6.2 2

$$\sqrt{x+3} - \sqrt{x-1} > \sqrt{2x-1}$$

$x \geq -3$, $x \geq 1$, $x \geq \frac{1}{2}$ } $x \geq 1$

$$\sqrt{x+3} > \sqrt{2x-1} + \sqrt{x-1} \quad /()^2$$

obě strany nezáporné

jsou nezáporné

$$x+3 > (2x-1) + 2\sqrt{(2x-1)(x-1)} + (x-1) \quad /()^2$$

$$\underbrace{-2x+5}_{>0 \Rightarrow x<\frac{5}{2}} > 2\sqrt{2x^2-3x+1}$$

$$4x^2 - 20x + 25 > 4(2x^2 - 3x + 1)$$

$$0 > 4x^2 + 8x - 21$$

$$x_{1,2} = \frac{-8 \pm \sqrt{64+16\cdot 21}}{8} = \frac{-8 \pm \sqrt{16(4+21)}}{8}$$

$$= \frac{-8 \pm 4\cdot 5}{8} = -1 \pm \frac{5}{2} = \left\langle \begin{array}{l} \frac{3}{2} \\ -\frac{7}{2} \end{array} \right.$$

$x < \frac{5}{2}$

$$x \in \left(-\frac{7}{2}, \frac{3}{2}\right)$$

$x \geq 1$

Závěr: $x \in \left[1, \frac{3}{2}\right)$

Vietovy vztahy:

kvadratický pol. s kořeny x_1, x_2

$$a(x-x_1)(x-x_2)=0, \quad a\neq 0$$

$$a[x^2-(x_1+x_2)x+x_1x_2]=0$$

$$ax^2\underbrace{-a(x_1+x_2)}_{b}x+\underbrace{ax_1x_2}_{c}=0$$

$$ax^2+bx+c=0$$

$$\boxed{\begin{aligned} x_1+x_2&=-\frac{b}{a}\\ x_1\cdot x_2&=\frac{c}{a}\end{aligned}}$$

6.3 Označme x_1, x_2 řešení rovnice $3x^2+8x+4=0$. Aniž rovnici řešíte, určete číslo:

1. $x_1^2+x_2^2=(x_1+x_2)^2-2x_1x_2=\sqrt{\frac{40}{9}}$
$$=\left(-\frac{8}{3}\right)^2-2\cdot\frac{4}{3}=\frac{64}{9}-\frac{8}{3}=\frac{64-24}{9}$$

2. $x_1^3 + x_2^3 = (x_1+x_2)^3 - 3x_1x_2(x_1+x_2) =$

$(x_1+x_2)^3 = x_1^3 + 3x_1^2x_2 + 3x_1x_2^2 + x_2^3$

$8^3 = (2^3)^3 = 2^9$

$= x_1^3 + x_2^3 + 3x_1x_2(x_1+x_2)$

$= \left(-\frac{8}{3}\right)^3 - 3\cdot\frac{4}{3}\cdot\left(-\frac{8}{3}\right) = \frac{-8^3}{27} + \frac{32}{3} =$

$= -\frac{2^9}{27} + \frac{2^5\cdot 9}{27} = \frac{2^5(-2^4+9)}{27} =$

$= \frac{2^5(-16+9)}{27} = -\frac{7\cdot 32}{27}$

3. $\frac{1}{x_1} + \frac{1}{x_2} = \frac{x_1+x_2}{x_1x_2} = \frac{-\frac{8}{3}}{\frac{4}{3}} = -2$

4. $x_1 - x_2 = m \quad /\ (\,)^2$

$x_1^2 - 2x_1x_2 + x_2^2 = m^2$

$\frac{40}{9} - 2\cdot\frac{4}{3} = m^2$

$\frac{40}{9} - \frac{8\cdot 3}{9} = m^2$

$\frac{40-24}{9} = \frac{16}{9} = m^2 \Rightarrow m = \pm\frac{4}{3}$

5. $x_1^2x_2+x_1x_2^2=x_1x_2(x_1+x_2)$

$$=\frac{4}{3}\cdot\left(-\frac{8}{3}\right)=-\frac{32}{3}$$

6. $x_1^2-x_2^2=(x_1+x_2)(x_1-x_2)=$

$$=-\frac{8}{3}\cdot\left(\pm\frac{4}{3}\right)=\pm\frac{32}{9}$$

Viétovy vztahy pro pol. 3 stupně

$a(x-x_1)(x-x_2)(x-x_3)=0$ $\quad x_1, x_2, x_3$ kořeny

$a\left(x^2-(x_1+x_2)x+x_1x_2\right)(x-x_3)=0$

$a\left[x^3-x_3x^2-(x_1+x_2)x^2+x_3(x_1+x_2)x+x_1x_2x-x_1x_2x_3\right]=$

$=a\left[x^3-(x_1+x_2+x_3)x^2+(x_1x_3+x_2x_3+x_1x_2)x-x_1x_2x_3\right]=$

$$=ax^3\overbrace{-a(x_1+x_2+x_3)}^{b}x^2+\underbrace{a(x_1x_2+x_2x_3+x_1x_3)}_{c}x\underbrace{-ax_1x_2x_3}_{d}=0$$

$$ax^3+bx^2+cx+d=0$$

$$\boxed{\begin{aligned} x_1+x_2+x_3 &= -\frac{b}{a} \\ x_1x_2+x_1x_3+x_2x_3 &= \frac{c}{a} \\ x_1x_2x_3 &= -\frac{d}{a} \end{aligned}}$$

7.1 1. $a>0$, $n\in\mathbb{Z}$,

- $n\geq 1$: $a^n = \underbrace{a\cdots a}_{n}$
- $n=0$: $a^0=1$
- $n\leq -1$: $a^n = \frac{1}{a^{-n}} = \underbrace{\frac{1}{a}\cdots\frac{1}{a}}_{-n}$

Z. $a>1$, $n<m$ celá čísla

pak $\underbrace{a^n < a^m}$

- $0\leq n<m$: $a^m = \underbrace{a\cdots a}_{n}\cdot\underbrace{a\cdots a}_{m-n} = a^n\cdot \underbrace{a^{m-n}}_{>1}$

$$\Rightarrow a^m > a^n$$

- $n<m\leq 0$: $a^{-m} < a^{-n}$ $\Big/ \frac{1}{a^{-n}}\cdot\frac{1}{a^{-m}}$

$-m<-n$

platí $\frac{1}{a^{-n}} < \frac{1}{a^{-m}}$

$$\frac{1}{\frac{1}{a^{-n}}} < \frac{1}{\frac{1}{a^{-m}}} \Rightarrow a^n < a^m$$

- $n<0\leq m$: $a^n = \frac{1}{a^{-n}}$ $\quad a^{-n}>1$

$$\Rightarrow a^n < 1 \leq a^m$$

3. $a \in \mathbb{R}_+$, $x = \frac{p}{q}$, $p \in \mathbb{Z}$, $q \in \mathbb{N}$

Definujeme $a^x = a^{\frac{p}{q}} := \sqrt[q]{a^p}$

- $f(x) := a^q$, $q \in \mathbb{N}$ má inverzní funkci $g(y) = \sqrt[q]{y}$
- $f(x) = y$ f injektivní
  $x = g(y)$ g inverzní funkce

$x = \frac{kp}{kq}$, $k \in \mathbb{N}$: $(a^p)^k = a^{kp}$, $k, p \in \mathbb{Z}$

$$(a^q)^k = a^{kq}$$

$$\sqrt[k]{\sqrt[q]{a}} = \sqrt[kq]{a}$$

$$a^x = a^{\frac{kp}{kq}} = \sqrt[kq]{a^{kp}} = \sqrt[q]{\sqrt[k]{(a^p)^k}} = \sqrt[q]{a^p}$$

tedy $a^{\frac{p}{q}}$ je dobře definované

4. $a \in \mathbb{R}_+$, $x, y \in \mathbb{Q}$, dokažte, že $\underbrace{a^x \cdot a^y = a^{x+y}}$ a $\underbrace{(a^x)^y = a^{xy}}$

$x = \frac{p_1}{q}$ $y = \frac{p_2}{q}$ po převedení na společného jmenovatele

$$a^x \cdot a^y = \underline{a^{\frac{p_1}{q}} \cdot a^{\frac{p_2}{q}}} = \sqrt[q]{a^{p_1}} \cdot \sqrt[q]{a^{p_2}} =$$

$$= \sqrt[q]{a^{p_1} \cdot a^{p_2}} = \sqrt[q]{a^{p_1+p_2}} = \underline{a^{\frac{p_1+p_2}{q}}}$$

$$\underbrace{b_1^q}_{c_1} \cdot \underbrace{b_2^q}_{c_2} = (b_1 \cdot b_2)^q \quad / \sqrt[q]{\ }$$

$$c_1 \cdot c_2 = c_1 c_2 \quad / \sqrt[q]{\ }$$

$$\sqrt[q]{c_1} \cdot \sqrt[q]{c_2} = \sqrt[q]{c_1 c_2}$$

$$(a^x)^y = \left(a^{\frac{p_1}{q}}\right)^{\frac{p_2}{q}} = \sqrt[q]{\left(\sqrt[q]{a^{p_1}}\right)^{p_2}} =$$

$$= \sqrt[q]{\sqrt[q]{a^{p_1 p_2}}} = \sqrt[q]{a^{\frac{p_1 p_2}{q}}} = a^{\frac{p_1 p_2}{q^2}} = a^{xy}$$

je třeba uhnout pro p_2 záporné

5. $x \in \mathbb{R}$: $a^x := \sup\{a^y \mid y \in \mathbb{Q}, y \le x\}$

$a \in \mathbb{R}_+$

$M \subseteq \mathbb{R}$

• h je horní závora M, jestliže $h \ge m \ \forall m \in M$

$\sup M := \min\{h \mid h$ je horní závora $M\}$

Př: $\sup \underbrace{(0,1)}_{\text{nemá maximum}} = 1$

množina horních závor je $[1,\infty)$

<u>G.</u> $a \in \mathbb{R}, a > 1$, pak <u>a^x je rostoucí</u>

$x_1 < x_2 \qquad x_1, x_2 \in \mathbb{R}$

$x_1 < \frac{p}{q} < x_2 \qquad \frac{p}{q},\ p \in \mathbb{Z},\ q \in \mathbb{N}$

$a^{x_1} = \sup \underbrace{\{a^y \mid y \in \mathbb{Q}, y \leq x_1\}}$

$\frac{p}{q}$ horní závora $\quad x_1 < \frac{c}{d} < \frac{p}{q}$

$a^{x_1} \leq a^{\frac{p}{q}} \qquad a^{x_1} \leq a^{\frac{c}{d}} < a^{\frac{p}{q}}$

$x_1 = \frac{p_1}{q} < x_2 = \frac{p_2}{q} \rightsquigarrow p_1 < p_2$

$a^{\frac{p_1}{q}} = \sqrt[q]{a^{p_1}} < a^{\frac{p_2}{q}} = \sqrt[q]{a^{p_2}}$

$a^{p_1} < a^{p_2} \quad / \sqrt[q]{\ }$

$\sqrt[q]{a^{p_1}} < \sqrt[q]{a^{p_2}}$

7.2 : 2. $f(x) = a^x$, $x \in \mathbb{R}$, $a > 0$

$D(f) = \mathbb{R}$

$H(f) = \mathbb{R}_+$

$a \neq 1$

- $a > 1$ rostoucí
- $a \in (0,1)$ klesající
- $a = 1$ konstantní

a^x injektivní a tedy má inverzní funkci

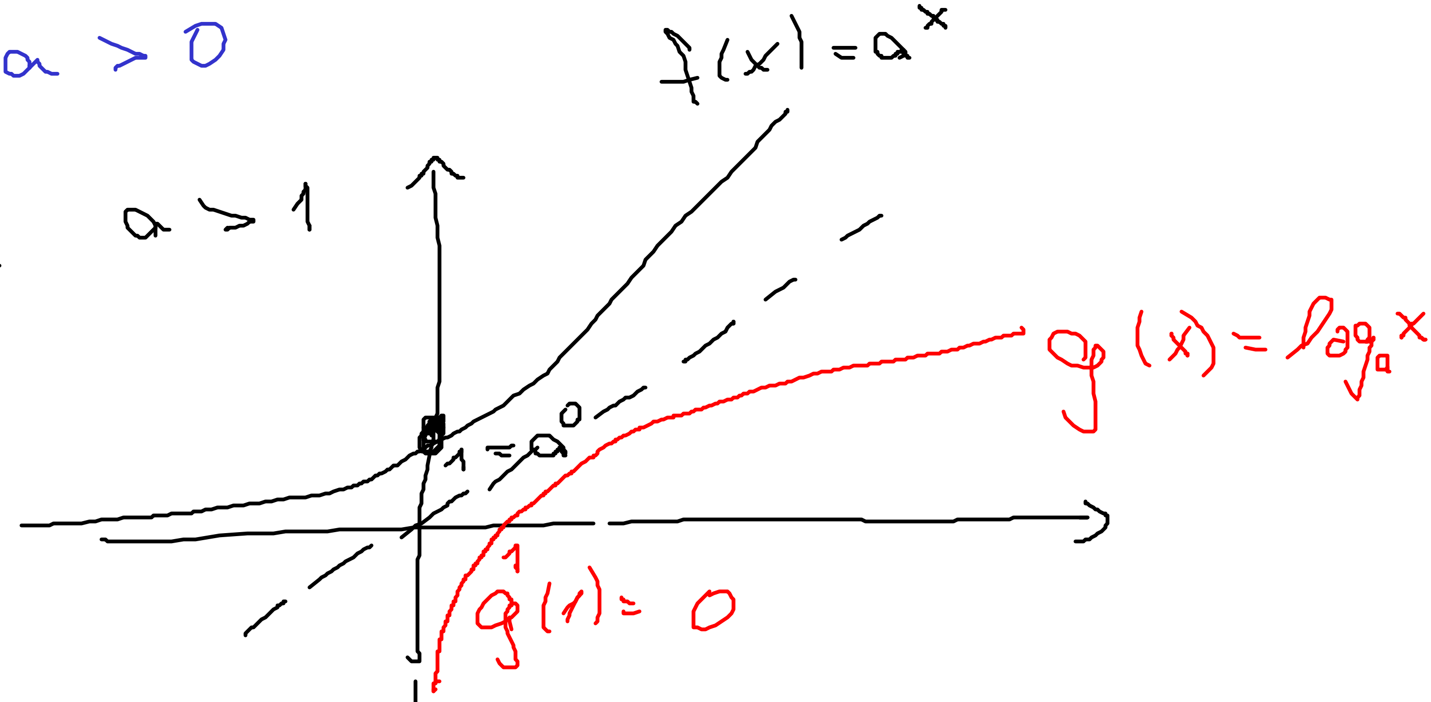
$g(y) = \log_a y$ $\quad D(g) = \mathbb{R}_+$

$H(g) = \mathbb{R}$

$a \neq 1$

$a > 0$

3. $a > 1$



$a \in (0,1)$