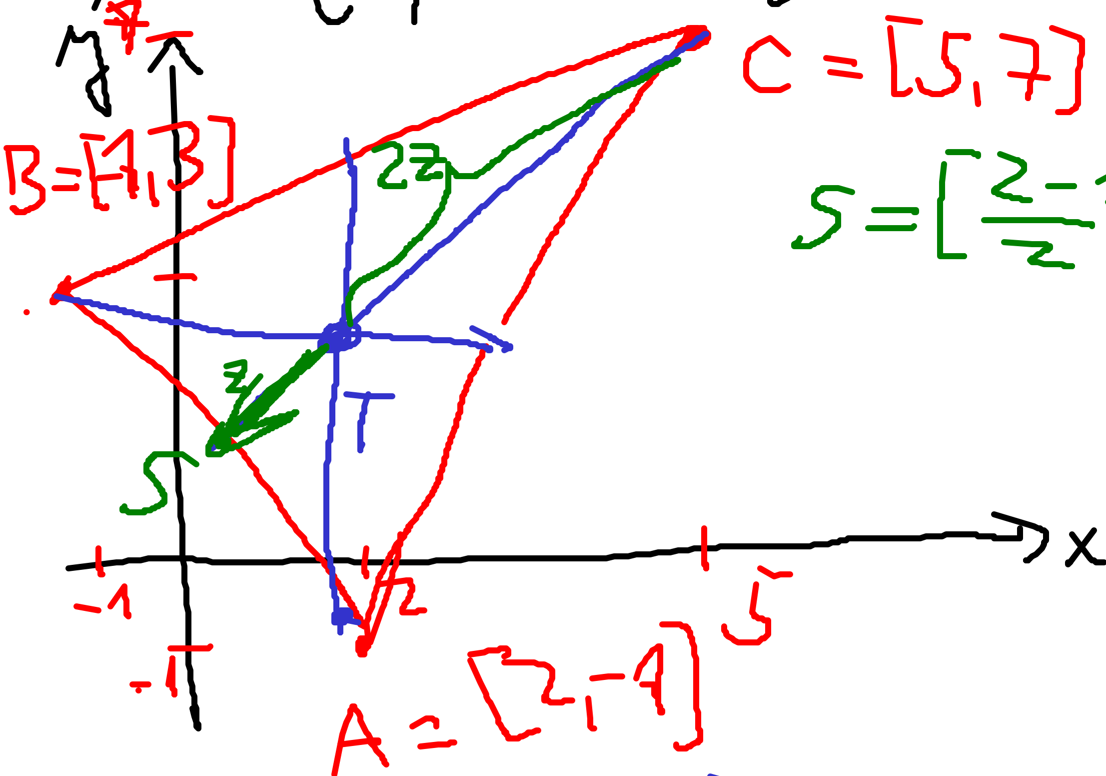
2.1 Těžiště trojúhelníka

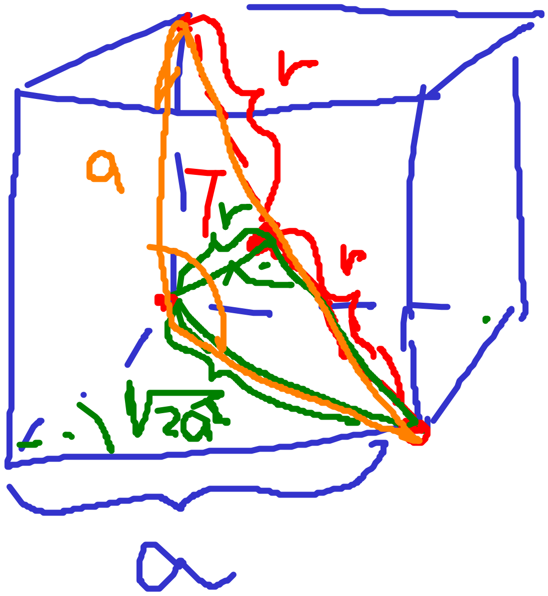
$A = [2, -1] \quad B = [-1, 3] \quad C = [5, 7]$



$$S = \left[\frac{2-1}{2}, \frac{-1+3}{2}\right] = \left[\frac{1}{2}, 1\right]$$

$$T = C + \frac{2}{3}\overrightarrow{CS} =$$
$$= [5, 7] + \frac{2}{3}\left(\frac{1}{2} - 5, 1 - 7\right) =$$
$$= [5, 7] + \frac{2}{3}\left(-\frac{9}{2}, -6\right) =$$
$$= [5, 7] + (-3, -4) = \underline{\underline{[2, 3]}}$$

2.2 $S = 72\,cm^2$ je povrch krychle vepsané do kulové plochy o poloměru r. Určete hodnotu r.



$$S = 6a^2$$
$$72 = 6a^2$$
$$12 = a^2$$
$$\underline{\underline{a = 2\sqrt{3}}}$$

T těžiště "střed" krychle (průsečík tělesových úhlopříček).

$$a^2 + \left(\sqrt{2a^2}\right)^2 = (2r)^2$$
$$a^2 + 2a^2 = 4r^2$$
$$3a^2 = 4r^2 \Rightarrow r^2 = \frac{3}{4}a^2$$
$$r^2 = \frac{3}{4}\cdot 12 = 9 \Rightarrow \underline{\underline{r = 3}}$$

2.3 $M \subseteq \mathbb{R}$ takovou, že

$|2x+1| < x+3$

$x - \frac{1}{2}$

$\langle -\infty, -\frac{1}{2})$ $\langle -\frac{1}{2}, \infty)$

$-(2x+1) < x+3$ $2x+1 < x+3$

$-4 < 3x$ $x < 2$

$-\frac{4}{3} < x$

$(-\frac{4}{3}, -\frac{1}{2})$ $\langle -\frac{1}{2}, 2)$

Celkem $(-\frac{4}{3}, 2)$

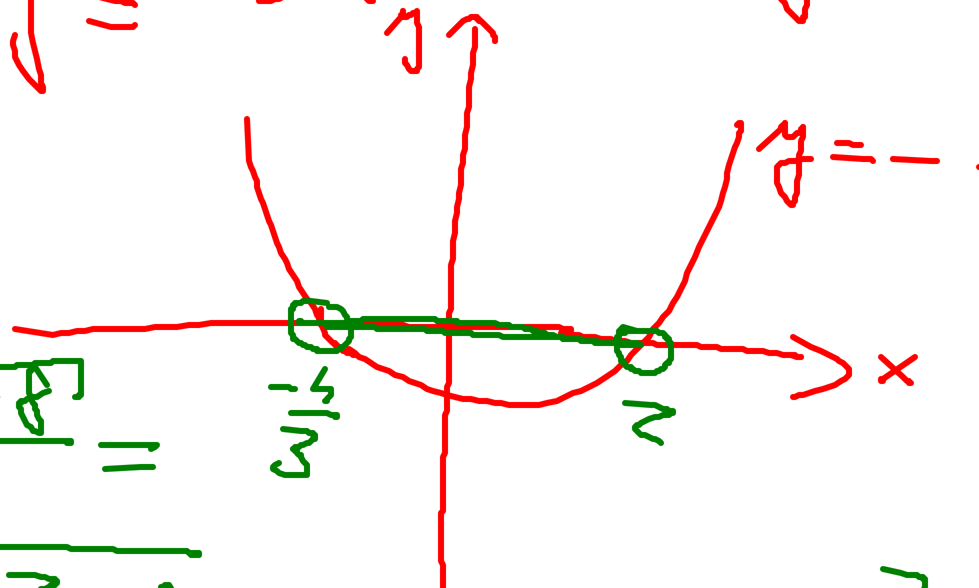
Jinak; $x \geq -3$

$|2x+1| < x+3 \quad /()^2$

$4x^2+4x+1 < x^2+6x+9$

$$\underbrace{3x^2 - 2x - 8 < 0}_{= 0}$$

$$y = 3x^2 - 2x - 8$$



$$x_{1,2} = \frac{2 \pm \sqrt{4 + 4 \cdot 3 \cdot 8}}{6} = \frac{2 \pm 2\sqrt{1 + 3 \cdot 8}}{6} = \frac{2 \pm 2 \cdot 5}{6} = \begin{cases} 2 \\ -\frac{8}{6} \end{cases}$$

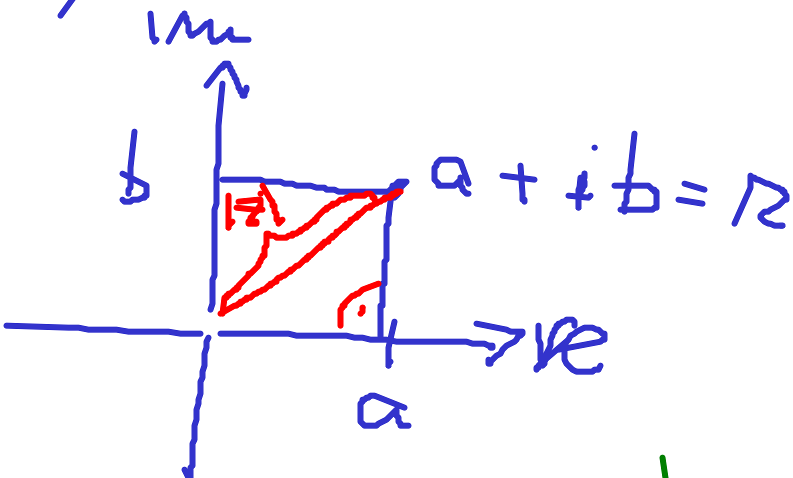
$$\left(-\frac{4}{3}, 2\right)$$

2.4 $z + |z| = 5 + (2+i)^2$

z kompl. i:

$z = a + ib$, $a, b \in \mathbb{R}$

$i^2 = -1$



$|z| = \sqrt{a^2+b^2}$

$z = a + ib$

L: $z + |z| = \underline{a + ib + \sqrt{a^2+b^2}}$

P: $5 + (2+i)^2 = 5 + (4 + 4i + \overset{-1}{i^2})$

$= \underline{5 + (3 + 4i)}$

$\underline{a} + \underline{i\,b} + \underline{\sqrt{a^2+b^2}} = \underline{8} + \underline{4i} \Rightarrow b = 4$

$a + \sqrt{a^2+16} = 8$

$\sqrt{a^2+16} = 8 - a \quad / ()^2$

$$a^2 + 16 = 64 - 16a + a^2$$

$$16a = 64 - 16 = 48$$

$$a = 3$$

Zinen: $z = 3 + 4i$

$$z^2 = (3+4i)^2 = 9 + 24i - 16$$

$$= -7 + 24i$$

2.5 $x^{2\log x+3{,}5}=100\sqrt{x}$ $/\cdot x^{-\frac{1}{2}}$

a, b dve riešenia, $a<b$

Určte ab^2.

$x^{2\log x+\frac{7}{2}-\frac{1}{2}}=100$

$\sqrt{x}\cdot x^{-\frac{1}{2}}=x^{\frac{1}{2}}\cdot x^{-\frac{1}{2}}=x^0=1$

$\boxed{x^{2\log x+3}=100}$ $/\log=\log_{10}$

$\log c^d=d\log c$

$\log\left(x^{2\log x+3}\right)=2$

$(2\underbrace{\log x}_{y}+3)\underbrace{\log x}_{y}=2$

$(2y+3)y=2y^2+3y=2$

$2y^2+3y-2=0$

$$y_{1,2} = \frac{-3 \pm \sqrt{3^2 + 4 \cdot 2 \cdot 2}}{4} =$$

$$= \frac{-3 \pm \sqrt{9 + 16}}{4} = \frac{-3 \pm 5}{4} = \begin{cases} -2 \\ \frac{1}{2} \end{cases}$$

- $\log x = y$

$\log x_1 = -2 \quad / \, 10^{(\,)}$

$$\left.\begin{array}{l} x_1 = 10^{-2} = \frac{1}{100} \\ \log x_2 = \frac{1}{2} \quad / \, 10^{(\,)} \\ x_2 = 10^{\frac{1}{2}} = \sqrt{10} \end{array}\right\} \begin{array}{l} a = \frac{1}{100} \\ \\ b = \sqrt{10} \end{array}$$

$$ab^2 = \frac{1}{100} \cdot (\sqrt{10})^2 = \frac{1}{100} \cdot 10$$

$$= \underline{\underline{\frac{1}{10}}}$$

2.6 $\sin x + \cos x = \sqrt{2}$

$x \in [0, 2\pi]$

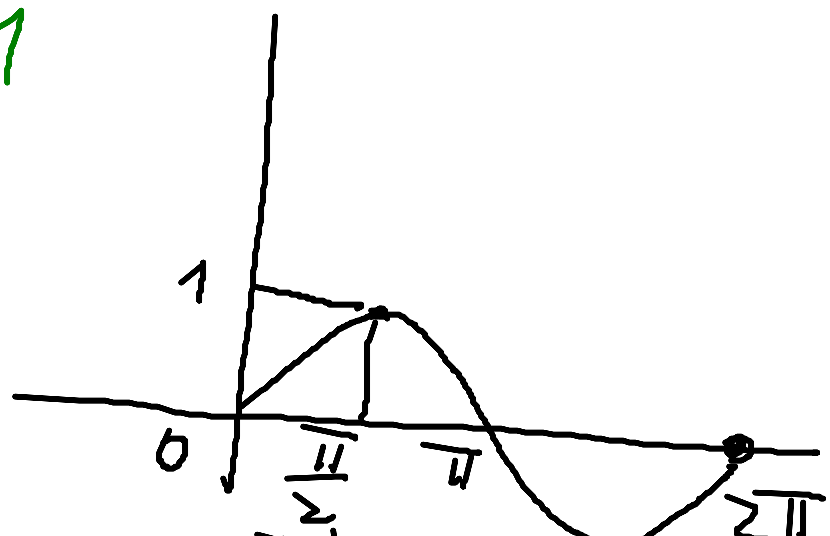
Určete hodnotu c, která je součtem všech řešení.

$$(\sin x + \cos x)^2 = (\sqrt{2})^2$$

$$\underline{\sin^2} x + 2\sin x\cos x + \underline{\cos^2} x = 2$$

$$\underbrace{2\sin x\cos x}_{\sin(2x)} + 1 = 2$$

$$\sin(2x) = 1$$



$$2x = \frac{\pi}{2}$$

$$x = \frac{\pi}{4}$$

$$\left(\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} = \cos\frac{\pi}{4}\right)$$

$$\Rightarrow c = \frac{\pi}{4}$$

2.7 Určete počet všech lichých pěticiferných čísel, která neobsahují ve svém zápisu cifru 9.

_ _ _ _ _

8 9 9 9 4

Výsledek: $8 \cdot 9 \cdot 9 \cdot 9 \cdot 4$

2.8 Kuželosečka

$$\boxed{3x^2+5y^2+6x-20y+8=0}$$

a, b délky poloos

Určete $c = a^2 + b^2$.

"Úprava na čtverec"

$$3(x^2+2x)+5(y^2-4y)+8=0$$

$$3[(x+1)^2-1]+5[(y-2)^2-4]+8=0$$

$$3(x+1)^2+5(y-2)^2-3-20+8=0$$

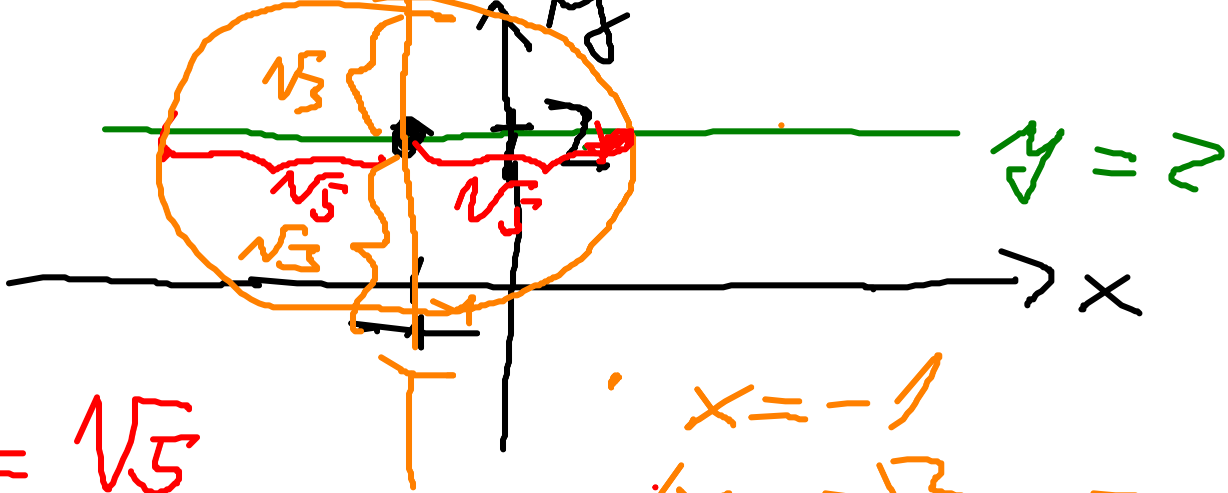
$$3(x+1)^2+5(y-2)^2=15 \quad /15$$

$$\frac{(x+1)^2}{5}+\frac{(y-2)^2}{3}=1$$

- $y=2 \rightsquigarrow (x+1)^2=5$

$$x+1=\pm\sqrt{5}$$

$$x=-1\pm\sqrt{5}$$



$x = -1$

$(y-2)^2 = 3$

$y - 2 = \pm\sqrt{3}$

$y = 2 \pm \sqrt{3}$

$a = \sqrt{5}$

$b = \sqrt{3}$

$a^2 + b^2 = 5 + 3 = 8$

2.9 Mějme reálná čísla $a_1, \dots, a_n \in \mathbb{R}$

- n liché $\quad M = \{a_1, \dots, a_n\}$

číslo $a_i \in M$ je medián množiny M, jestliže polovina hodnot $M \setminus \{a_i\}$ je $\leq a_i$ a polovina $\geq a_i$

Př. $M = \{1, 2, 5, 7, 9, 10, 11\}$

$\rightsquigarrow$ medián 7

Př. $M_1 = \{1, \underline{3}, 5\} \Rightarrow$ medián $=$ průměr

$M_2 = \{1, \underline{2}, 5\} \Rightarrow$ medián $<$ průměr

$M_3 = \{1, \underline{4}, 5\} \Rightarrow$ medián $>$ průměr

- M sudé, Nechť $a, b \in M$ takové, že $a < b$ a
  - polovina z $M \setminus \{a, b\}$ je $\le a$
  - polovina z $M \setminus \{a, b\}$ je $\ge b$

Pak medián je $\frac{a+b}{2}$

2.10. $a, b > 0, \ a, b \in \mathbb{R}$

$$A(a,b) = \frac{a+b}{2}$$

$$G(a,b) = \sqrt{ab}$$

$$H(a,b) = \frac{2}{\frac{1}{a}+\frac{1}{b}}$$

Ukažte nerovnosti

$$A(a,b) \geq G(a,b) \geq H(a,b)$$

$\to$ doma

$$\frac{a+b}{2} \geq \sqrt{ab} \quad / (\ )^2$$

$$(a+b)^2 \geq (2\sqrt{ab})^2$$

$$(a+b)^2 \geq 4ab$$

$$a^2+2ab+b^2 \geq 4ab$$

$$a^2-2ab+b^2 \geq 0 \qquad (a-b)^2 \geq 0$$

rovnost platí pro $a = b$