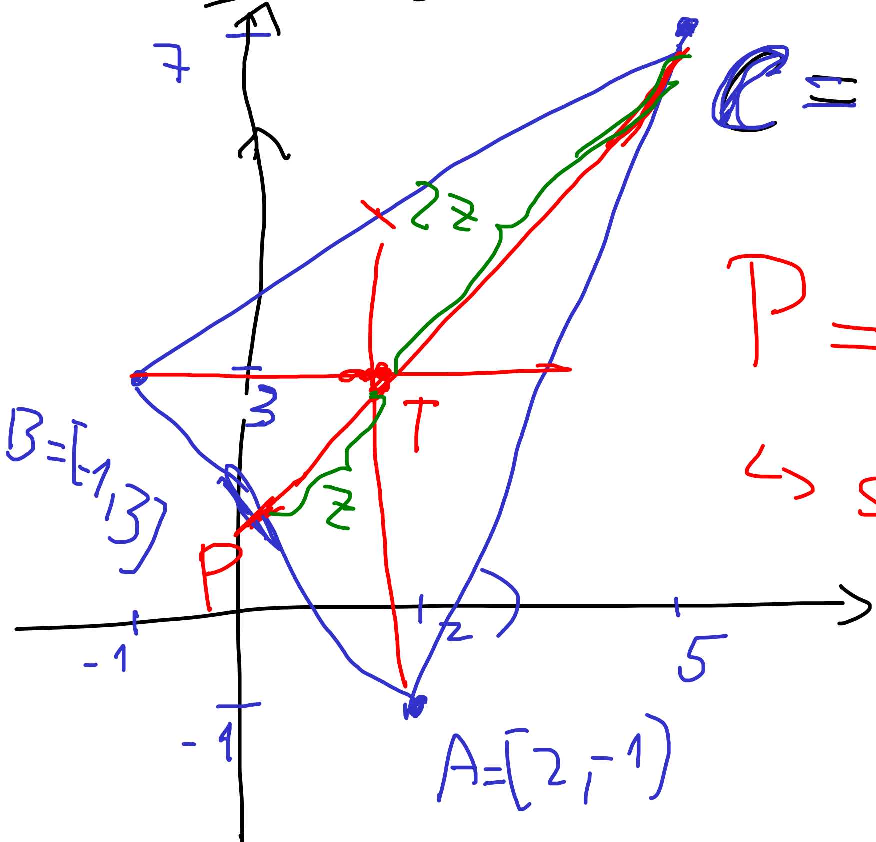
2.1 $\triangle ABC$ $\quad A=[2,-1]$, $B=[-1,3]$



$C=[5,7]$

$$P=\left[\frac{2+(-1)}{2}, \frac{-1+3}{2}\right]=\left[\frac{1}{2}, 1\right]$$

↳ střed hrany AB

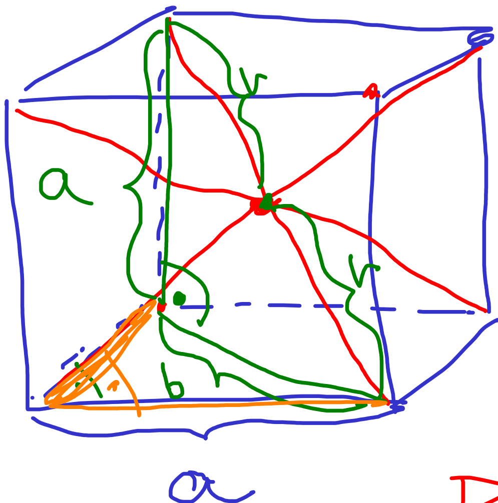
$$T = C + \frac{2}{3}\,\overrightarrow{CP} =$$

$$= [5,7] + \frac{2}{3}\left(\frac{1}{2}-5, 1-7\right) =$$

$$= [5,7] + \frac{2}{3}\left(-\frac{9}{2}, -6\right) =$$

$$= [5,7] + (-3,-4) = [2,3]$$

2.2 Mějme krychli vepsanou do kulové plochy o pol. r, přičemž povrch krychle je 72 cm². Určete hodnotu r.



$72 = 6a^2$

$12 = a^2$

Střed kulové plochy bude ve středu (těžišti) krychle.

$a^2 + a^2 = b^2$

$b^2 = 2a^2 \rightarrow b = \sqrt{2}\,a$

$a^2 + (\sqrt{2}\,a)^2 = (2r)^2$

$a^2 + 2a^2 = 4r^2$

$3\underbrace{a^2}_{12} = 4r^2 \Rightarrow 3 \cdot 12 = 4r^2$

$9 = r^2 \quad r = 3$

2.3 M je množina reálných čísel $x \in \mathbb{R}$ takových, že $|2x+1| < x+3$.

- $2x+1 = 0 \rightsquigarrow x = -\frac{1}{2}$
- $x \in (-\infty, -\frac{1}{2}]$ $\qquad$ $x \in [-\frac{1}{2}, \infty)$

$|2x+1| = -(2x+1) < x+3$

$0 < 3x+4$

$3x > -4$

$x > -\frac{4}{3}$

interval

$(-\frac{4}{3}, -\frac{1}{2}]$

$|2x+1| = 2x+1 < x+3$

$x < 2$

interval

$[-\frac{1}{2}, 2)$

sjednocení

$M = (-\frac{4}{3}, 2)$

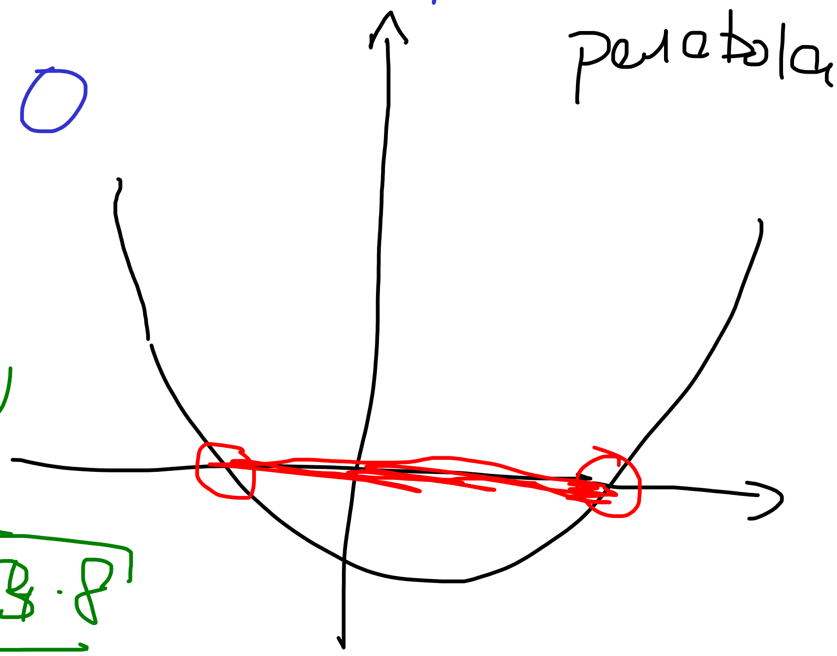
Jiné řešení $|2x+1| < x+3 \quad /(\,)^2$

$\Updownarrow$ ekvivalentní

$$(2x+1)^2 < (x+3)^2$$

$$4x^2+4x+1 < x^2+6x+9$$

$$3x^2-2x-8<0$$



$$3x^2-2x-8=0$$

$$x_{1,2} = \frac{2 \pm \sqrt{4-4\cdot 3\cdot(-8)}}{2\cdot 3} = \frac{2 \pm \sqrt{4}\cdot\sqrt{1+3\cdot 8}}{6}$$

$$= \frac{2 \pm 2\cdot 5}{6} = \frac{1\pm 5}{3} = \begin{cases} 2 \\ -\frac{4}{3} \end{cases}$$

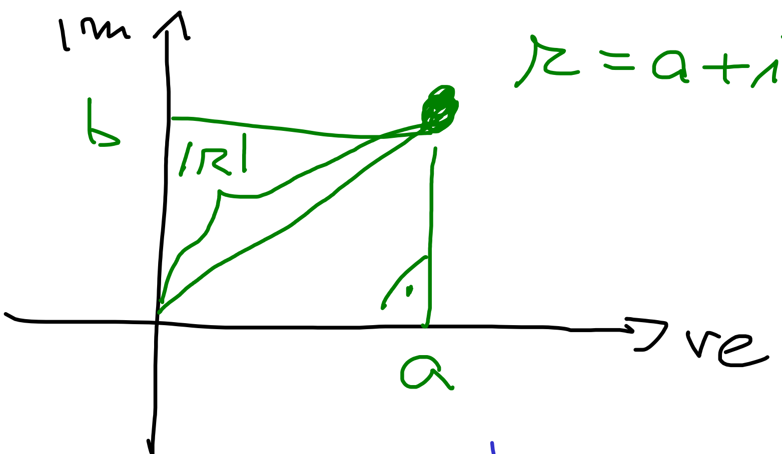
$$M = \left(-\frac{4}{3}, 2\right)$$

Z.4 $z + |z| = 5 + (2+i)^2$

$z \in \mathbb{C}$

$i^2 = -1$

$\mathbb{C} = \{ a + ib \mid a, b \in \mathbb{R} \}$



$z = a + ib$

$|z| = \sqrt{a^2+b^2}$

$z = a + ib$

$(a+ib) + \sqrt{a^2+b^2} = 5 + (4 + 4i - 1)$

$\underbrace{a + \sqrt{a^2+b^2}}_{re} + \underbrace{ib}_{im} = \underbrace{8}_{re} + \underbrace{4i}_{im}$

$a + \sqrt{a^2+b^2} = 8$

$\boxed{b = 4}$ $\Rightarrow a + \sqrt{a^2+4^2} = 8$

$\sqrt{a^2+16} = 8 - a \quad /()^2$

$\cancel{a^2} + 16 = 64 - 16a + \cancel{a^2}$

$16a = 64 - 16 = 48$

$a = 3$ $z = 3 + 4i$

$$z^2 = (3+4i)^2 = 9 + 24i - 16$$

$$= \underline{\underline{-7 + 24i}}$$

2.5 $a < b$, $a, b \in \mathbb{R}$

jsou rešenih vornic

$$x^{2\log x + 3,5} = 100\sqrt{x} \quad / \cdot x^{-\frac{1}{2}}$$

Vučete čislo $k = ab^2$

$$\sqrt{x} = x^{\frac{1}{2}}$$

- $x^{2\log x + \frac{7}{2} - \frac{1}{2}} = 100$

$$\underbrace{x^{2\log x + 3}} = 100 \quad / \log(\ldots)$$

$$\log(c^{d}) = d\log c$$

$$(2\underbrace{\log x}_{y} + 3)\underbrace{\log x}_{y} = 2$$

$$(2y+3)y = 2 \rightarrow \boxed{2y^2 + 3y - 2 = 0}$$

$$y_{1,2} = \frac{-3 \pm \sqrt{9 - 4\cdot 2\cdot(-2)}}{4} = \frac{-3 \pm \sqrt{9+16}}{4} = \frac{-3 \pm 5}{4} = \begin{cases} -2 = y_1 \\ \frac{1}{2} = y_2 \end{cases}$$

- $\log x_1 = -2 \quad / 10^{()}$

$$x_1 = 10^{-2} = \frac{1}{100}$$

- $\log x_2 = \frac{1}{2} \quad / 10^{()}$

$$x_2 = 10^{\frac{1}{2}} = \sqrt{10}$$

$$\Rightarrow \quad a = \frac{1}{100} \qquad b = \sqrt{10}$$

$$ab^2 = \frac{1}{100} \cdot (\sqrt{10})^2 = \frac{10}{100} = \frac{1}{10}$$

2.6 $\cos x + \sin x = \sqrt{2}$ $/ (\ )^2$

c! = součet všech řešení této rovnice na intervalu $[0, 2\pi]$

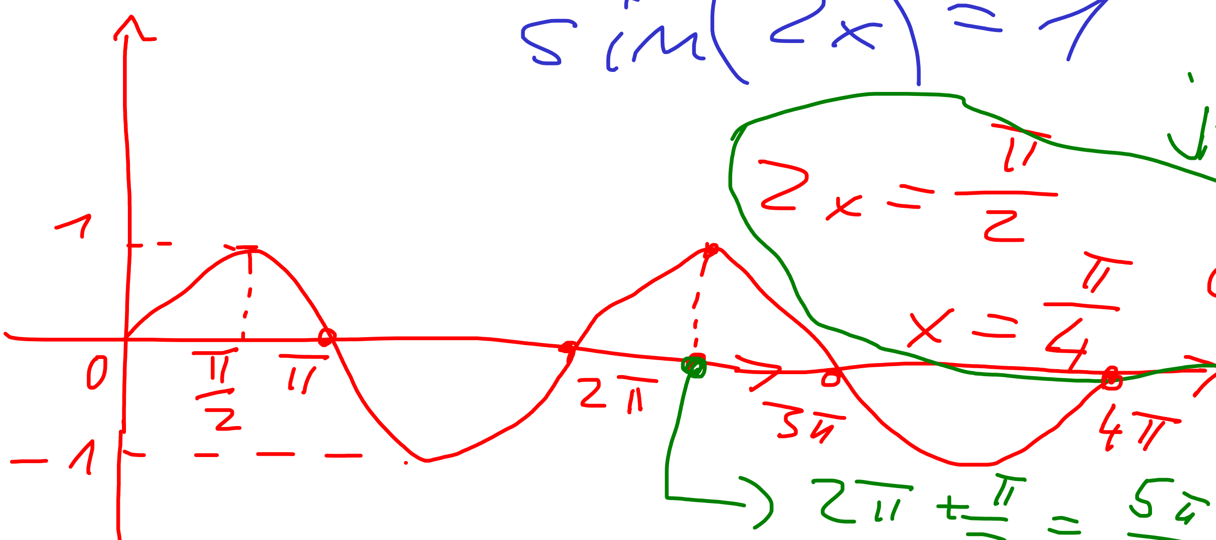
• $(\cos x + \sin x)^2 = (\sqrt{2})^2$

$$\cos^2 x + 2\sin x \cos x + \sin^2 x = 2$$

$$1 + 2\sin x \cos x = 2$$

$$2\sin x \cos x = 1$$

$$\sin(2x) = 1$$



$$2x = \frac{\pi}{2}$$

$$x = \frac{\pi}{4}$$

jedno řešení

$$\cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\to 2\pi + \frac{\pi}{2} = \frac{5\pi}{2}$$

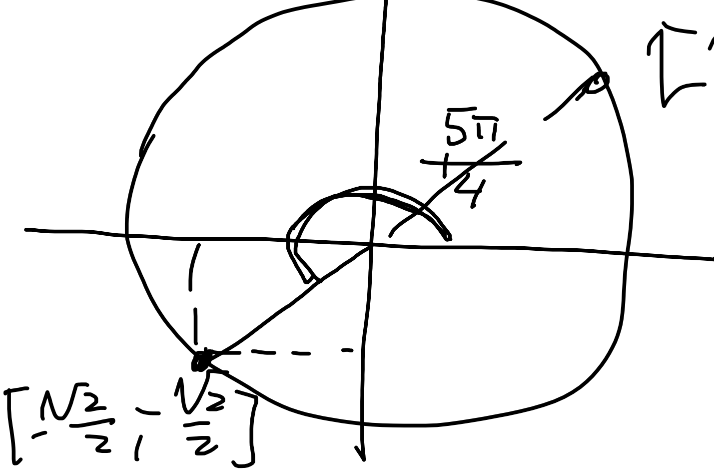
$$x \in [0, 2\pi]$$

$$2x \in [0, 4\pi]$$

$$2x = \frac{5\pi}{2}$$

$$x = \frac{5\pi}{4}$$

$$x = \pi + \frac{\pi}{4}$$



$$\cos\frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$\sin\frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$$

$\Rightarrow x = \frac{5\pi}{4}$ není řešení

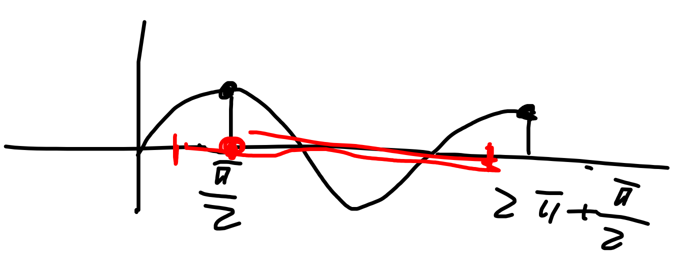
Závěr: $x = \frac{\pi}{4}$ jediné řešení $\Rightarrow c = \frac{\pi}{4}$

$\boxed{\text{Jinak}}$

$$\underbrace{\frac{\sqrt{2}}{2}}_{\cos\frac{\pi}{4}}\sin x + \underbrace{\frac{\sqrt{2}}{2}}_{\sin\frac{\pi}{4}}\cos x = 1$$

$$\sin\left(x + \frac{\pi}{4}\right) = 1$$

$$x \in [0, 2\pi] \Rightarrow x + \frac{\pi}{4} \in \left[\frac{\pi}{4}, 2\pi + \frac{\pi}{4}\right]$$



$x + \frac{\pi}{4} = \pi$

$x + \frac{\pi}{4} = \frac{\pi}{2}$

$x = \frac{\pi}{4}$

Z. 6 Určete počet všech lichých pěticiferných čísel, která ve svém zápisu neobsahují cifru 9.

$$8 \cdot 9^3 \cdot 4$$

_ _ _ _ _

8 9 9 9 4

2.8 kuželosečke

$$3x^2+5y^2+6x-20y+8=0$$

a, b dolžini poloos $\rightsquigarrow c = a^2+b^2$

$$3(x^2+2x)+5(y^2-4y)+8=0$$

$$3[(x+1)^2-1]+5[(y-2)^2-4]+8=0$$

$$3(x+1)^2+5(y-2)^2-3-20+8=0$$

$$3(x+1)^2+5(y-2)=15 \quad /15$$

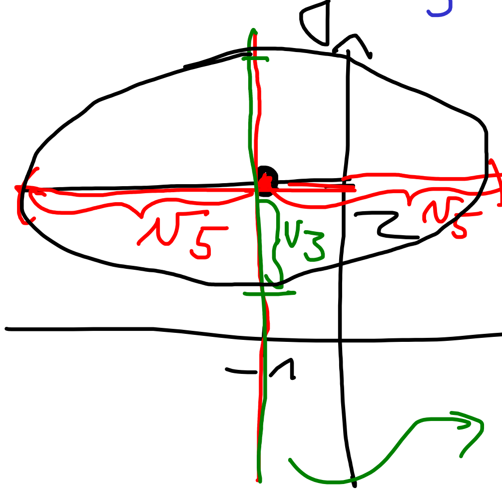
$$\frac{(x+1)^2}{5}+\frac{(y-2)^2}{3}=1$$

$\rightsquigarrow a=\sqrt{5}$

$b=\sqrt{3}$

$(x+1)^2=5$

$x+1=\pm\sqrt{5}$



$y=2$ $\quad x=-1\pm\sqrt{5}$

$$\boxed{a^2+b^2=5+3=8}$$

$x=-1 \Rightarrow (y-2)^2=3$

$y=2\pm\sqrt{3}$

2.9 Co je to medián?

$a_1, \dots, a_n$ n reálných čísel

• n liché

a_i je medián n-tice $a_1, \dots, a_n$ jestliže polovina z čísel $a_1, \dots, a_{i-1}, a_{i+1}, \dots, a_n$ je $\leq a_i$ a polovina $\geq a_i$

Př: • $1, 2, 3 \Rightarrow$ průměr 2
medián 2

• $1, 2, 6 \Rightarrow$ průměr 3
medián 2

• $1, 5, 6 \Rightarrow$ průměr 4
medián 5

• n sudé

mějme čísla $a_i < a_j$ taková, že polovina z čísel

$a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_{j-1}, a_j, a_{j+1}, \ldots, a_n$

byle $\le a_i$ a połowa $\ge a_j$

Pak mediana $:= \frac{a_i + a_j}{2}$

2.10 $A(a,b) = \frac{a+b}{2}$ $a,b>0$

$G(a,b) = \sqrt{ab}$

$H(a,b) = \frac{2}{\frac{1}{a}+\frac{1}{b}}$

$A \geq G \geq H$ → doma

↳ $\frac{a+b}{2} \geq \sqrt{ab}$ $/()^2$

$(a+b)^2 \geq (2\sqrt{ab})^2$

$a^2+2ab+b^2 \geq 4ab$

$a^2-2ab+b^2 \geq 0$

$(a-b)^2 \geq 0$ platí

Pozn: rovnost nastane pouze pro $a=b$