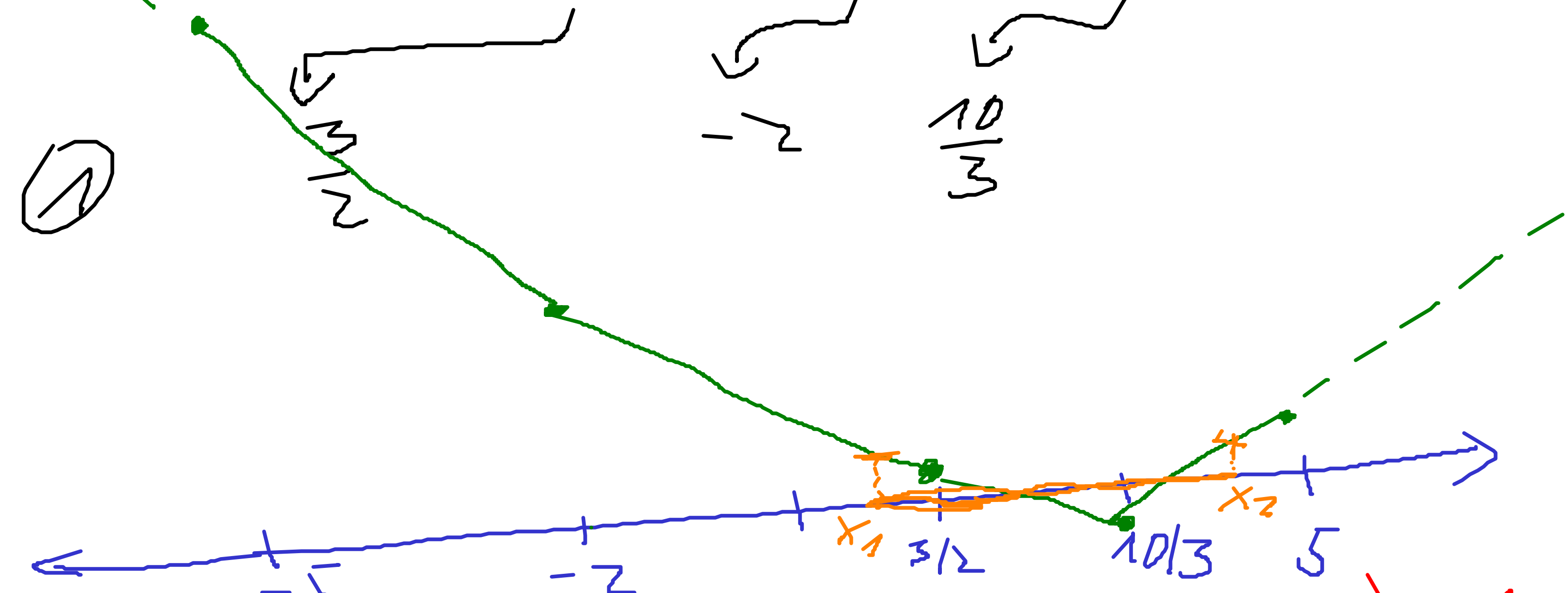
5.1 $f(x) = |2x-3| - |x+2| + |10-3x| - 1$

$\frac{3}{2}$ $\qquad -2 \qquad \frac{10}{3}$

(1)



$x \in [-5,-2]$: $f(x) = -(2x-3) + (x+2) + (10-3x) - 1$
$= -4x + 14$ $\quad f(-5) = 34 \quad f(-2) = 22$

$x \in [-2, 3/2]$: $f(x) = -(2x-3) - (x+2) + (10-3x) - 1$
$= -6x + 10$ $\quad f\left(\frac{3}{2}\right) = 1 \quad f(-2) = 22$

$x \in \left[3/2, \frac{10}{3}\right]$: $f(x) = (2x-3) - (x+2) + (10-3x) - 1$
$= -2x + 4$ $\quad f\left(\frac{3}{2}\right) = 1 \quad f\left(\frac{10}{3}\right) = -\frac{8}{3}$

$x \in \left[\frac{10}{3}, 5\right]$: $f(x) = (2x-3) - (x+2) - (10-3x) - 1$
$= 4x - 16$ $\quad f\left(\frac{10}{3}\right) = \frac{40}{3} - \frac{48}{3} = -\frac{8}{3}$

$f(5) = 4$

(2) Obor hodnot funkce $f: \mathbb{R} \to \mathbb{R}$

$H(f) = [-\frac{8}{3}, \infty)$

③ Funkce f je rostoucí na $[\frac{10}{3}, \infty)$

— " — klesající na $(-\infty, \frac{10}{3}]$

④ Určete, pro která reálná $x \in \mathbb{R}$ platí $f(x) < 2$.

$$-6x + 10 = 2$$

$$8 = 6x \qquad x_1 = \frac{4}{3}$$

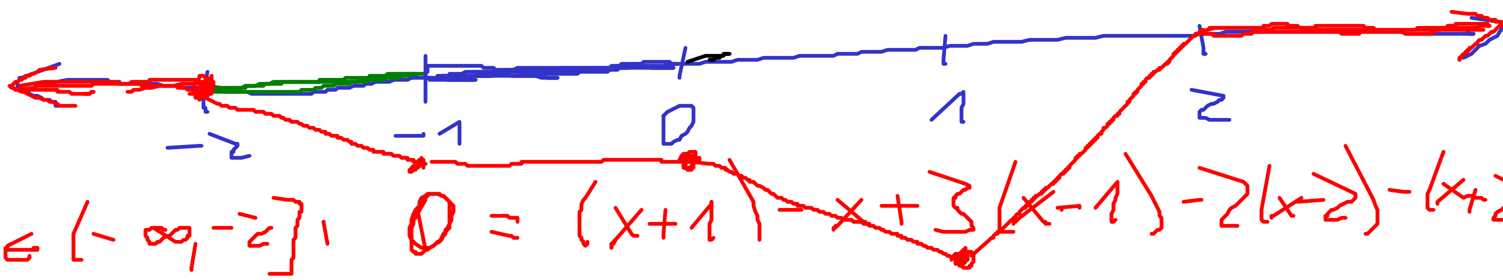
$$4x - 16 = 2$$

$$4x = 18 \qquad x_2 = \frac{9}{2}$$

Odpověď: $(\frac{4}{3}, \frac{9}{2})$

5.2 (1) $|x+1| - |x| + 3|x-1| - 2|x-2| - |x+2| = 0$

$-1 \qquad 0 \qquad 1 \qquad 2 \qquad -2$



$x \in (-\infty, -2]$: $0 = (x+1) - x + 3(x-1) - 2(x-2) - (x+2)$

$x \in [2, \infty)$: $0 = 0$

$x \in [-2, -1]$: $-(x+1) + x - 3(x-1) + 2(x-2) - (x+2) = 0$

$-2x - 4 = 0 \rightsquigarrow x = -2$

$x \in [-1, 0]$: $(x+1) + x - 3(x-1) + 2(x-2) - (x+2) = 0$

$-2 = 0$ nema rešen

$x \in [0, 1]$: $(x+1) - x - 3(x-1) + 2(x-2) - (x+2) = 0$

$-2x - 2 = 0 \rightsquigarrow x = -1$ ni v interval

$x \in [1, 2]$: $(x+1) - x + 3(x-1) + 2(x-2) - (x+2) = 0$

$4x - 8 = 0 \rightsquigarrow x = 2$

Zaver: $x \in (-\infty, -2] \cup [2, \infty)$

5.2 (2) $\dfrac{|x^2-4x|+3}{x^2+|x-5|}=1$

$$|x(x-4)|+3-x^2-|x-5|=0$$

$|x(x-4)| \to 0, 4$; $|x-5| \to 5$

$$-3x-2=0 \Rightarrow x=-\frac{2}{3}$$

$x \in (-\infty, 0]$: $x(x-4)+3-x^2+(x-5)=0$

$$-3x-2=0$$

$x \in [0,4]$: $-x(x-4)+3-x^2+(x-5)=0$

$$\underbrace{-2x^2+5x-2}_{x=4 \rightsquigarrow -2\cdot 16+20-2=-14}=0$$

$$x_{1,2}=\frac{-5\pm\sqrt{25-16}}{-4}=\frac{-5\pm 3}{-4}=\begin{cases}2\\ \frac{1}{2}\end{cases}$$

v int. $[0,4]$

$x \in [4;5]$: $x(x-4)+3-x^2+(x-5)=0$

$$\underbrace{-3x-2}_{x=5 \rightsquigarrow -15-2=-17}=0$$

$x \in [5, \infty)$; $x(x-4)+3-x^2-(x-5)=0$

$$-5x+8=0$$

Závěr: $x \in \left\{-\frac{2}{3}, \frac{1}{2}, 2\right\}$

5.3 $g(x) = \big||x+1| - |x-1|\big|$

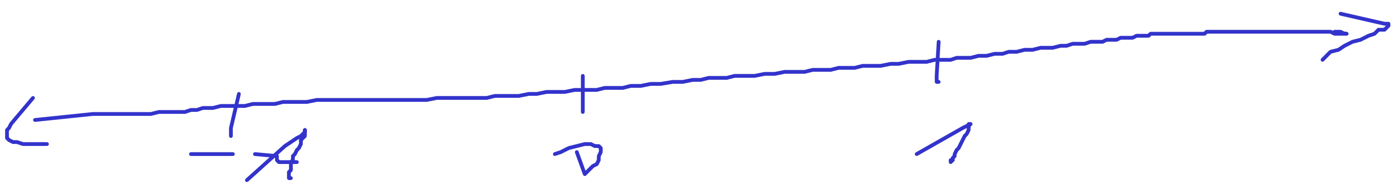
-1 $\quad 0 \quad$ $+1$

Dále $|x+1| = |x-1| \quad /()^2$

$$(x+1)^2 = (x-1)^2$$

$$x^2+2x+1 = x^2-2x+1$$

$$4x = 0 \Rightarrow x = 0$$



5.5 Najděte polynom s celočíselnými koeficienty t. ž.

(1) má kořeny $0, 1, -\frac{1}{2}$

$$2x(x-1)\left(x+\frac{1}{2}\right)$$

(2) jediný reálný koreň je -1, ale stupeň pol. > 1

$(x+1)^4$ nebo $(x+1)(x^2+1)$

(3) $(x-1)^3$

(4) koreny jsou $\sqrt{2}, -1$, případně další reálný koreň

$$(x+1)\underbrace{(x-\sqrt{2})(x+\sqrt{2})}_{x^2-2}$$

5.6 $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

$a_i \in \mathbb{Z}$

Nech ať $f(x)$ má racionální koreň $\frac{p}{q}$, $p \in \mathbb{Z}$, $q \in \mathbb{N}$, $(p,q)=1$

$$f\left(\frac{p}{q}\right) = a_n\left(\frac{p}{q}\right)^n + a_{n-1}\left(\frac{p}{q}\right)^{n-1} + \dots + a_1\frac{p}{q} + a_0 = 0 \quad / \cdot q^n$$

$$\underbrace{a_n p^n + a_{n-1} p^{n-1} q + \dots + a_1 p q^{n-1}}_{p(\text{---})} + a_0 q^n = 0$$

$\Rightarrow p \mid a_0 q^n$

$$\underbrace{(p,q) \Rightarrow p \mid a_0}_{q \mid (\ldots - \ldots)}$$

$$\Rightarrow q \mid a_n p^n \Rightarrow q \mid a_n$$

5.7/1) $f(x) = 2x^3 + x^2 - 4x - 3 = 0$

$\frac{p}{q}$ koren $p \in \cancel{Z}$, $q \in N$, $(p,q) = 1$

$\Rightarrow p \mid 3$, $q \in 2$

$p \in \{\pm 1, \pm 3\}$, $q \in \{1, 2\}$

$\frac{p}{q} \in \{\pm 1, \pm \frac{1}{2}, \pm 3, \pm \frac{3}{2}\}$

	2	1	-4	-3
1	2	3	-1	$-4 \neq 0$
-1	2	-1	-3	0
-1	2	-3	0	
$\frac{3}{2}$	2	0		

Hornerovo schema

$$f(x) = (x+1)\underbrace{(2x^2 - x - 3)}$$

$$x_{1,2} = \frac{1 \pm \sqrt{1+24}}{4}$$

$$x_{1,2} = \frac{1 \pm 5}{4} = \left\langle \begin{matrix} \frac{3}{2} \\ -1 \end{matrix} \right.$$