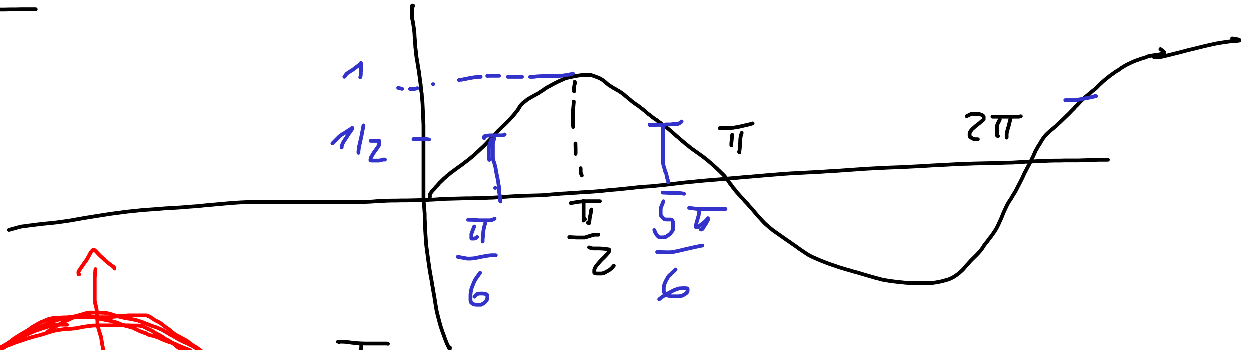
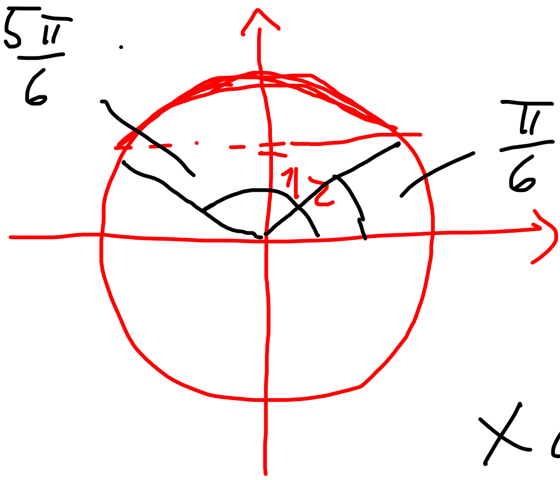
11.8 (a) $\sin x > \frac{1}{2}$





Závěr:

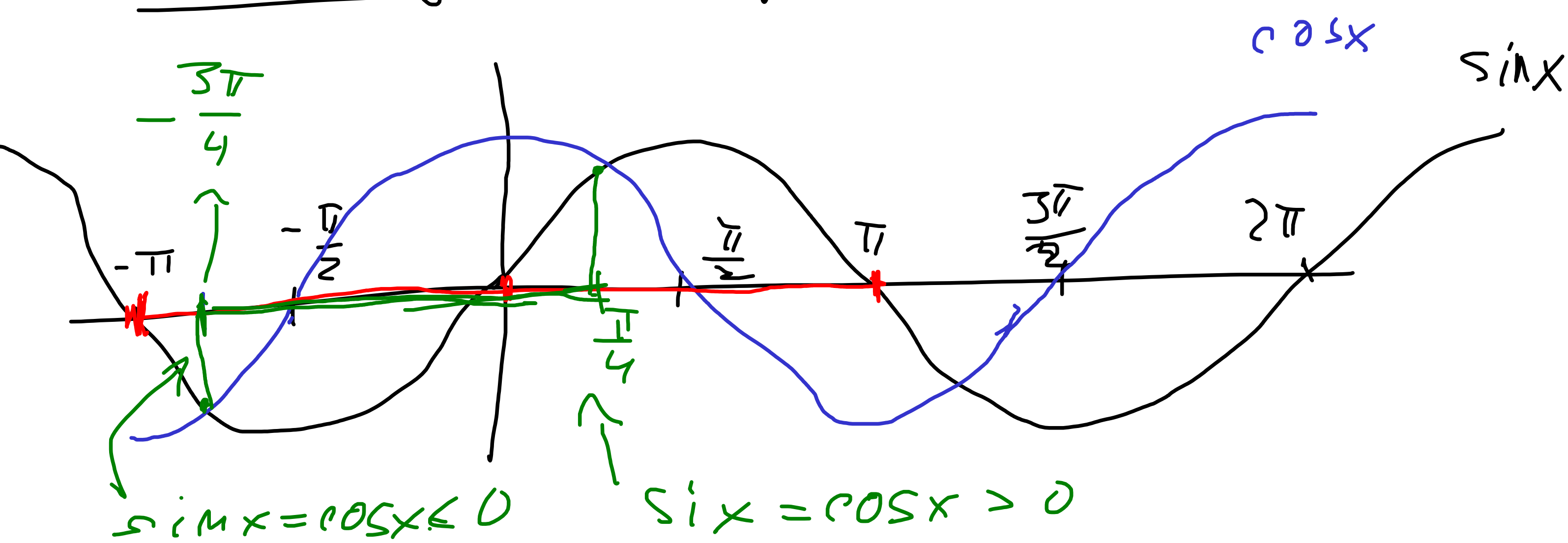
$$x \in \left(\frac{\pi}{6}+2k\pi, \frac{5\pi}{6}+2k\pi\right)$$

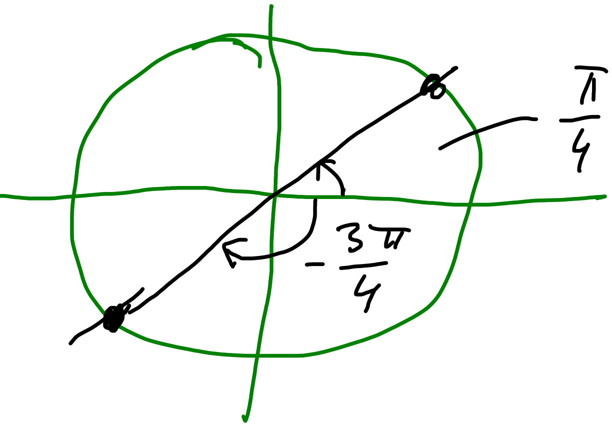
(b) $\sin x < \cos x$

Pozn.: $\cos x > 0$ $\quad$ $\cos x = 0$ $\quad$ $\cos x < 0$

$\operatorname{tg} x < 1$ $\quad$ $\operatorname{tg} x > 1$

Přímo graficky





Závěr:

$$x \in \left(-\frac{3\pi}{4} + 2k\pi, \frac{\pi}{4} + 2k\pi\right)$$
$$k \in \mathbb{Z}$$

11.9 1 $\sin 3x < \sin x$ 9.3 (3)

$$\sin 3x - \sin x < 0$$
$$2 \sin \frac{3x - x}{2} \cos \frac{3x + x}{2} < 0$$
$$\sin x \cos(2x) < 0$$

$\sin x > 0 \wedge \cos 2x < 0$	$\sin x < 0 \wedge \cos 2x > 0$
• $x \in (0, \pi) + 2k\pi$	• $x \in (-\pi, 0) + 2k\pi$
• $2x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) + 2k\pi$	• $2x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) + 2k\pi$
$x \in \left(\frac{\pi}{4}, \frac{3\pi}{4}\right) + k\pi$	$x \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right) + k\pi$
Tedy $x \in \left(\frac{\pi}{4}, \frac{3\pi}{4}\right) + 2k\pi$	$k = 0 \rightsquigarrow \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$
	$k = -1 \rightsquigarrow \left(-\frac{5\pi}{4}, -\frac{3\pi}{4}\right)$
	Tedy $x \in \left(-\frac{\pi}{4}, 0\right) + 2k\pi$
	nebo $x \in \left(-\pi, -\frac{3\pi}{4}\right) + 2k\pi$

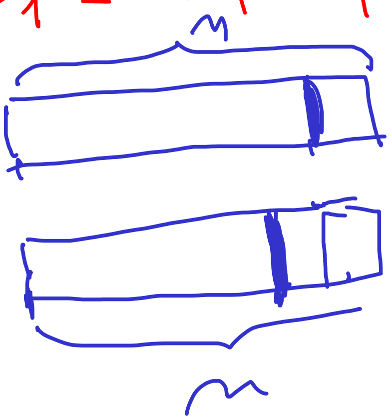
12. Rekurentní vztahy

12.1 (a) Kolik způsobů mohu vyjít 10 schodů, pokud dělám pouze kroky o jeden nebo dva schody?

P_n – počet způsobů, jak vyjít 10 schodů, dělám-li pouze kroky o 1 nebo 2 schody.

$P_{10} = ?$

$P_1 = 1, \quad P_2 = 2, \quad P_3 = 3, \dots$



$$P_n = P_{n-1} + P_{n-2}$$

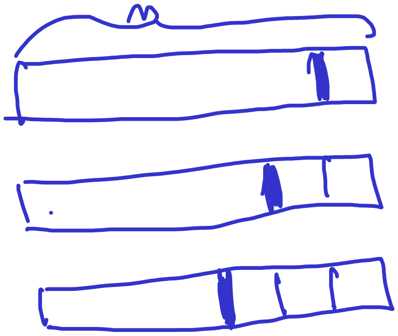
Fibonacciho posloupnost

$1, 2, 3, 5, 8, 13, 21, 34, 55, 89$

$89 = P_{10}$

Z: Mohu dělat kroky o 1, 2 nebo 3 schody.

$P_1 = 1, \quad P_2 = 2, \quad P_3 = 4$



$$P_n = P_{n-1} + P_{n-2} + P_{n-3}$$

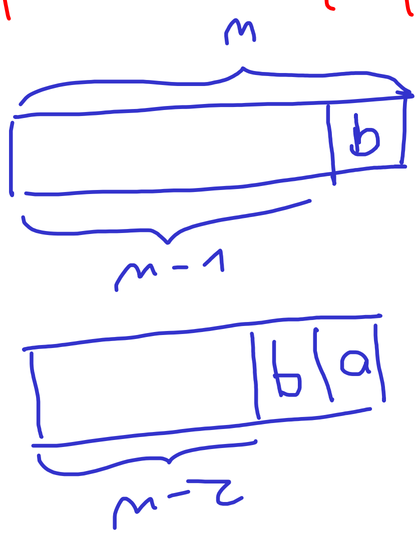
1, 2, 4, 7, 13, 24, 44, 81, 149, 274

$149 = P_9$, $\boxed{274 = P_{10}}$

12.2: Určete, kolik je řetězců délky $n = 10$ nad abecedou $\{a, b\}$, které neobsahují podřetězec "aa".

P_n - počet takových řetězců délky n.

$P_1 = 2$, $P_2 = 4 - 1 = 3$



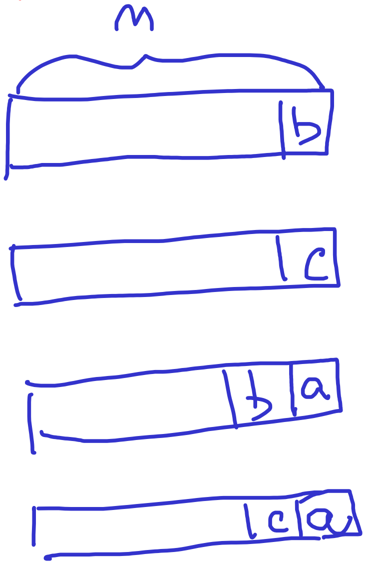
$$P_n = P_{n-1} + P_{n-2}$$

2, 3, 5, 8, 13, 21, 34, 55, 89, 144

$\boxed{144 = P_{10}}$

(b) totéž pro obecedu $\{a, b, c\}$

$P_1 = 3$, $P_2 = 3\cdot 3 - 1 = 8$ --

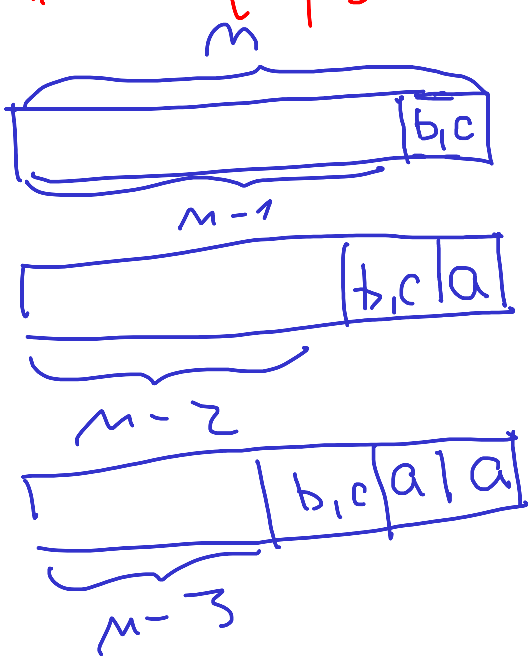


$$P_n = P_{n-1} + P_{n-1} + P_{n-2} + P_{n-2} = 2P_{n-1} + 2P_{n-2}$$

3, 8, 22, 60, 164, 2·224=448, ...

(c) abeceda $\{a, b, c\}$, neobsahuje podřetězec "aaa"

$P_1 = 3$, $P_2 = 9$, $P_3 = 3^3 - 1 = 26$,



$$P_n = 2P_{n-1} + 2P_{n-2} + 2P_{n-3}$$

3, 9, 26, 2·38=76, ...

Explicitní vztah pro

p_n splňující $p_n = p_{n-1} + p_{n-2}$

$p_1 = 1, \; p_2 = 2$

• najdeme všechny posloupnosti p_n splňující $p_n - p_{n-1} - p_{n-2} = 0$

"uhodneme" p_n ve tvaru $p_n = \lambda^n$, $\lambda \neq 0$

dosazení: $\lambda^n - \lambda^{n-1} - \lambda^{n-2} = 0$

$$\underbrace{\lambda^{n-2}}_{\neq 0} \underbrace{(\lambda^2 - \lambda - 1)}_{=0} = 0$$

kořeny: $\lambda_{1,2} = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$

Obecné řešení rovnice

$p_n - p_{n-1} - p_{n-2} = 0$ je tvaru

$$\boxed{p_n = A \cdot \left(\frac{1+\sqrt{5}}{2}\right)^n + B \cdot \left(\frac{1-\sqrt{5}}{2}\right)^n, \; A, B \in \mathbb{R}}$$

Použijeme počáteční podmínky

$$P_1 = A\,\frac{1+\sqrt{5}}{2} + B\,\frac{1-\sqrt{5}}{2} = 1 \quad / -\frac{1+\sqrt{5}}{2}$$

$$P_2 = A\left(\frac{1+\sqrt{5}}{2}\right)^2 + B\left(\frac{1-\sqrt{5}}{2}\right)^2 = 2$$

$$B\left[\left(\frac{1-\sqrt{5}}{2}\right)^2 - \frac{1+\sqrt{5}}{2}\cdot\frac{1-\sqrt{5}}{2}\right] = 2 - \frac{1+\sqrt{5}}{2}$$

$$B\cdot\frac{1-\sqrt{5}}{2}\underbrace{\left(\frac{1-\sqrt{5}}{2} - \frac{1+\sqrt{5}}{2}\right)}_{-\sqrt{5}} = \frac{3-\sqrt{5}}{2}$$

$$B\,\frac{5-\sqrt{5}}{2} = \frac{3-\sqrt{5}}{2}$$

$$B = \frac{3-\sqrt{5}}{5-\sqrt{5}}\cdot\frac{5+\sqrt{5}}{5+\sqrt{5}} = \frac{15+3\sqrt{5}-5\sqrt{5}-5}{25-5}$$

$$= \frac{10-2\sqrt{5}}{20} = \frac{5-\sqrt{5}}{10}$$

$$A\,\frac{1+\sqrt{5}}{2} + \frac{5-\sqrt{5}}{10}\cdot\frac{1-\sqrt{5}}{2} = 1$$

$$A\,\frac{1+\sqrt{5}}{2} + \frac{5-5\sqrt{5}-\sqrt{5}+5}{20} = 1$$

$$A\,\frac{1+\sqrt{5}}{2} + \frac{10-6\sqrt{5}}{20} = 1$$

$$A\,\frac{1+\sqrt{5}}{2} = 1 - \frac{5-3\sqrt{5}}{10} = \frac{5+3\sqrt{5}}{10}$$

$$A = \frac{5+3\sqrt{5}}{5(1+\sqrt{5})} \cdot \frac{1-\sqrt{5}}{1-\sqrt{5}} =$$

$$= \frac{5-5\sqrt{5}+3\sqrt{5}-3\cdot 5}{5(1-5)} = \frac{-10-2\sqrt{5}}{-20}$$

Závěr: $A = \frac{5+\sqrt{5}}{10}$ $\quad B = \frac{5-\sqrt{5}}{10}$

$$p_n = \frac{5+\sqrt{5}}{10}\left(\frac{1+\sqrt{5}}{2}\right)^n + \frac{5-\sqrt{5}}{10}\left(\frac{1-\sqrt{5}}{2}\right)^n$$