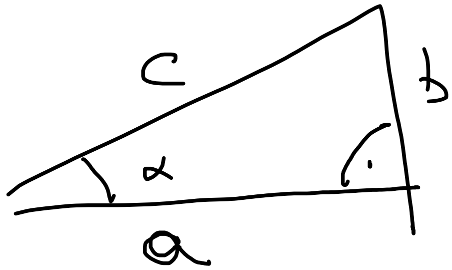
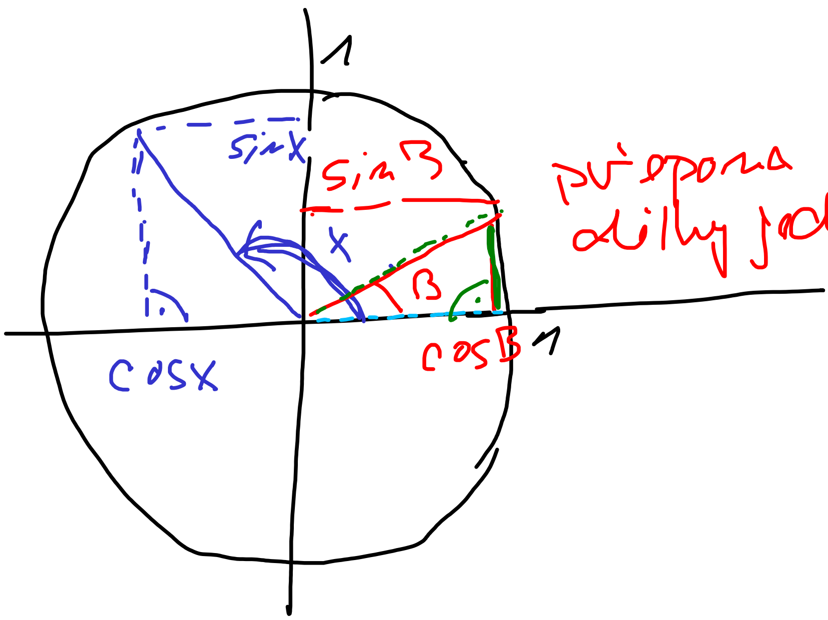


$$\cos\alpha = \frac{a}{c}$$

$$\sin\alpha = \frac{b}{c}$$

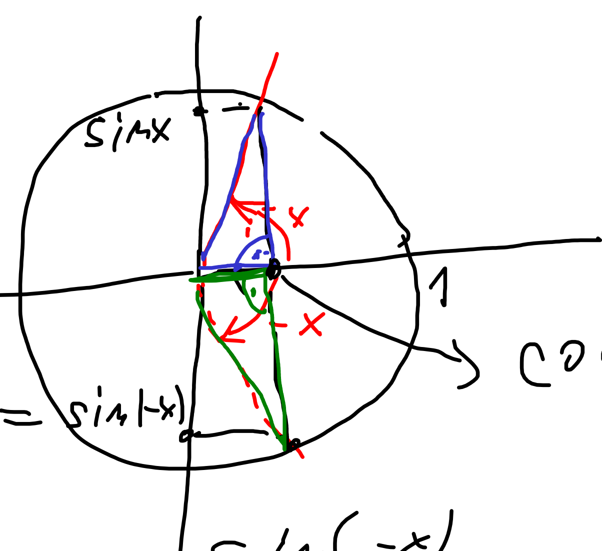
$$\alpha \in \left[0, \frac{\pi}{2}\right)$$

$$\operatorname{tg} x = \frac{\sin x}{\cos x}$$



9.1 (1) $\sin^2 x + \cos^2 x = 1$

(2)



$$\cos x = \cos(-x)$$

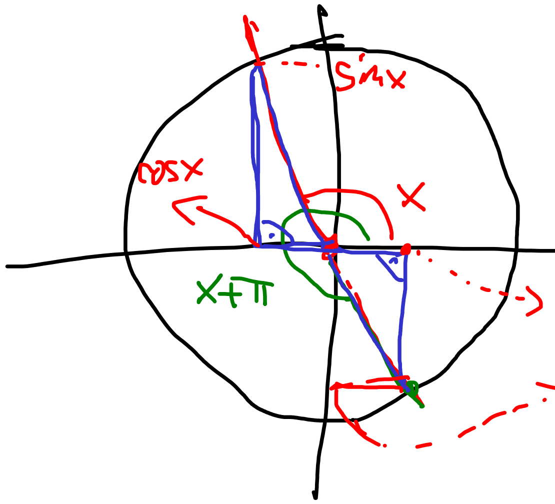
$$-\sin x = \sin(-x)$$

$$\operatorname{tg}(-x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin x}{\cos x} = -\operatorname{tg} x$$

(3) $\sin(x + 2\pi) = \sin x$

$\cos(x + 2\pi) = \cos x$

přímo z definice na jednotkové kružnici



$$tg(x+\pi) = \frac{\sin(x+\pi)}{\cos(x+\pi)} = \frac{-\sin x}{-\cos x} =$$

$$= tg\, x$$

$$\cos(x+\pi) = -\cos x$$

$$\sin(x+\pi) = -\sin x$$

(*)

4

5

$$\sin x = -\sin(-x) = -\sin(2\pi - x)$$

$$\sin x = -\sin(-x) = -(-\sin(\pi - x))$$

$$= \sin(\pi - x)$$

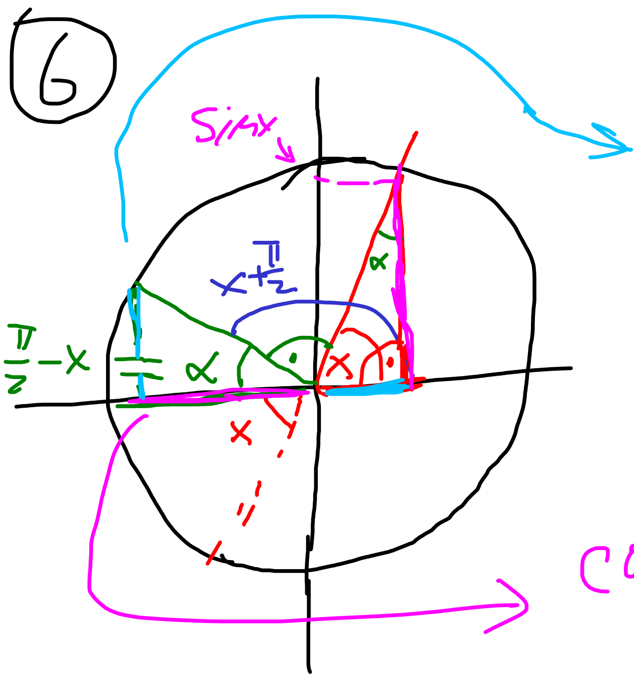
(*)

$$tg\, x = -tg(-x) = -tg(\pi - x)$$

(*)

$$\alpha = \pi - \left(\frac{\pi}{2} + x\right) = \frac{\pi}{2} - x$$

6



$$\sin\left(x+\frac{\pi}{2}\right) = \cos x$$

$$\sin\left(\frac{\pi}{2}-x\right) = \cos(-x) = \cos x$$

$$\cos\left(\frac{\pi}{2}-x\right) = -\sin(-x) = \sin x$$

$$\cos\left(x+\frac{\pi}{2}\right) = -\sin x$$

9.2

$$\boxed{e^{ix} = \cos x + i\sin x}$$

$\forall r_1, r_2 \in \mathbb{C}$

$r = a + ib \in \mathbb{C}$, $\boxed{e^{r_1} \cdot e^{r_2} = e^{r_1 + r_2}}$

$$e^{a+ib} = e^a \cdot e^{ib} \rightarrow \text{rotace}$$

výlet do analýzy

- Taylorův rozvoj

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{3!}x^3 + \dots$$

$x \in \mathbb{C}$ $\nearrow$ funguje pro pravou stranu

$|x| = 1$

$x, y \in \mathbb{R}$

$$\left.\begin{aligned} e^{ix} &= \cos x + i\sin x \\ e^{iy} &= \cos y + i\sin y \end{aligned}\right\} \text{vynásobit}$$

$$e^{ix} \cdot e^{iy} = (\cos x \cos y - \sin x \sin y) + i(\sin x \cos y + \cos x \sin y)$$

$$\|$$

$$e^{i(x+y)} = \cos(x+y) + i\sin(x+y)$$

9.3

(1) $\sin 2x = \sin(x+x)$

$= \sin x \cos x + \cos x \sin x =$

$= 2 \sin x \cos x$

(2) $\sin x + \sin y = - - - .$

$x = \alpha + \beta$ $\Leftrightarrow$ $\alpha = \frac{x+y}{2}$

$y = \alpha - \beta$ $\quad$ $\beta = \frac{x-y}{2}$

$\sin x = \sin(\alpha+\beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$

$\sin y = \sin(\alpha-\beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$

$\alpha + (-\beta)$

$\sin x + \sin y = 2 \sin\alpha \cos\beta$

$= 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$

$y \Leftrightarrow -y$

$\sin x - \sin y = 2 \cos\alpha \sin\beta$

$y \Leftrightarrow -y$

$= 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$

9.4 (1) $\operatorname{tg}(x+y) = \dfrac{\sin(x+y)}{\cos(x+y)} =$

$$= \frac{(\sin x\cos y + \cos x\sin y)\cdot\frac{1}{\cos x\cos y}}{(\cos x\cos y - \sin x\sin y)\cdot\frac{1}{\cos x\cos y}}$$

$$= \frac{\operatorname{tg}x + \operatorname{tg}y}{1 - \operatorname{tg}x\operatorname{tg}y}$$

$\operatorname{tg}x$ je definovaný pro $\cos x \neq 0$,

tj. $x \neq \frac{\pi}{2} + k\pi$, $k \in \mathbb{Z}$

Součtový vzorec má smysl pro

$x, y \neq \frac{\pi}{2} + k\pi$, $k \in \mathbb{Z}$

$x+y \neq \frac{\pi}{2} + k\pi$, $k \in \mathbb{Z}$

(2) $\operatorname{tg}(x-y) = \operatorname{tg}(x+(-y))$

(3) $\operatorname{tg}\left(x+\frac{\pi}{2}\right) = \dfrac{\sin\left(x+\frac{\pi}{2}\right)}{\cos\left(x+\frac{\pi}{2}\right)} = \dfrac{\cos x}{-\sin x}$

$$= -\frac{1}{\operatorname{tg}x} = -\operatorname{cotg}x$$

9.5

(1) $\sin x = \dfrac{2 \cdot tg\frac{x}{2}}{1 + tg^2\frac{x}{2}} =$

$= \dfrac{2\,\dfrac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{1 + \dfrac{(\sin\frac{x}{2})^2}{(\cos\frac{x}{2})^2}} \quad \dfrac{/\cdot(\cos\frac{x}{2})^2}{/\cdot(\cos\frac{x}{2})^2} = \dfrac{2\sin\frac{x}{2}\cos\frac{x}{2}}{(\cos\frac{x}{2})^2 + (\sin\frac{x}{2})^2} =$

$= \sin\left(\frac{x}{2} + \frac{x}{2}\right) = \sin x$

$\frac{x}{2} \neq \frac{\pi}{2} + k\pi$

$x \neq \pi + 2k\pi = (2k+1)\pi$

(2) - - .

(3) $tg\,x = tg\left(\frac{x}{2} + \frac{x}{2}\right) = \dfrac{2\,tg\frac{x}{2}}{1 - (tg\frac{x}{2})^2}$

9.6

$1 + tg\,x + tg^2 x + tg^3 x =$

$= 1 + \dfrac{\sin x}{\cos x} + \dfrac{\sin^2 x}{\cos^2 x} + \dfrac{\sin^3 x}{\cos^3 x} =$

$= \dfrac{\cos^3 x + \sin x\cos^2 x + \sin^2 x\cos x + \sin^3 x}{\cos^3 x}$

$= \dfrac{\cos x(\cos^2 x + \sin^2 x) + \sin x(\cos^2 x + \sin^2 x)}{\cos^3 x}$

$$= \frac{\cos x + \sin x}{\cos^3 x}$$

$x \neq \frac{\pi}{2} + k\pi$

(2) $$tg\left(x+\frac{\pi}{4}\right) = \frac{tg x + tg\frac{\pi}{4}}{1 - tg x \, tg\frac{\pi}{4}}$$

$tg\frac{\pi}{4} = 1$

$$= \frac{1 + tg x}{1 - tg x} = \frac{1 + \frac{\sin x}{\cos x}}{1 - \frac{\sin x}{\cos x}} \quad \begin{matrix} / \cdot \cos x \\ / \cdot \cos x \end{matrix}$$

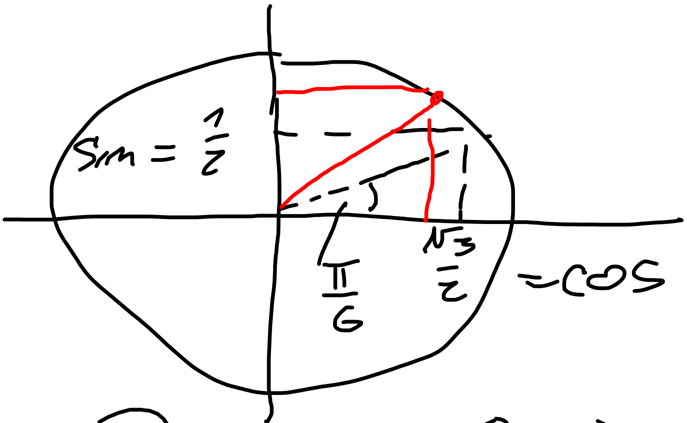
$$= \frac{\cos x + \sin x}{\cos x - \sin x} \quad \begin{matrix} / \cos x + \sin x \\ / \cos x + \sin x \end{matrix}$$

$$= \frac{\cos^2 x + 2\sin x \cos x + \cos^2 x}{\cos^2 x - \sin^2 x} =$$

$$= \frac{1 + \sin 2x}{\cos 2x}$$

(9.7)

	0	$\frac{\pi}{6}$ $=30^\circ$	$\frac{\pi}{4}$ $=45^\circ$	$\frac{\pi}{3}$ $=60^\circ$	$\frac{\pi}{2}$ $=90^\circ$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tg	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\times$



$$15^\circ = \frac{\pi}{12}$$

① $\cos 15^\circ = ?$

$$15^\circ = 45^\circ - 30^\circ$$

$$\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{4} + \left(-\frac{\pi}{6}\right)\right) =$$

$$= \cos\frac{\pi}{4}\cos\left(-\frac{\pi}{6}\right) - \sin\frac{\pi}{4}\sin\left(-\frac{\pi}{6}\right)$$

$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\,\frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$x = \frac{\pi}{12}$$

$$\cos(\underbrace{2x}_{\frac{\pi}{6}}) = \cos^2 x - \sin^2 x = \frac{\sqrt{3}}{2}$$

$$\cos^2 x - \left(1 - \cos^2 x\right) = \frac{\sqrt{3}}{2}$$

$$2\cos^2 x = \frac{\sqrt{3}}{2} + 1$$

$$\cos^2 x = \frac{\sqrt{3}+2}{4}$$

$$\cos x = \frac{\sqrt{\sqrt{3}+2}}{2}$$

$$\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)^2 = \frac{6+2+2\sqrt{2}\sqrt{6}}{16} = \frac{8+4\sqrt{3}}{16} = \frac{2+\sqrt{3}}{4}$$

(2) $tg\,75^\circ = tg\,(45^\circ + 30^\circ) = \ldots$

(3) $tg\,(20^\circ + 40^\circ) = \dfrac{tg\,20^\circ + tg\,40^\circ}{1 - tg\,20^\circ\,tg\,40^\circ} = \sqrt{3}$

$tg\,20^\circ + tg\,40^\circ = \sqrt{3}\,(1 - tg\,20^\circ\,tg\,40^\circ)$

$tg\,20^\circ + tg\,40^\circ + \sqrt{3}\,tg\,20^\circ\,tg\,40^\circ = \sqrt{3}$