11.1 Určete nejmenší periodu zadané funkce:

<u>Připomenutí:</u>

- funkc $f(x)$ je <u>periodická s periodou ℓ</u> $\in \mathbb{R}_+$, jestliže $\forall x \in \mathbb{R}$: $f(x+\ell) = f(x)$
- nejmenší perioda funkc $f(x)$ je nejmenší ℓ t. ž. ℓ je perioda $f(x)$

<u>1.</u> $f(x) = \underbrace{\sin x}_{\text{perioda } 2\pi} + \underbrace{\cos x}_{\text{perioda } 2\pi}$

$\Rightarrow$ $f(x)$ má periodu 2π

$\rightsquigarrow$ existuje menší periodu?

$\rightsquigarrow$ kandidáti na menší periodu jsou $\frac{2\pi}{k}$, $k \in \mathbb{Z}_+$

$f(x) = \sin x + \cos x =$ ← 9.3 cast 2

$= \sin x + \sin\left(x+\frac{\pi}{2}\right) =$

$= 2 \sin \frac{x+\left(x+\frac{\pi}{2}\right)}{2} \cos \frac{x-\left(x+\frac{\pi}{2}\right)}{2}$

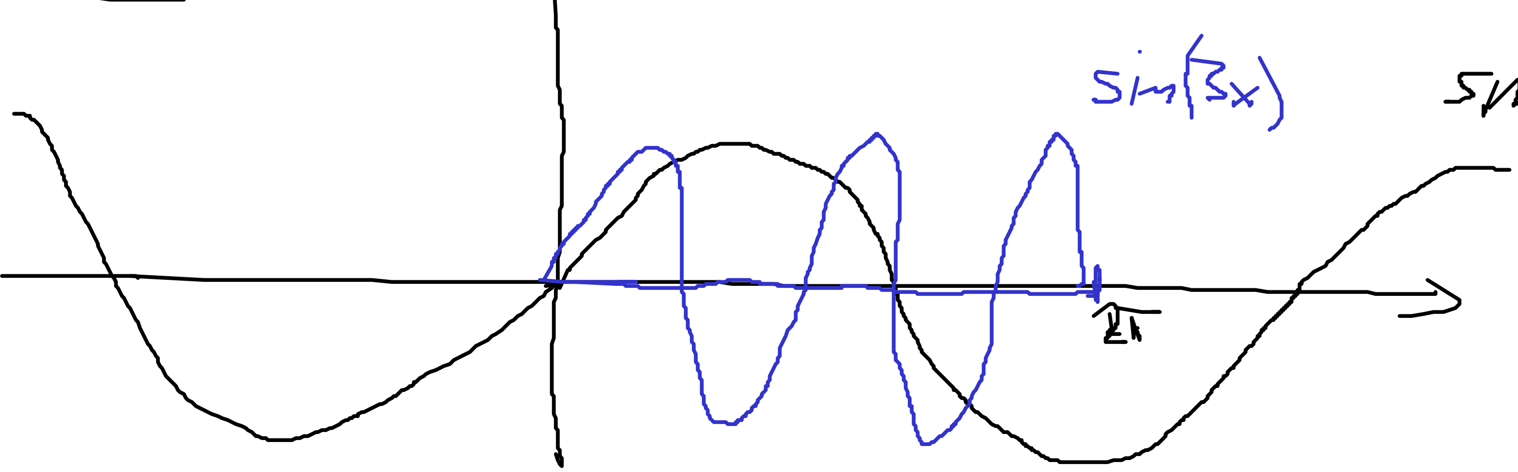
$= 2 \sin\left(x+\frac{\pi}{4}\right) \cos\left(-\frac{\pi}{4}\right)$

$= \sqrt{2} \sin\left(x+\frac{\pi}{4}\right)$

$\Rightarrow 2\pi$ je nejmenší perioda

$$\boxed{\sin x + \cos x = \sqrt{2} \sin\left(x+\frac{\pi}{4}\right)}$$

2. $f(x) = \sin 3x$



$\Rightarrow$ nejmenší perioda $\frac{2\pi}{3}$

Algebraické zdůvodnění:

$\underline{f(x+\ell)} = \sin(3(x+\ell)) = \sin(3x) = \underline{f(x)}$

$\Updownarrow$

$\forall x \in \mathbb{R}: \quad \sin(3x + \underbrace{3\ell}) = \sin(3x)$

$\Rightarrow 3\ell = 2k\pi \quad , k \in \mathbb{Z}_+$

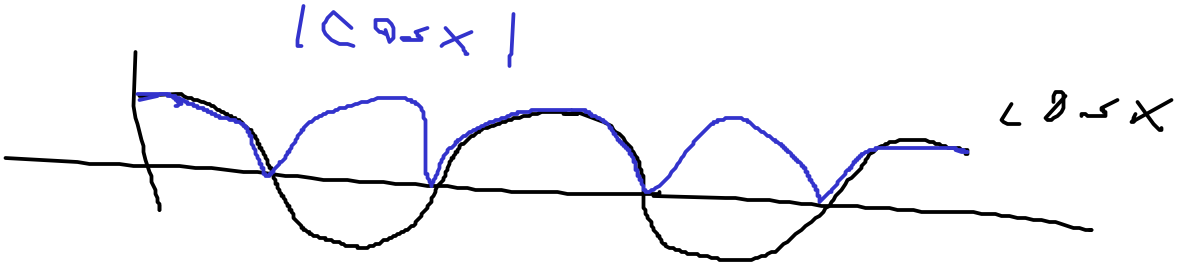
všechny periody $\longrightarrow$ $\ell = \frac{2k\pi}{3}$

$\Downarrow$

$k = 1: \ \ell = \frac{2\pi}{3}$ nejmenší perioda

3. $f(x) = |\cos(2x)|$

- $\cos(2x)$ má nejm. periodu $\frac{2\pi}{2} = \pi$
- $f(x)$ nejm. perioda $\frac{\pi}{2}$

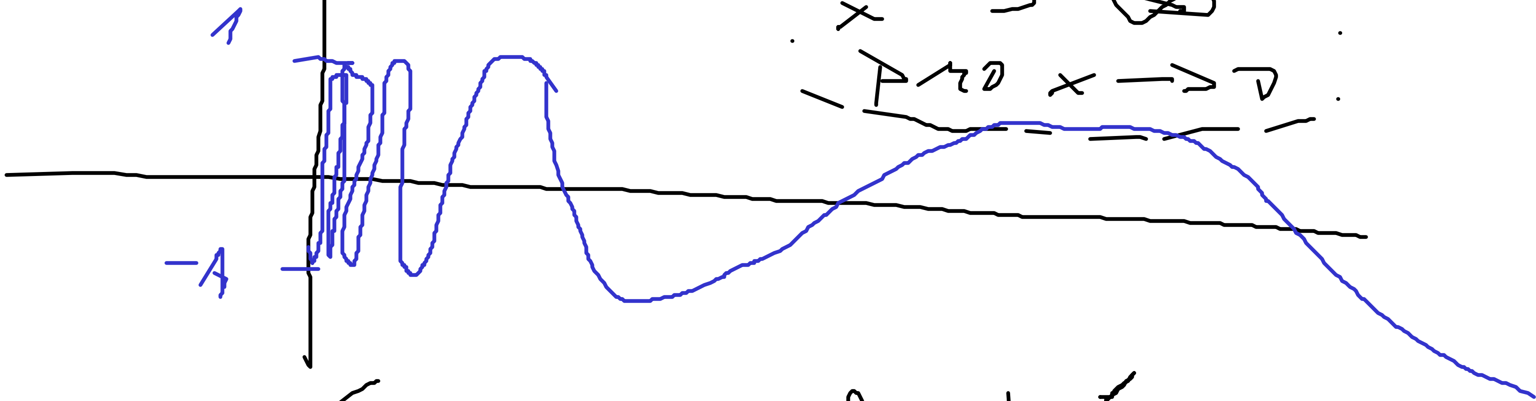


4. $f(x) = \sin \frac{1}{x}$

má graf

$\sin \frac{1}{x} \to 0$ pro $x \to \infty$

$\frac{1}{x} \to \infty$ pro $x \to 0$

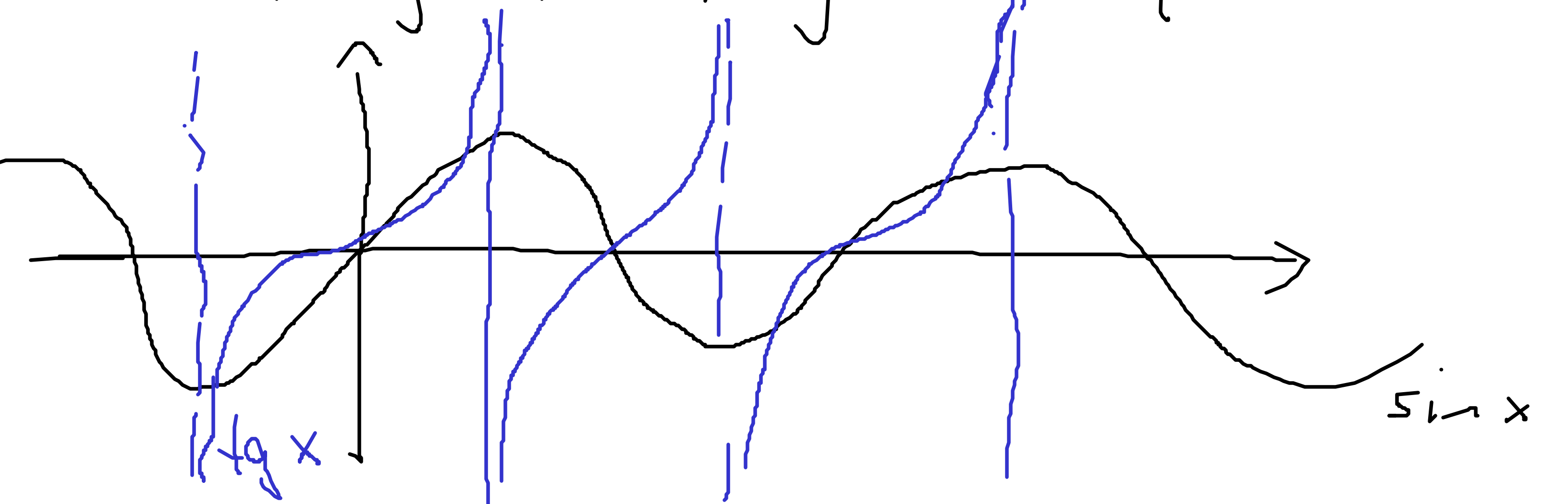


není periodická

5. $f(x) = \sin + tg\, x$

$\to$ má periodu 2π

$\to$ je to nejmenší perioda?



→ jediný kandidát na menší periodu je π
→ není to perioda

11.2 Rozhodněte sudost/lichost

1. $f(x) = x \cdot \sin x$

$$f(-x) = (-x)\underbrace{\sin(-x)}_{-\sin x} = x\sin x$$

⇒ sudá

2. $f(x) = |\sin x + \cos x|$

$$f(0) \neq 0$$

$$f(x) = \sqrt{2}\,|\underbrace{\sin\left(x+\frac{\pi}{4}\right)}_{\substack{\text{ani sudá} \\ \text{ani lichá}}}|$$

11.3 Najděte funkci splňující následující:

1. $f(x)$ má periodu 3π a obor hodnot $[1,2]$

$\sin(kx)$ má periodu $\frac{2\pi}{k}$

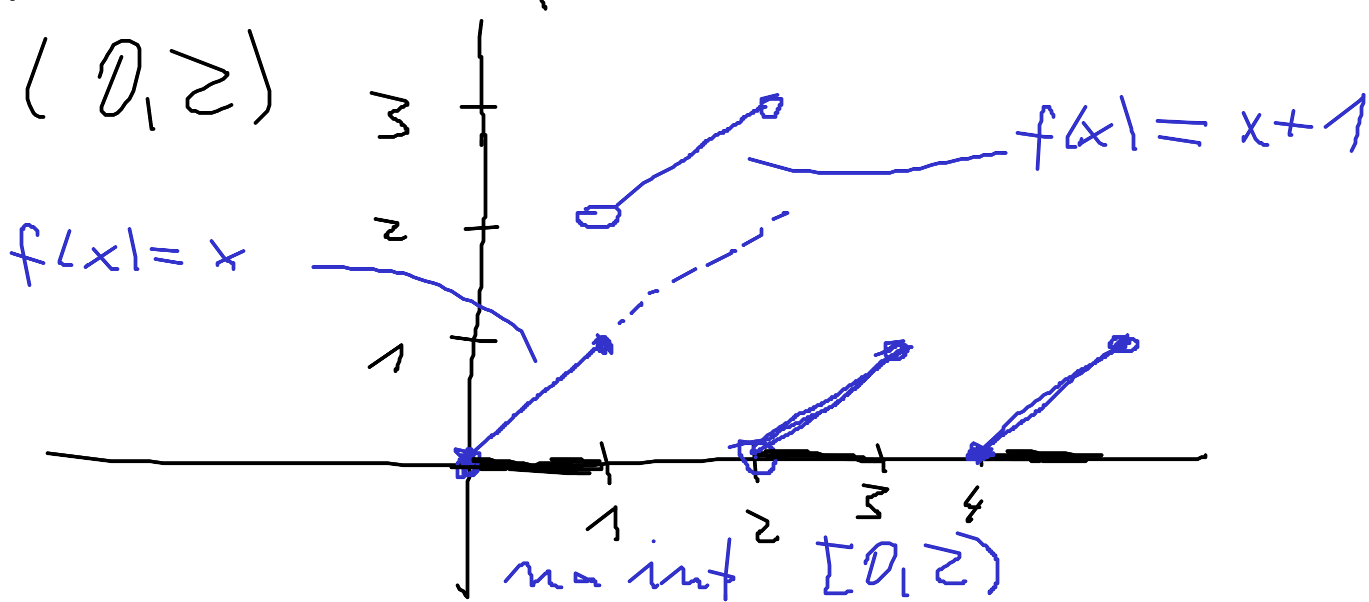
$\Downarrow$ $\qquad \frac{2\pi}{k} = 3\pi \Rightarrow k = \frac{2}{3}$

$\sin\left(\frac{2}{3}x\right)$ má periodu 3π

$\hookrightarrow$ má obor hodnot $[-1,1]$

$$f(x) = \underbrace{\frac{1}{2}\sin\left(\frac{2}{3}x\right)}_{\text{obor hodnot } \left[-\frac{1}{2},\frac{1}{2}\right]} + \frac{3}{2}$$

3. perioda 2, obor hodnot $[0,1] \cup (2,3)$, rostoucí na $(0,2)$

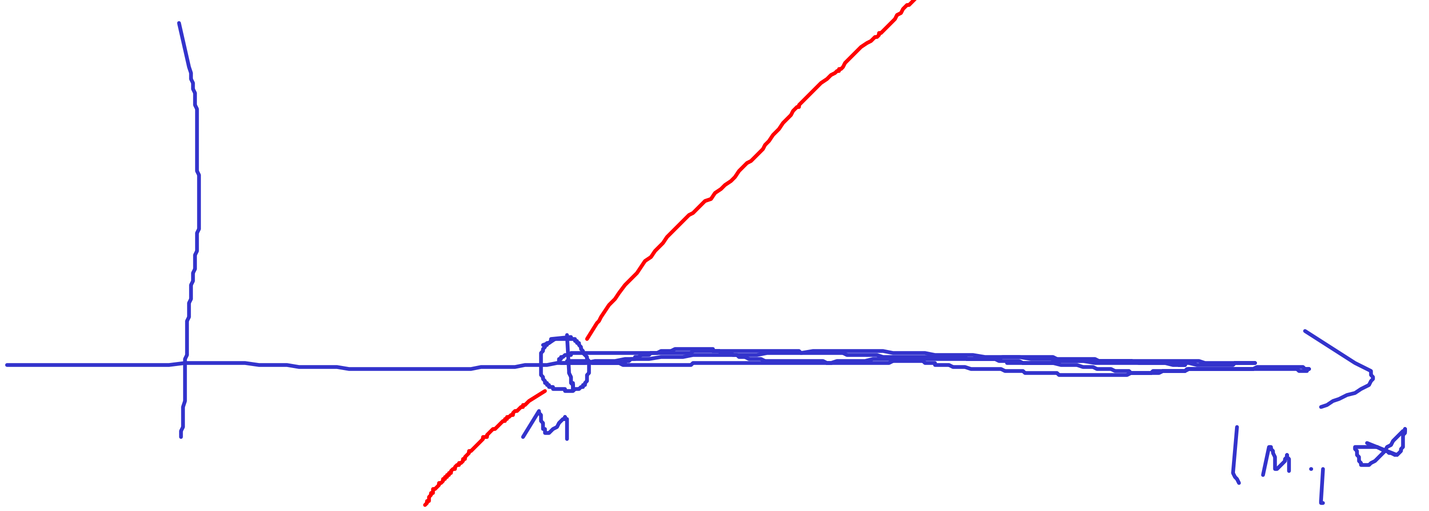


$$f(x) = \begin{cases} x-2k & x \in [2k, 2k+1] \\ x-2k+1 & x \in (2k+1, 2k+2) \end{cases}$$

11.4: Najděte příklad funkce s def. oborem otevřený int. I, pro kterou je obor hodnot na I roven $[0,\infty)$ $\forall$.

$f(I) = [0, \infty)$

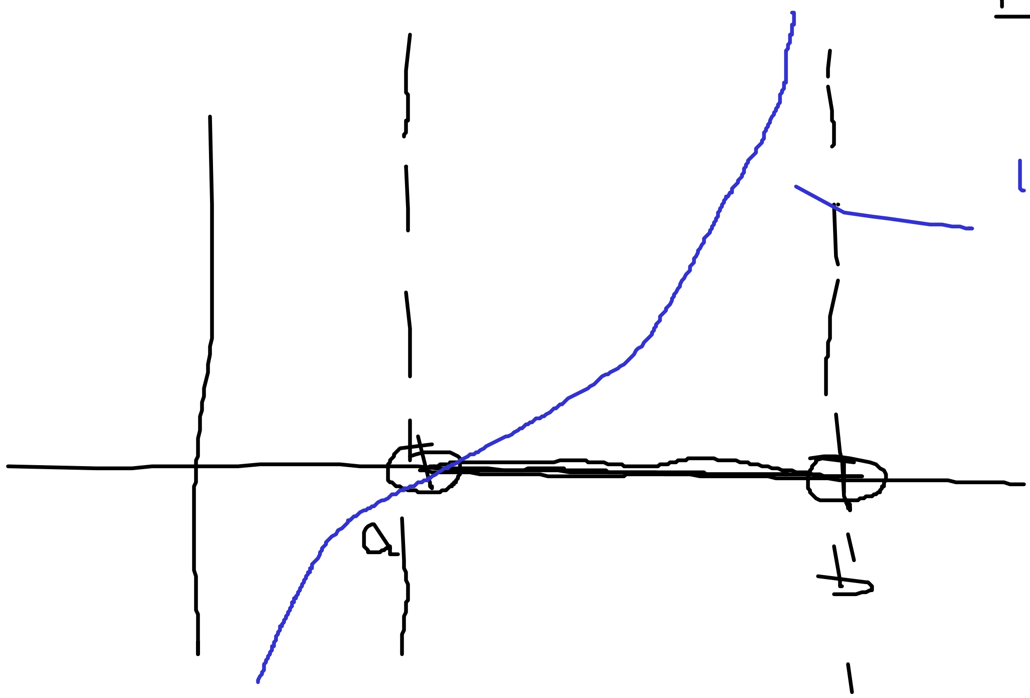
1. $I = (n, \infty)$ $n \in \mathbb{N}$ pevné zvolené



$f(x) = x - n$

4. $I = (a, b)$ $a, b \in \mathbb{R}$

$a < b$



$\rightsquigarrow$ perioda $2(b-a)$

$\mathrm{tg}(kx)$ je $\dfrac{\pi}{k} = 2(b-a)$

$k = \dfrac{\pi}{2(b-a)}$

$\rightsquigarrow$ $\mathrm{tg}\left(\dfrac{\pi}{2(b-a)}x\right)$ má "správnou" periodu

$f(x) = \mathrm{tg}\left(\dfrac{\pi}{2(b-a)}(x-a)\right)$

$f(a) = 0$

výsledek

11.6: <u>1</u>: $\sin x = \sin 2x$

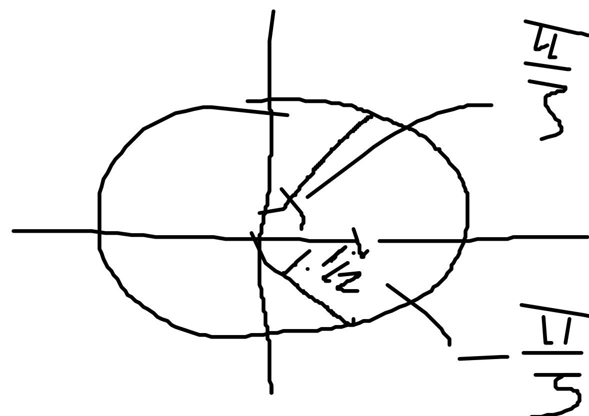
$$\sin x = 2\sin x\cos x$$

$$0 = \sin x \, (\underbrace{2\cos x - 1})$$

$\sin x = 0$

$x = k\pi,\ k \in \mathbb{Z}$

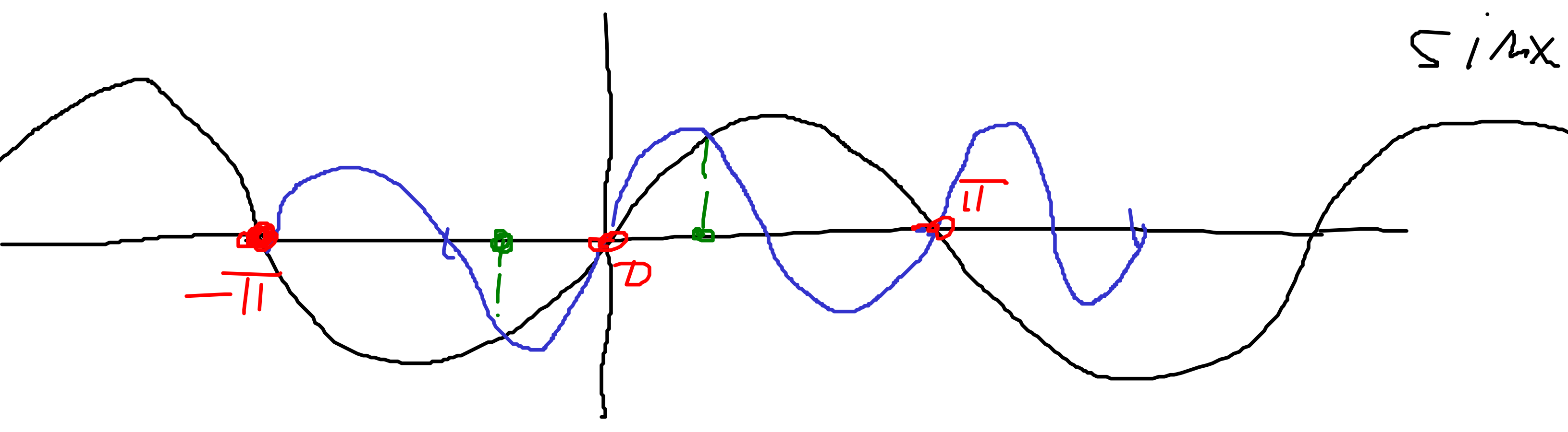
$\cos x = \frac{1}{2}$



$x = \pm\frac{\pi}{3} + 2k\pi,\ k \in \mathbb{Z}$

Zajedno

$x \in \{ k\pi,\ \pm\frac{\pi}{3} + 2k\pi \mid k \in \mathbb{Z} \}$



2. $\sin 3x + \cos 3x = 0$

$$\rightsquigarrow \frac{\sin 3x}{\cos 3x} = -1 \quad + \text{ošetřit}$$

$$\operatorname{tg} 3x = -1 \qquad \cos 3x = 0$$

Jinak: $\sin 3x + \cos 3x =$

$$= \sqrt{2} \sin\left(3x + \frac{\pi}{4}\right) = 0$$

$$3x + \frac{\pi}{4} = k\pi, \quad k \in \mathbb{Z}$$

$$x = \frac{\left(k - \frac{1}{4}\right)\pi}{3} = \frac{(4k-1)\pi}{12}$$

11.7

2. $2\sin^2 x + 7\cos x - 5 = 0$

$$2(1 - \cos^2 x) + 7\cos x - 5 = 0$$

$$y = \cos x \Rightarrow -2y^2 + 7y - 3 = 0$$

$$2y^2 - 7y + 5 = 0$$

$$y_{1,2} = \frac{7 \pm \sqrt{49 - 4 \cdot 2 \cdot 3}}{4} = \frac{7 \pm \sqrt{49 - 24}}{4} = \frac{7 \pm 5}{4}$$

$$= \begin{cases} \dfrac{7+5}{4} = 3 = \cos x \quad \text{nejde} \\[2ex] \dfrac{7-5}{4} = \dfrac{1}{2} = \cos x \end{cases}$$

$$x = \pm \frac{\pi}{3} + 2k\pi$$

$$k \in \mathbb{Z}$$

Řešení v intervalu $[0, 2\pi)$:

$$\frac{\pi}{3} + 2k\pi \in [0, 2\pi)$$

$$\hookrightarrow k = 0 \longrightarrow x = \frac{\pi}{3}$$

$$-\frac{\pi}{3} + 2k\pi \in [0, 2\pi)$$

$$\hookrightarrow k = 1 \longrightarrow x = \frac{5\pi}{3}$$

4: $\sqrt{3}\cos x + \sin x = 2$

$$(\sin x)\frac{1}{2} + (\cos x)\frac{\sqrt{3}}{2} = 1$$

$$\sin(x+y) = \sin x \underbrace{\cos y}_{1/2} + \cos x \underbrace{\sin y}_{\sqrt{3}/2}$$

$$y = \frac{\pi}{3}$$

$$\sin\left(x+\frac{\pi}{3}\right) = 1$$

$$x + \frac{\pi}{3} = \frac{\pi}{2} + 2k\pi$$

$$x = \frac{\pi}{6} + 2k\pi = \left(2k + \frac{1}{6}\right)\pi$$

Řešení na $[0, 2\pi)$:

$$\rightarrow k = 0 \Rightarrow x = \frac{\pi}{6}$$