**8.2** (3) $6\cdot 9^x - 13\cdot 6^x + 6\cdot 4^x = 0 \quad / \frac{1}{4^x}$

$6\cdot\left(\frac{9}{4}\right)^x - 13\cdot\left(\frac{6}{4}\right)^x + 6 = 0$

$6\left(\left(\frac{3}{2}\right)^2\right)^x - 13\left(\frac{3}{2}\right)^x + 6 = 0$

$\left(\frac{3}{2}\right)^{2x} = \left(\left(\frac{3}{2}\right)^x\right)^2 \qquad y = \left(\frac{3}{2}\right)^x$

$6y^2 - 13y + 6 = 0$

$y_{1,2} = \frac{13 \pm \sqrt{169 - 4\cdot 36}}{12} = \frac{13 \pm \sqrt{169-144}}{12}$

$= \frac{13 \pm 5}{12} = \begin{cases} \frac{18}{12} = \frac{3}{2} \\ \frac{8}{12} = \frac{2}{3} \end{cases}$

- $y = \left(\frac{3}{2}\right)^x = \frac{3}{2} \rightsquigarrow x = 1$
- $y = \left(\frac{3}{2}\right)^x = \frac{2}{3} \rightsquigarrow x = -1$

$y = \pm 1$

④ $\left(\frac{3}{5}\right)^x + \frac{7}{5} = 2^x$

- $x = 1$ je rješenje
- LS klesajića funkcija } max jedno
  PS rastuća funkcija } rješenje

8.3

① $\log 5 + \log(x+10) = 1 - \log(2x-1) + \log(21x-20)$

($1 = \log 10$)

$$\log 5(x+10) = \log \frac{10(21x-20)}{2x-1}$$

$$5(x+10) = \frac{10(21x-20)}{2x-1}$$

$$(x+10)(2x-1) = 2(21x-20)$$

$$2x^2 + (20-1)x - 10 = 42x - 40$$

$$2x^2 - 23x + 30 = 0$$

$$x_{1,2} = \frac{23 \pm \sqrt{23^2 - 8\cdot 30}}{4}$$

$$23^2 - 8\cdot 30 = 529 - 240 = 289 = 17^2$$

$$= \frac{23 \pm 17}{4} = \begin{cases} \frac{40}{4} = 10 \\ \frac{6}{4} = \frac{3}{2} \end{cases}$$

$$x \in \left\{10, \frac{3}{2}\right\}$$

(2) $$\log_{0.5x} x^2 - 14 \log_{16x} x^3 + 40 \log_{4x} \underbrace{\sqrt{x}}_{x^{\frac{1}{2}}} = 0$$

$$2 \log_{\frac{1}{2}x} x - 42 \log_{16x} x + 20 \log_{4x} x = 0$$

$$2 \frac{1}{\log_x\left(\frac{1}{2}x\right)} - 42 \frac{1}{\log_x(16x)} + 20 \frac{1}{\log_x 4x} = 0$$

$$2 \frac{1}{\log_x \frac{1}{2} + \log_x x} - \frac{42}{\log_x 16 + \log_x x} + \frac{20}{\log_x 4 + \log_x x} = 0$$

$$2 \frac{1}{1 - \log_x 2} - \frac{42}{1 + 4\log_x 2} + \frac{20}{1 + 2\log_x 2} = 0$$

$$y = \log_x 2$$

$$\frac{2}{1-y} - \frac{42}{1+4y} + \frac{20}{1+2y} = 0 \quad / \cdot (1-y) \cdot (1+2y) \cdot (1+4y)$$

$$2(1+4y)(1+2y) - 42(1-y)(1+2y) + 20(1-y)(1+4y) = 0$$

kvadratická rovnice ...

**8.4** (1)

$$\frac{1}{3^x+5} \leq \frac{1}{3^{x+1}-1}$$

- $3^{x+1} > 1 \rightsquigarrow x+1 > 0$

$\rightsquigarrow x > -1$

$$\underbrace{3^{x+1}}_{3 \cdot 3^x} - 1 \leq 3^x + 5$$

$y = 3^x$

$$3y - 1 \leq y + 5$$

$$2y \leq 6$$

$$y \leq 3 \rightsquigarrow 3^x \leq 3$$

$x \leq 1$

Závěr: $x \in (-1, 1]$

(2) $8^x + 18^x - 2\cdot 27^x > 0$

$(2^3)^x + 2^x\cdot(3^2)^x - 2\cdot(3^3)^x > 0$

$2^{3x} + 2^x\cdot 3^{2x} - 2\cdot 3^{3x} > 0 \quad / \cdot \frac{1}{3^{3x}}$

$\left(\frac{2}{3}\right)^{3x} + \left(\frac{2}{3}\right)^x - 2 > 0$

$y := \left(\frac{2}{3}\right)^x \rightsquigarrow y^3 + y - 2 > 0$

↳ korion 1

| | 1 | 0 | 1 | -2 |
|---|---|---|---|---|
| (1) | 1 | 1 | 2 | 0 |

$y^3 + y - 2 = (y-1)(y^2+y+2)$

$D = 1 - 8 < 0$

$(y-1)(y^2+y+2) > 0$

$\geq 0 \qquad > 0$

$y > 1$

$y = \left(\frac{2}{3}\right)^x > 1 \Rightarrow \underbrace{x < 0}_{\text{výsledek}}$