\pi \Rightarrow k = 0 \rightsquigarrow x = \frac{\pi}{3}$
- $-\frac{\pi}{3} + 2k\pi \Rightarrow k = 1 \rightsquigarrow x = \frac{5\pi}{3}$

<u>Závěr</u>: dvě řešení

<u>11.7</u> 4

$$\sqrt{3}\cos x + \sin x = 2$$

$$\underbrace{\frac{\sqrt{3}}{2}}_{\sin\frac{\pi}{3}}\cos x + \underbrace{\frac{1}{2}}_{\cos\frac{\pi}{3}}\sin x = 1$$

$$\sin\left(\frac{\pi}{3} + x\right) = 1$$

$$x + \frac{\pi}{3} = \frac{\pi}{2} + 2k\pi$$

$$x = \frac{\pi}{6} + 2k\pi$$

Na intervalu $[0, 2\pi)$ je jediné řešení $x = \frac{\pi}{6}$