3.1 · definiční obor, obor hodnot

· graf

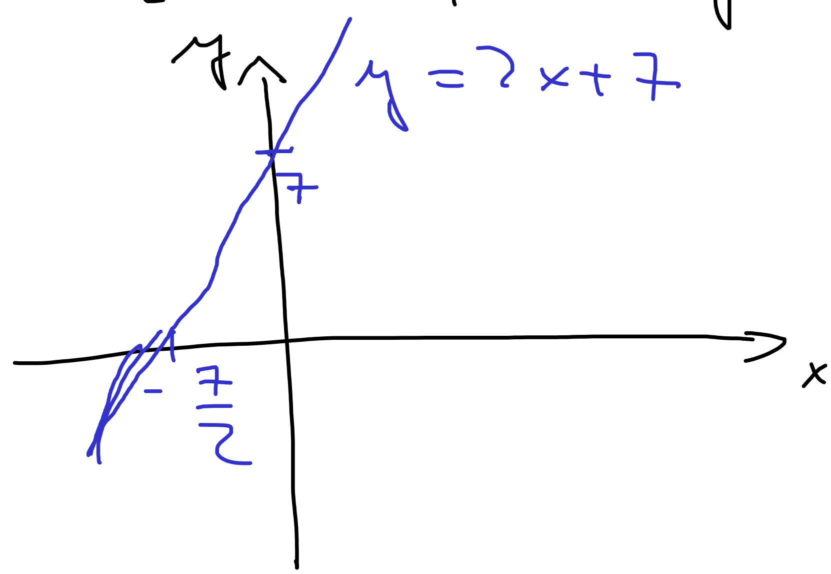
· injektivita / surjektivita

· $f:\mathbb{R}\to\mathbb{R}$ je injektivní (prostá), jestliže $\forall x_1, x_2 \in \mathbb{R},\ x_1 \neq x_2$
$\Rightarrow f(x_1) \neq f(x_2)$

· $f:\mathbb{R}\to\mathbb{R}$ je surjektivní, jestliže $\forall y \in \mathbb{R}\ \exists x \in \mathbb{R}: f(x) = y$

· funkce rostoucí / klesající

· $f(x) = 2x + 7 = y$



$f(0) = 7$

$f(x) = 2x + 7 = 0$

$x = -\frac{7}{2}$

$D(f) = \mathbb{R}$

$H(f) = \mathbb{R} = (-\infty, \infty)$

injektivní i surjektivní

rostoucí

• $f(x) = |\underbrace{3x+1}_{=0}| - x \rightsquigarrow x = -\frac{1}{3}$

$x \in (-\infty, -\frac{1}{3})$

$f(x) = -(3x+1) - x$

$= -4x - 1$

$x \in \langle -\frac{1}{3}, \infty)$

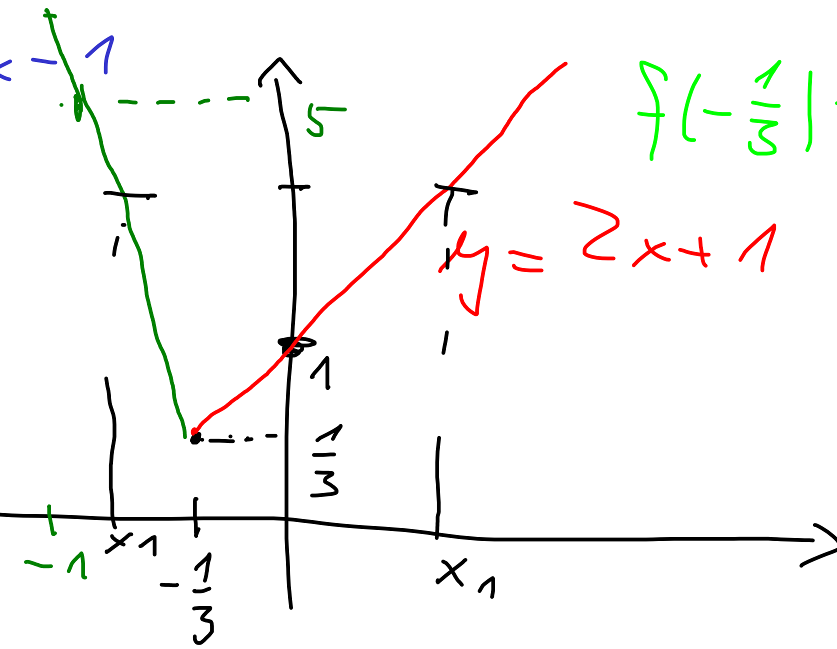
$f(x) = 3x + 1 - x$

$= 2x + 1$

$f(-\frac{1}{3}) = \frac{1}{3}$

$f(-1) = 5$

$f(-\frac{1}{3}) = \frac{1}{3}$



$D(f) = \mathbb{R}$

$H(f) = \langle -\frac{1}{3}, \infty)$

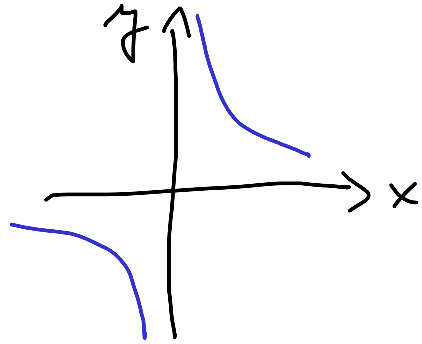
není inj. ani surjektivní

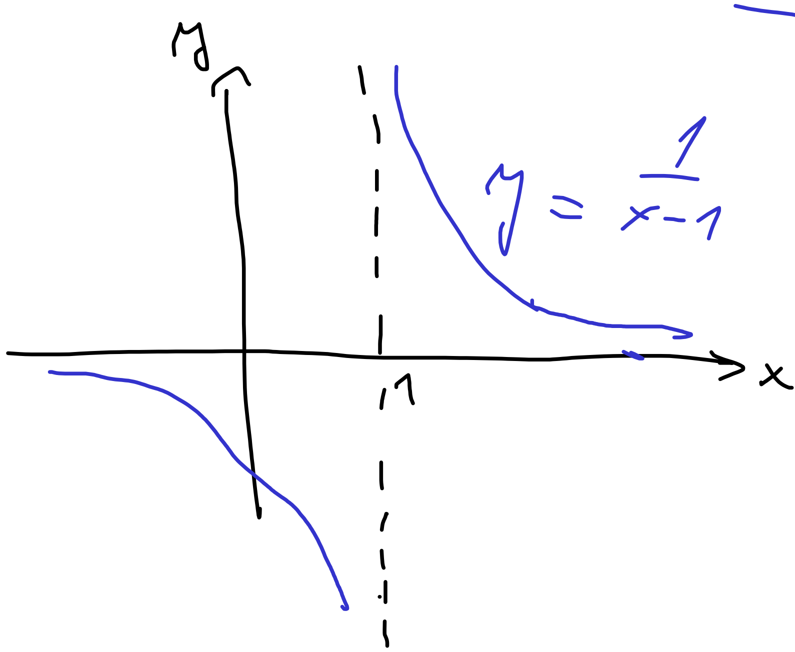
klesající na $(-\infty, -\frac{1}{3})$

rostoucí na $\langle -\frac{1}{3}, \infty)$

• $f(x) = \frac{1}{x-1}$

$f(x) = \frac{1}{x}$





$D(f) = \mathbb{R} \setminus \{1\}$

$H(f) = \mathbb{R} - \{0\}$

je injektivní

není surj.

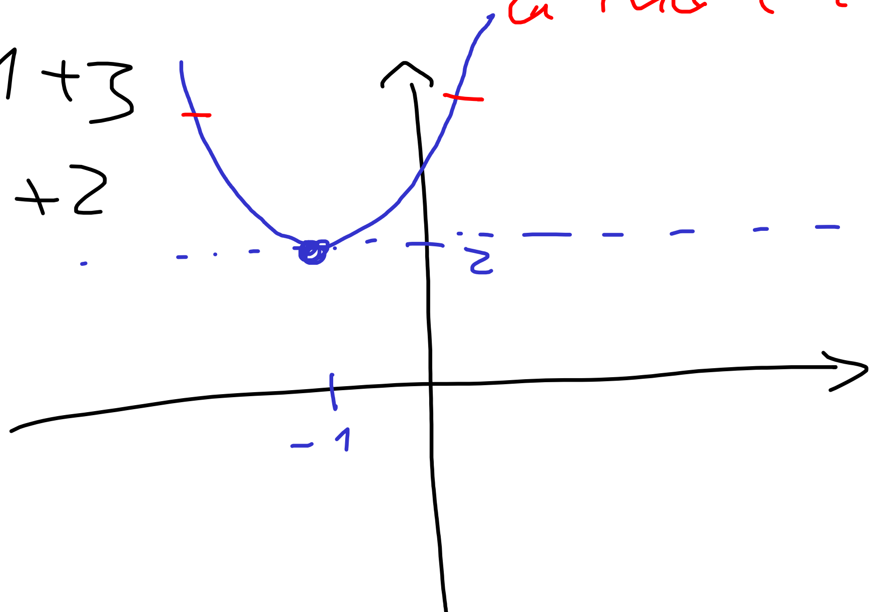
klesá na $(-\infty, 1)$

a na $(1, \infty)$

• $f(x) = x^2 + 2x + 3$

$= (x+1)^2 - 1 + 3$

$= (x+1)^2 + 2$



$D(f) = \mathbb{R}$

$H(f) = \langle 2, \infty)$

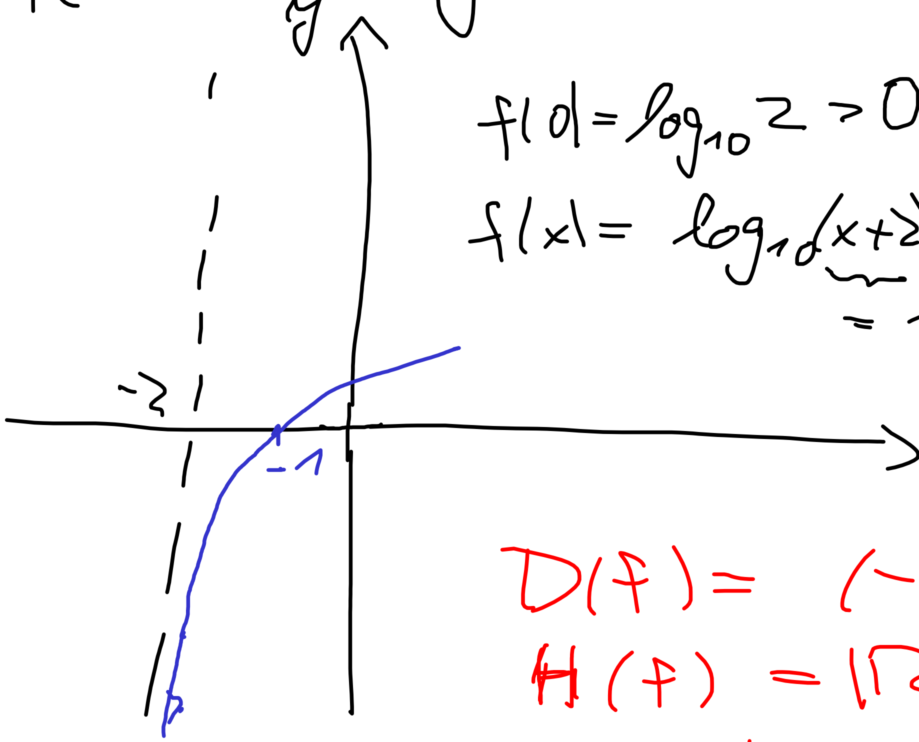
není inj.

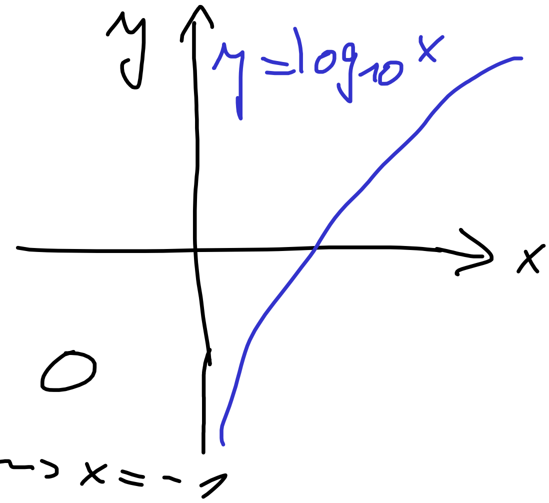
není surj.

klesající na $(-\infty, -1\rangle$

rostoucí na $\langle -1, \infty)$

$f(x) = \log_{10}(x+2) = y$





$f(0) = \log_{10} 2 > 0$

$f(x) = \log_{10}\underbrace{(x+2)}_{=1} = 0 \rightsquigarrow x = -1$

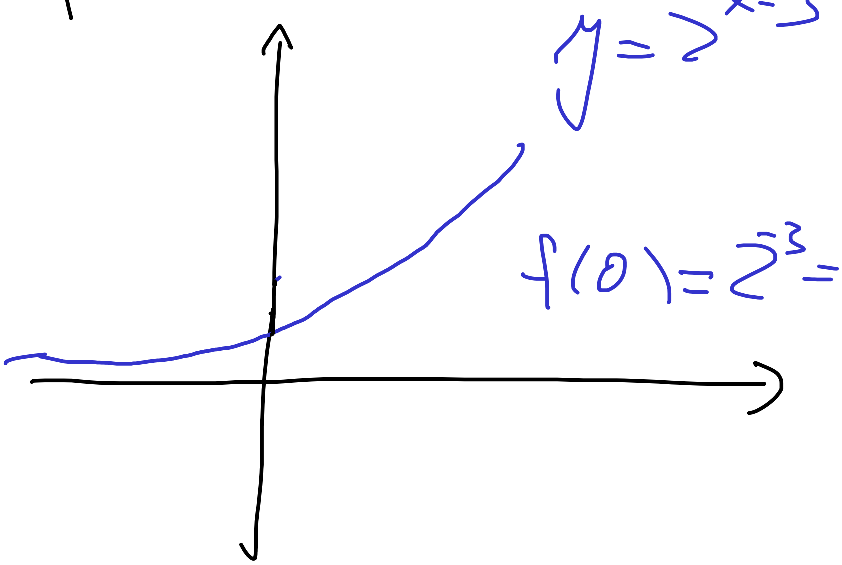
$D(f) = (-2, \infty)$

$H(f) = \mathbb{R}$

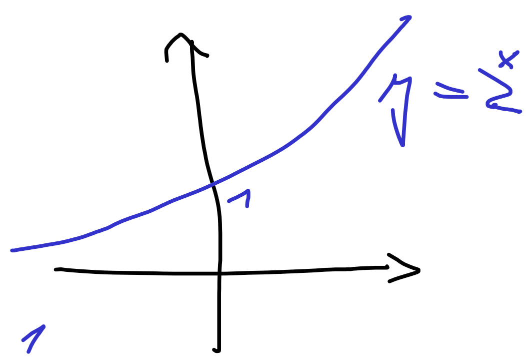
injektivní i surjektivní

rostoucí na $(-2, \infty)$

$f(x) = 2^{x-3}$



$f(0) = 2^{-3} = \frac{1}{8}$



$D(f) = \mathbb{R}$

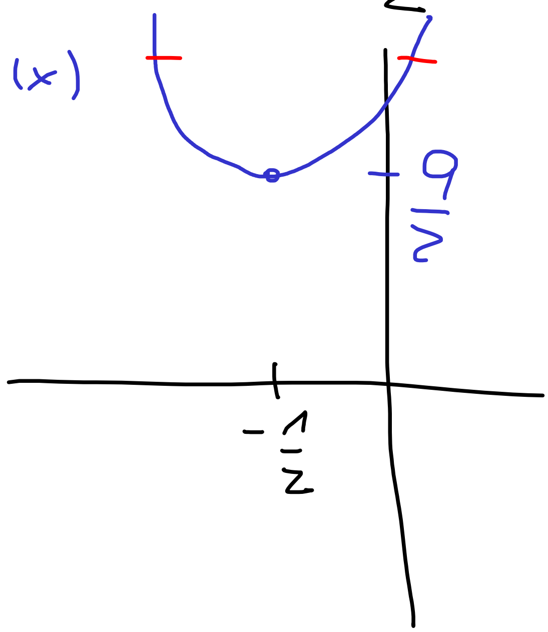
$H(f) = \mathbb{R}_+$

je inj, není surj

rostoucí na $\mathbb{R}$

$$\bullet\ f(x) = (x-1)^2 + (x+2)^2$$
$$= \underline{x^2} - 2x + 1 + \underline{x^2} + 4x + 4$$
$$= 2x^2 + 2x + 5$$
$$= 2(x^2 + x) + 5$$
$$= 2\left((x+\tfrac{1}{2})^2 - \tfrac{1}{4}\right) + 5$$
$$= 2(x+\tfrac{1}{2})^2 - \tfrac{1}{2} + 5$$
$$= 2(x+\tfrac{1}{2})^2 + \frac{9}{2}$$

$y = f(x)$



$D(f) = \mathbb{R}$

$H(f) = \langle \frac{9}{2}, \infty)$

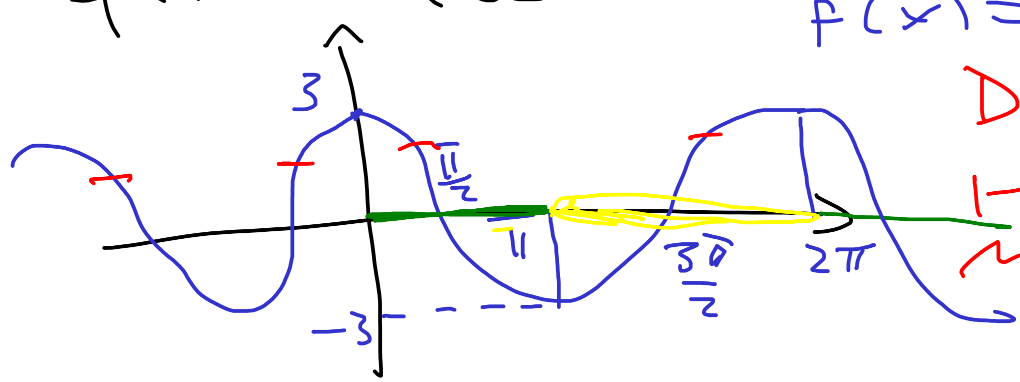
není injektivní

není surjektivní

klesající na $(-\infty, -\frac{1}{2}\rangle$

rostoucí na $\langle -\frac{1}{2}, \infty)$

$\bullet\ f(x) = 3\cos x$



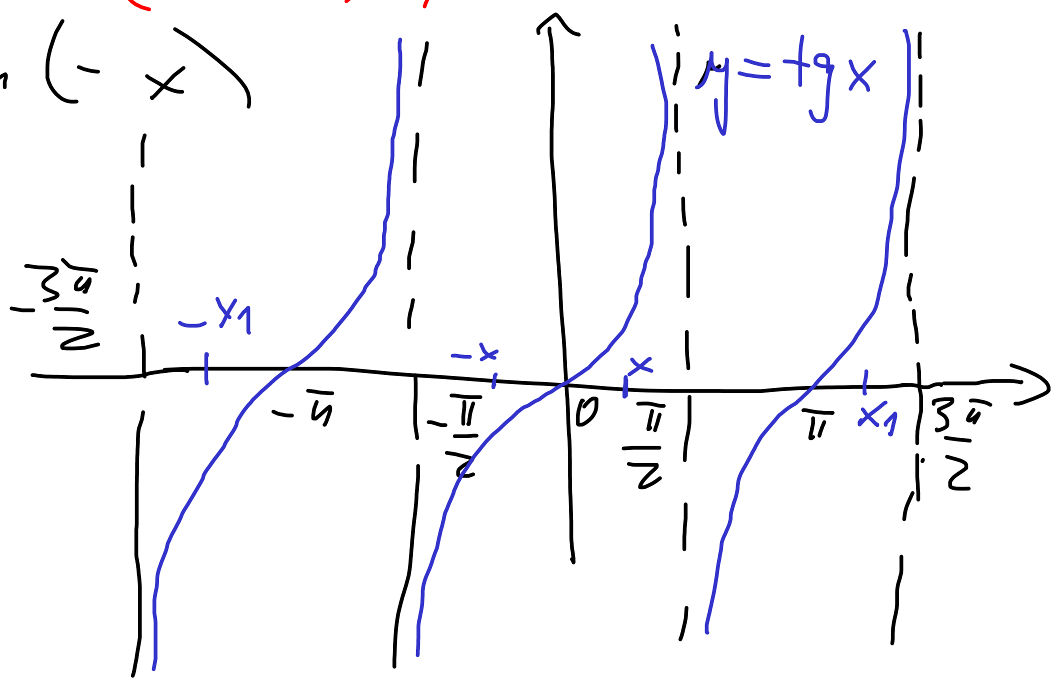
$f(x) = 3\cos x$

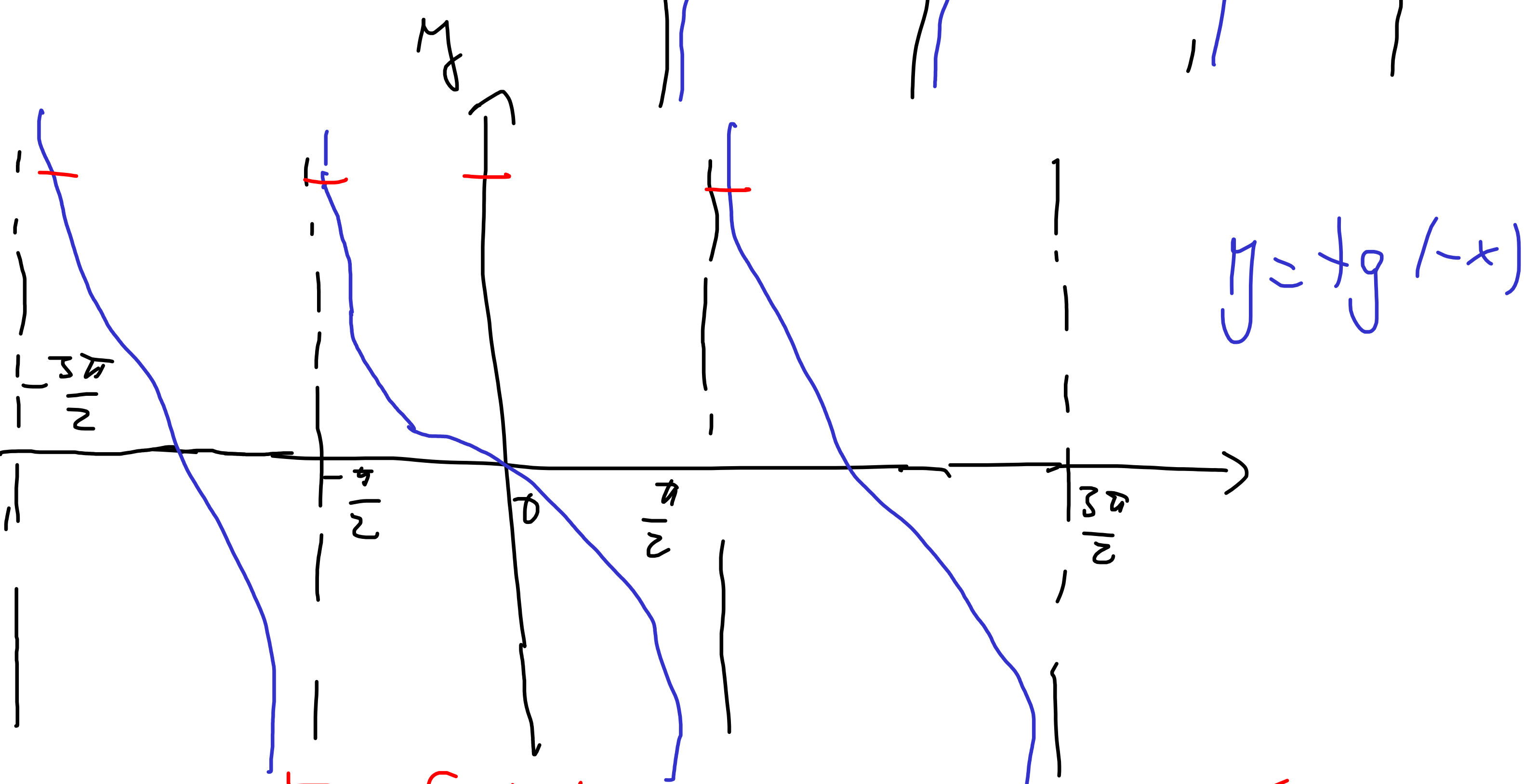
$D(f) = \mathbb{R}$

$H(f) = \langle -3, 3\rangle$

není inj ani surj

- klesajúca na $\langle 0, \pi \rangle$, obecnej na intervaloch $\langle 2k\pi, (2k+1)\pi \rangle$
$k \in \mathbb{Z}$
- rastúca $\langle (2k-1)\pi, 2k\pi \rangle$
- $f(x) = \tan(-x)$





$D(f) = \mathbb{R} \setminus \left\{ \frac{2k+1}{2}\pi \right\}$

$H(f) = \mathbb{R}$

nemá inj., je surj

klesá na $\left\langle \frac{2k-1}{2}\pi, \frac{2k+1}{2}\pi \right\rangle$

3.2 $f(x) = \dfrac{1}{\log_{10}(x^2-1)-1}$

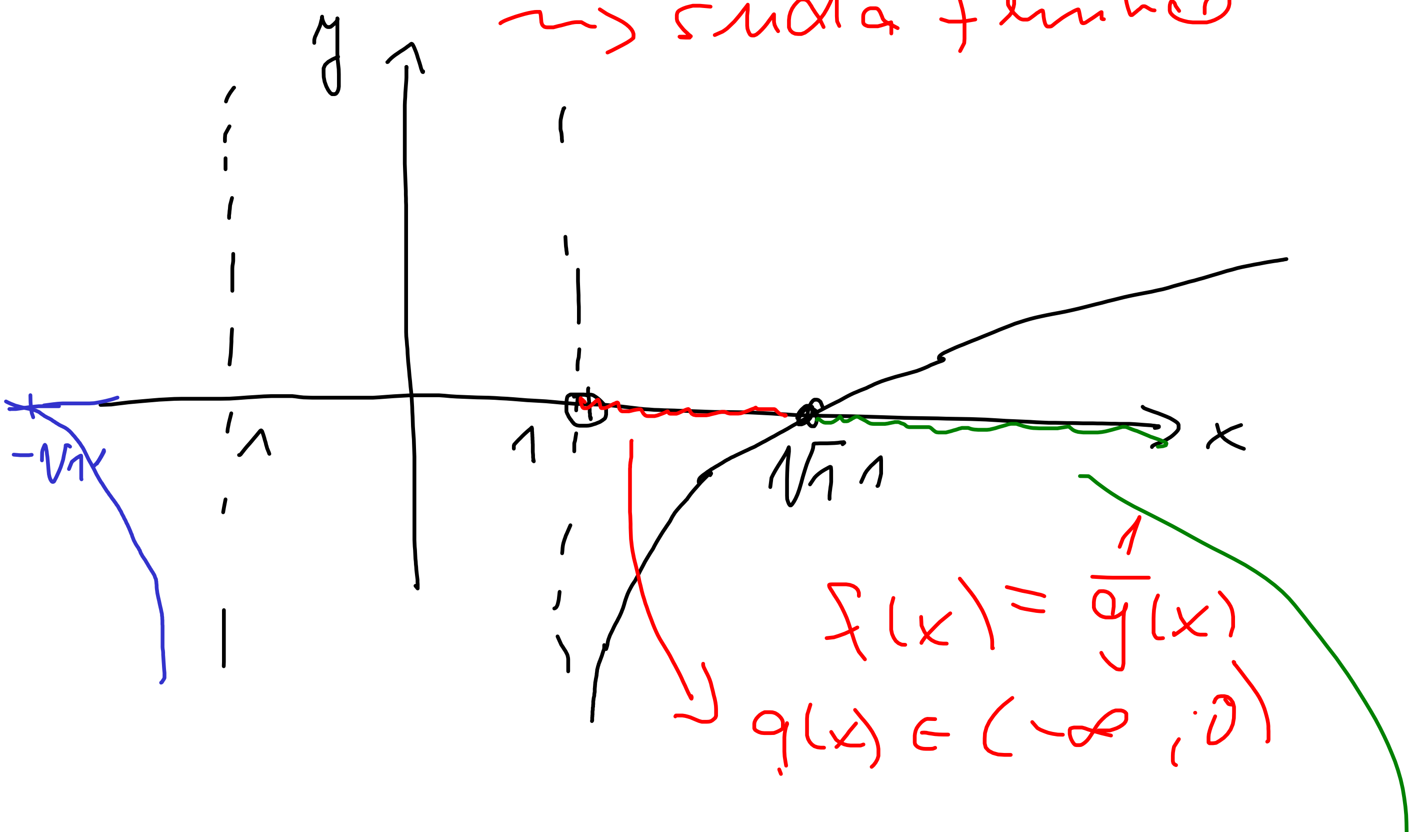
Určete D(f) a H(f)

- $\log_{10}(x^2-1) \neq 1$
  $x^2-1 \neq 10$
  $x^2 \neq 11$
  $x \neq \pm\sqrt{11}$

- $x^2-1 > 0$
  $x^2 > 1$
  $x \in (-\infty,-1) \cup$
  $\cup (1,\infty)$

$D(f) = (-\infty,-\sqrt{11}) \cup (-\sqrt{11},-1) \cup$
$\cup (1,\sqrt{11}) \cup (\sqrt{11},\infty)$

- obor hodnot:
  $g(x) = \log_{10}(x^2-1) - 1$
  $\rightsquigarrow$ sudá funkce



$f(x) = \dfrac{1}{g(x)}$

$\rightsquigarrow g(x) \in (-\infty, 0)$

$f(x) \in (-\infty, 0)$

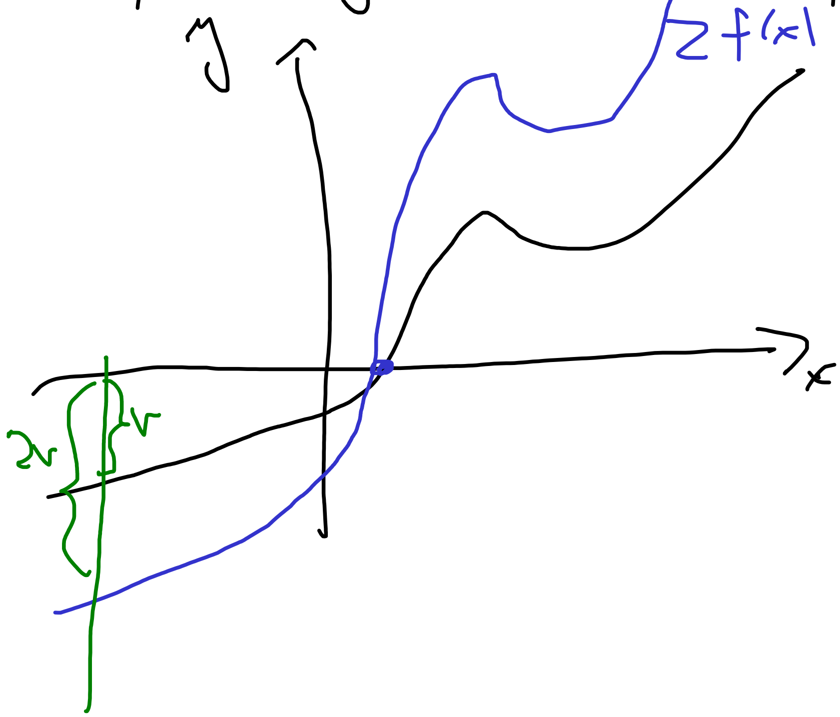
$H(f) = \mathbb{R} \setminus \{0\}$

$g(x) \in (0, \infty)$

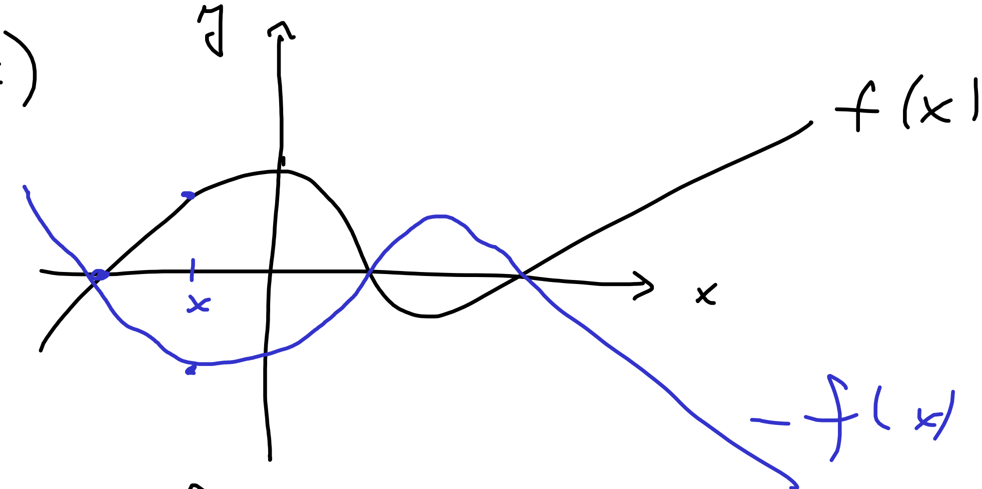
$f(x) \in (0, \infty)$

3.3 Jak se změní graf funkce $y = f(x)$, přejdeme-li k funkci

- $y = 2f(x)$



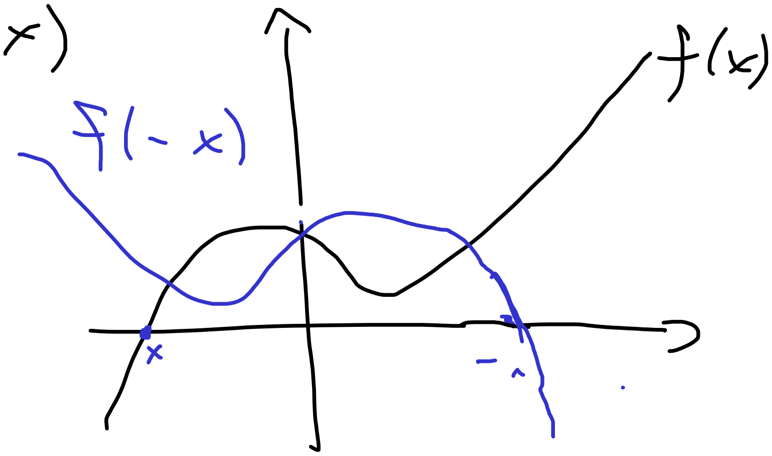
• $y = -f(x)$



• $y = f(-x)$

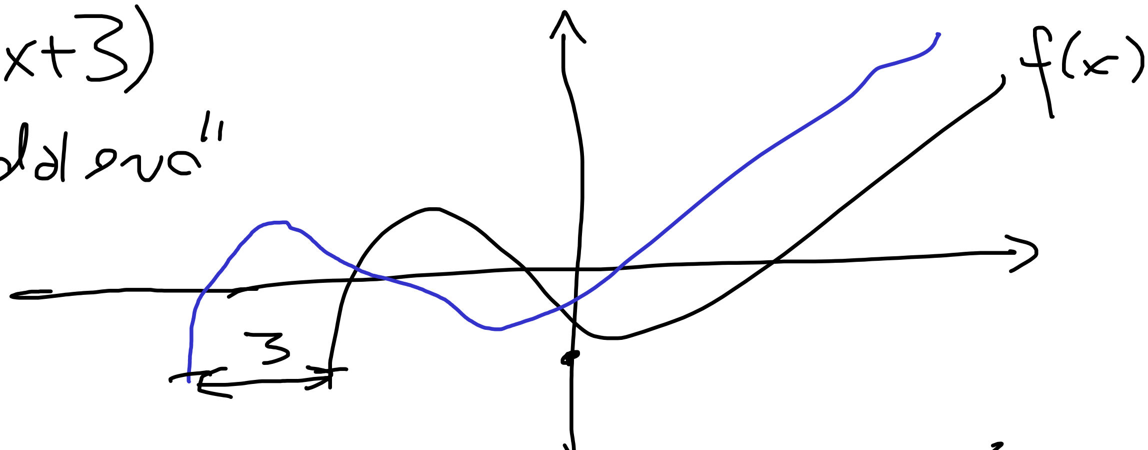
$f(a) = a$

$f(-a) = a$



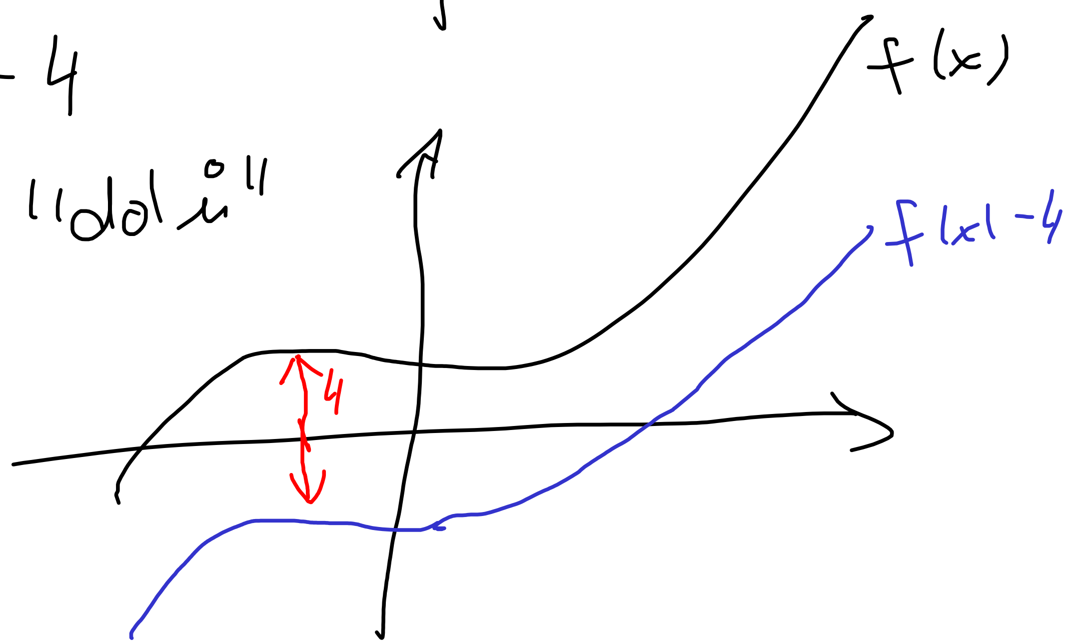
• $y = f(x+3)$

posun „doleva"

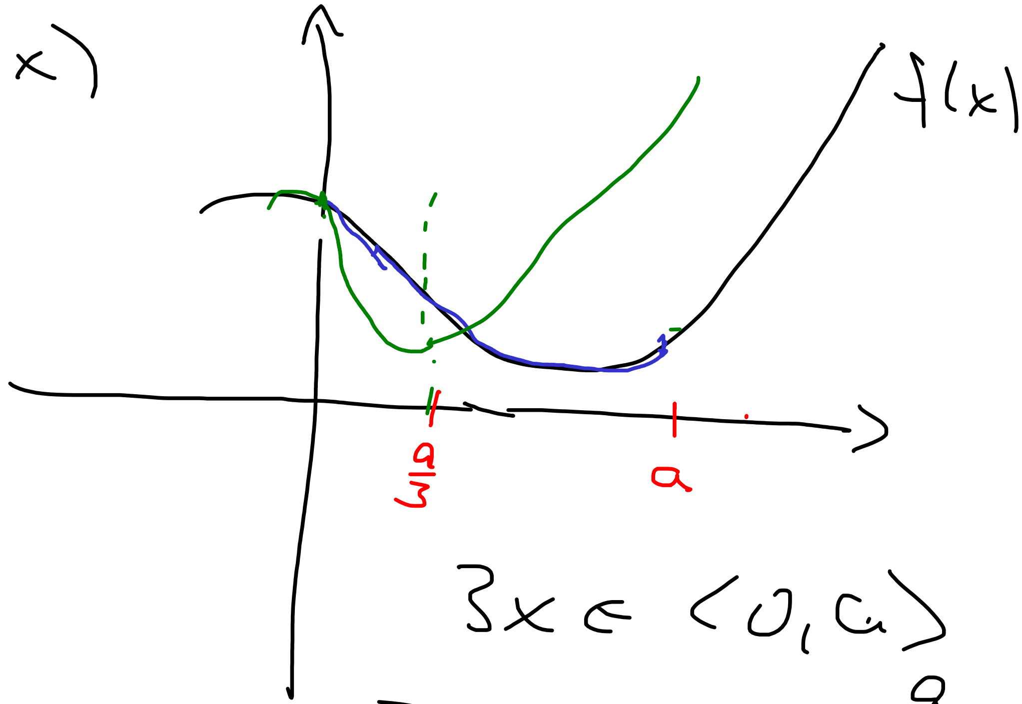


• $y = f(x) - 4$

posun „dolů"



$y = f(3x)$



$3x \in \langle 0, a \rangle$

$x \in \langle 0, \frac{a}{3} \rangle$

3.4 $g(x) = \frac{3}{x+5} + 2$

$g(x) = (f_4 \circ f_3 \circ f_2 \circ f_1)(x)$

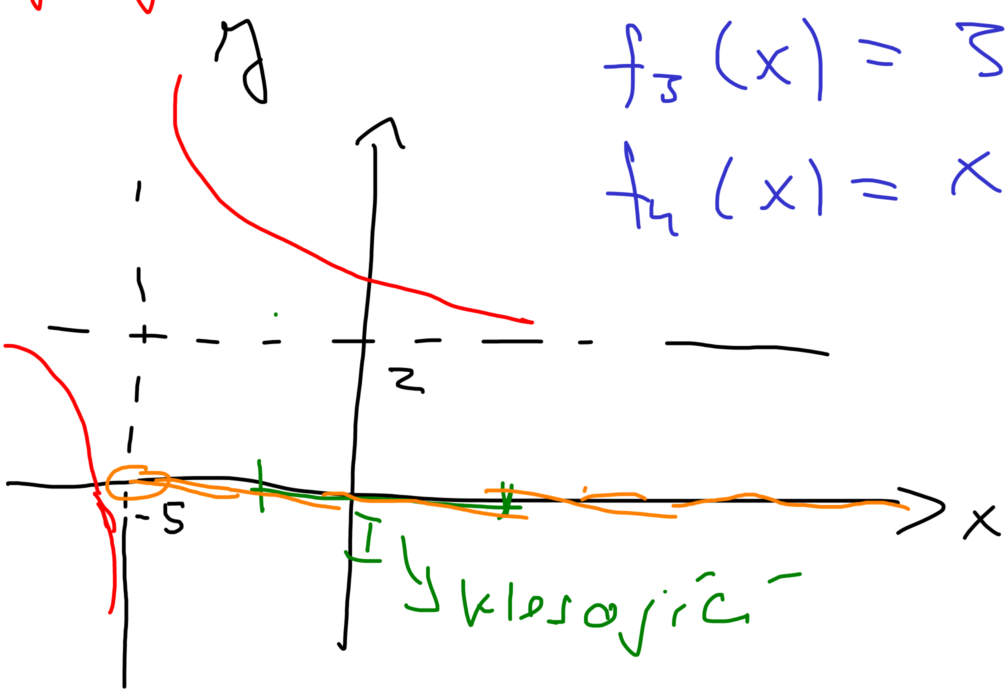
$f_1(x) = x+5$

$f_2(x) = \frac{1}{x}$

$f_3(x) = 3x$

$f_4(x) = x+2$

$y = g(x)$



$(-\infty, -5)$

$(-5, \infty)$

dva maximální intervaly, na nichž je f klesající

3.5

Def: Funkce $f: \mathbb{R} \to \mathbb{R}$ je <u>rostoucí na intervalu I</u>, jestliže pro $\forall x_1, x_2 \in I$, $x_1 < x_2$ platí $f(x_1) < f(x_2)$

Def: Interval I je <u>maximální</u>, na němž je f rostoucí, jestliže f je rostoucí na I a pro každý interval $J \supseteq I$ takový, že f je rostoucí na J, platí, že $J = I$.

$f \circ g(x)$

Věta: Nechť g je rostoucí funkce na intervalu I a f je rostoucí na intervalu J takovém, že $\{g(x) \mid x \in I\} \subseteq J$. Pak $f \circ g$ je rostoucí na I.

Dk: Chceme dokázat, že $f \circ g$ je rostoucí na I.

$x_1, x_2 \in I$ lib., $x_1 < x_2$

- $g(x_1), g(x_2) \in J$ t. ž. $g(x_1) \in g(x_2)$
- $f(g(x_1)) < f(g(x_2))$ /f

Tedy $f \circ g$ rostoucí na I.

3.6 Graf funkce

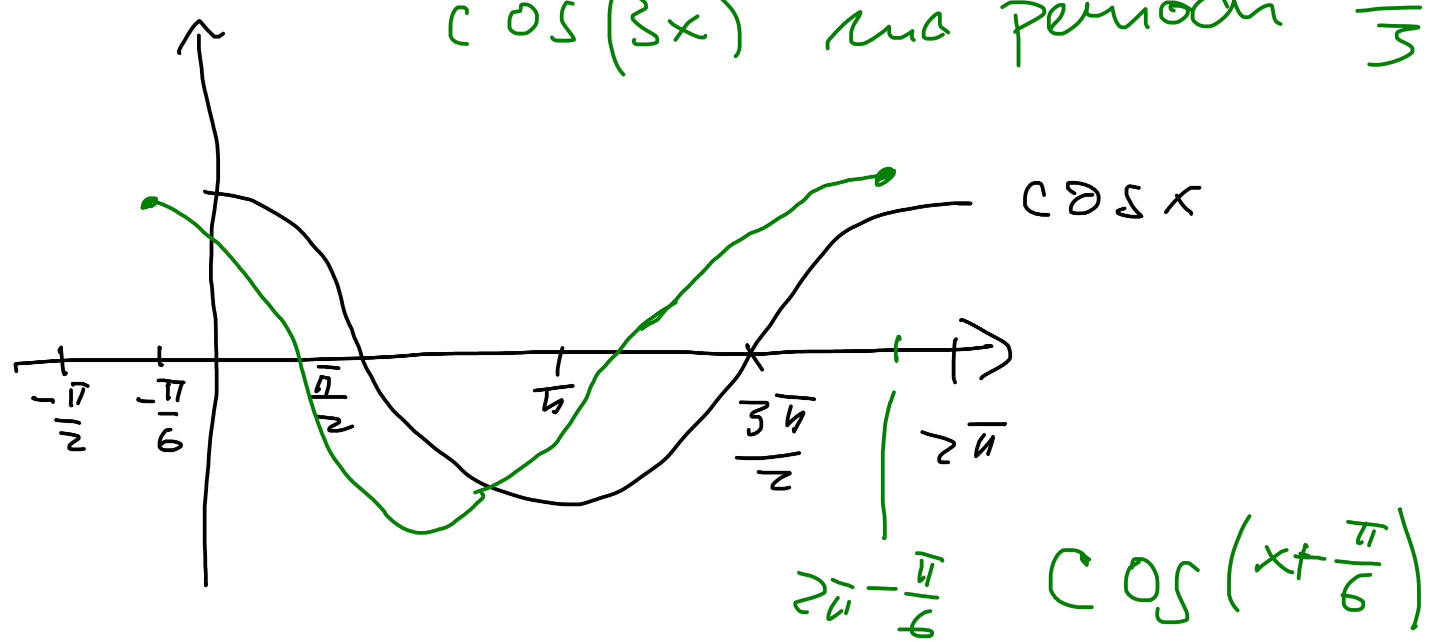
$$f(x) = 2\cos\left(3x + \frac{\pi}{2}\right) - 1$$

$g(x) := \cos\left(3x + \frac{\pi}{2}\right) = \cos\left(3\left(x + \frac{\pi}{6}\right)\right)$

$g(0) = 0$

- $\cos x$ má periodu 2π

$\cos(3x)$ má periodu $\frac{2\pi}{3}$



$\cos(3(x+\frac{\pi}{6}))$