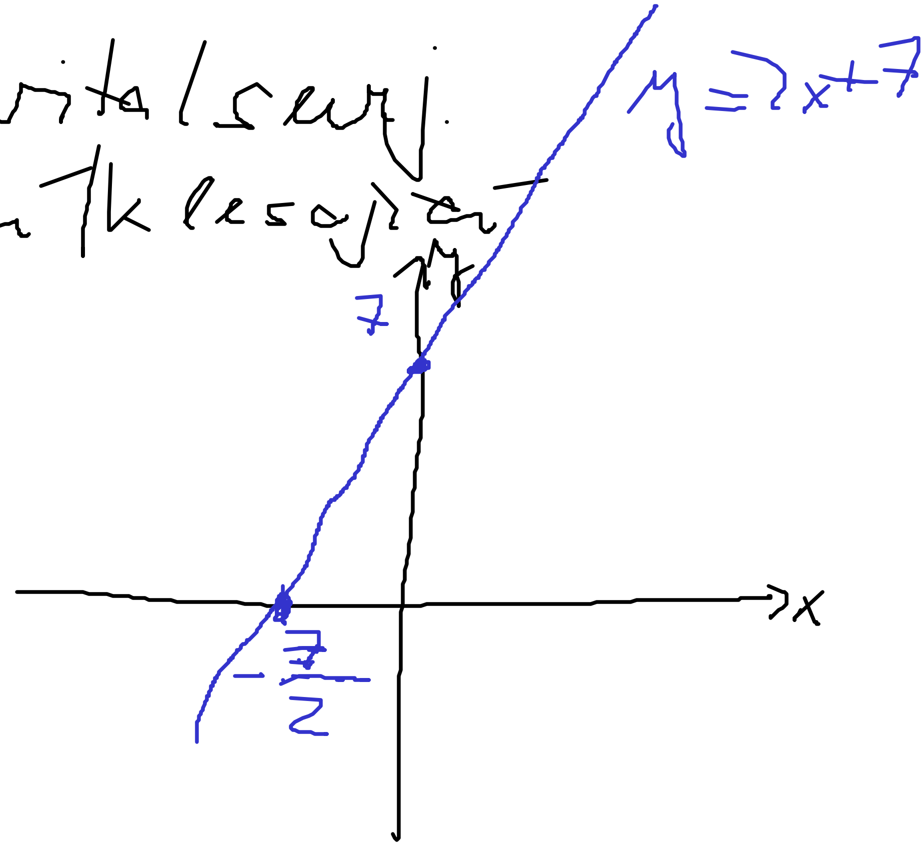
3.1 · Určete definiční obor a obor hodnot
· graf
· injektivita, surj
· rostoucí/klesající



· $f(x) = 2x+7$

$f(0) = 7$

$f(x) = 0$

$\Rightarrow x = -\frac{7}{2}$

· $D(f) = \mathbb{R}$

· $H(f) = \mathbb{R}$

· injektivní

· rostoucí

· injektivit.

$x \neq y \Rightarrow f(x) \neq f(y)$

· surjektivita

$\forall y \in \mathbb{R} \; \exists x \in D(f)$

t. ž. $f(x) = y$

$f(x) = |3x+1| - x$

$\underbrace{3x+1} = 0 \rightsquigarrow x = -\frac{1}{3}$

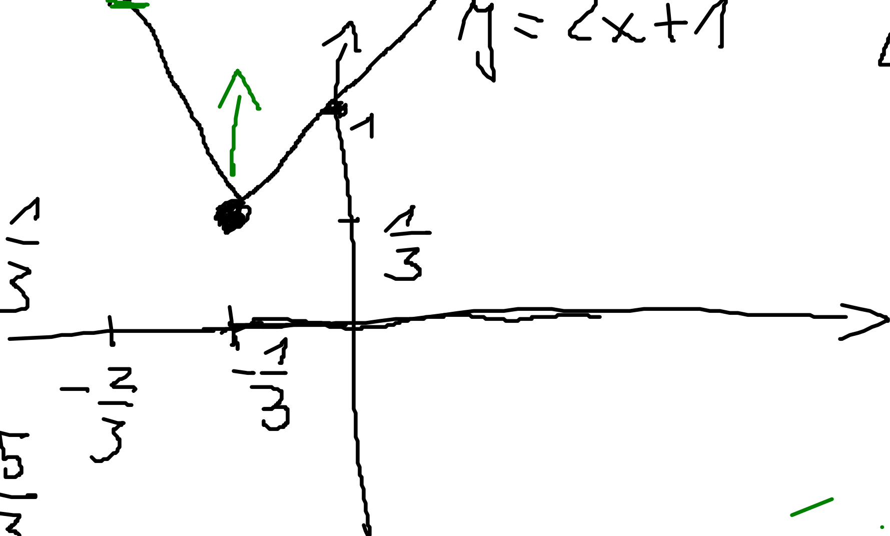
klesající — rostoucí

$x \in \left(-\infty, -\frac{1}{3}\right)$ — $x \in \left\langle -\frac{1}{3}, \infty\right)$

$f(x) = -(3x+1) - x$ — $f(x) = (3x+1) - x$

$f(x) = -4x - 1$ — $f(x) = 2x + 1$

$y = -4x - 1$ — $y = 2x + 1$



$f(0) = 1$

$f\left(-\frac{1}{3}\right) = \frac{1}{3}$

$f\left(-\frac{2}{3}\right) = \frac{5}{3}$

není injektivní

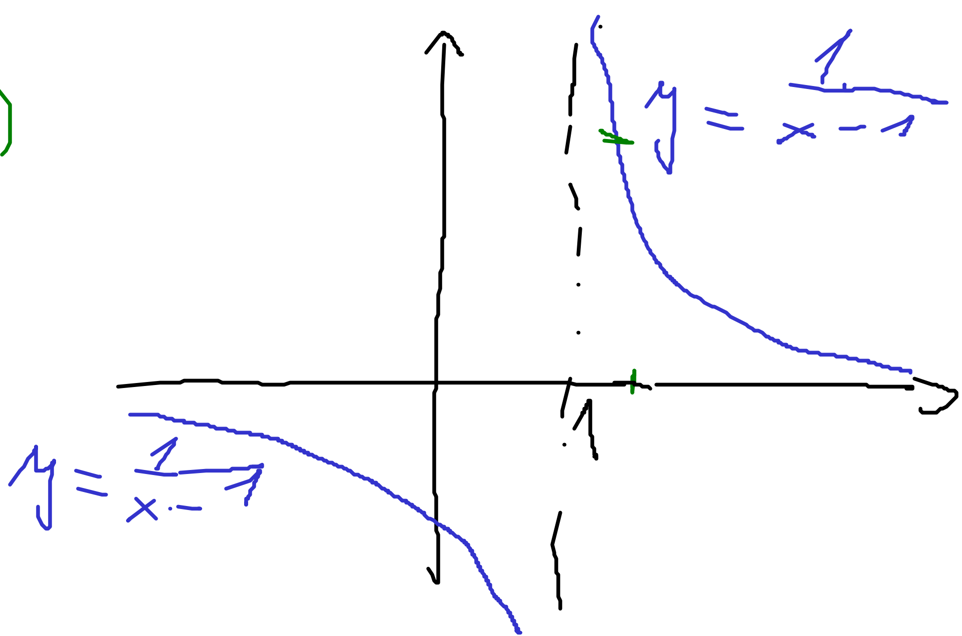
$D(f) = \mathbb{R}$

$H(f) = \left\langle \frac{1}{3}, \infty\right)$

- $f(x) = \dfrac{1}{x-1}$

$D(f) = \mathbb{R} \setminus \{1\}$

$H(f) = \mathbb{R} \setminus \{0\}$

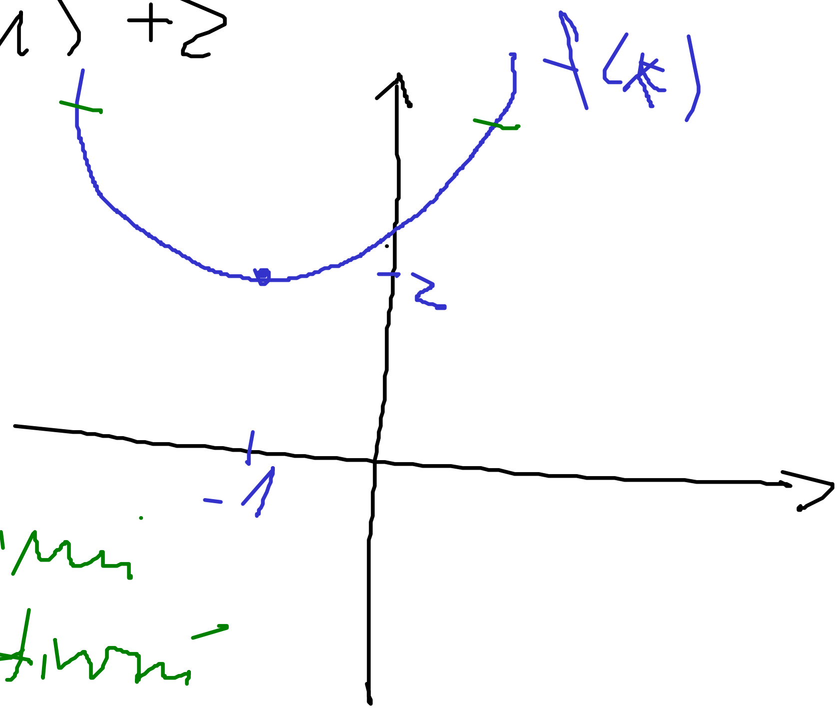


klesající je na $(-\infty,1)$ a $(1,\infty)$
je injektivní
není surjektivní

• $f(x) = x^2+2x+3$

$f(x) = (x+1)^2 - 1 + 3$

$= (x+1)^2 + 2$



$f(-1)=2$

$D(f)=\mathbb{R}$

$H(f)=\langle 2,\infty)$

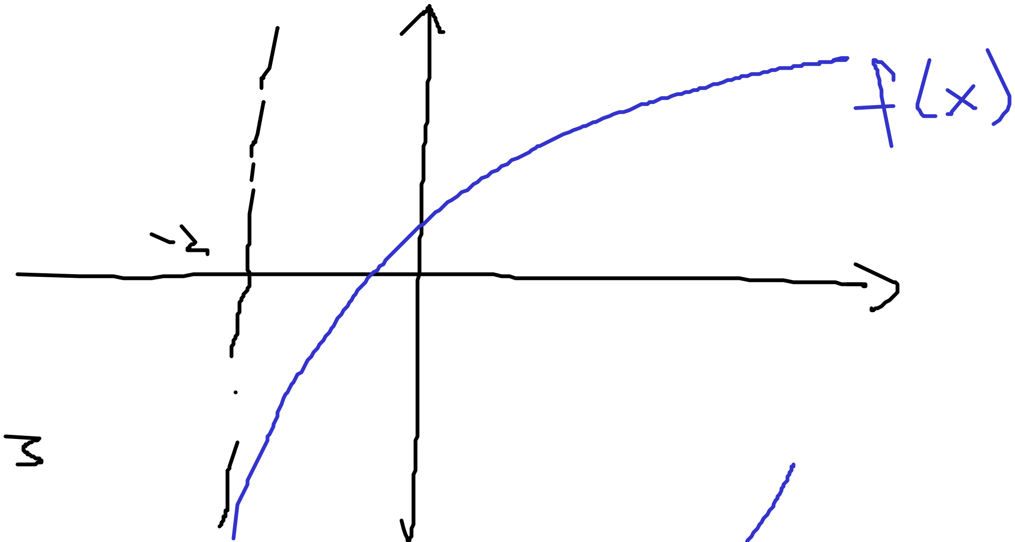
není injektivní
není surjektivní
klesající na $(-\infty,1\rangle$
rostoucí na $\langle 1,\infty)$

• $f(x) = \log_{10}\underbrace{(x+2)}_{=0 \text{ pro } x=-2}$

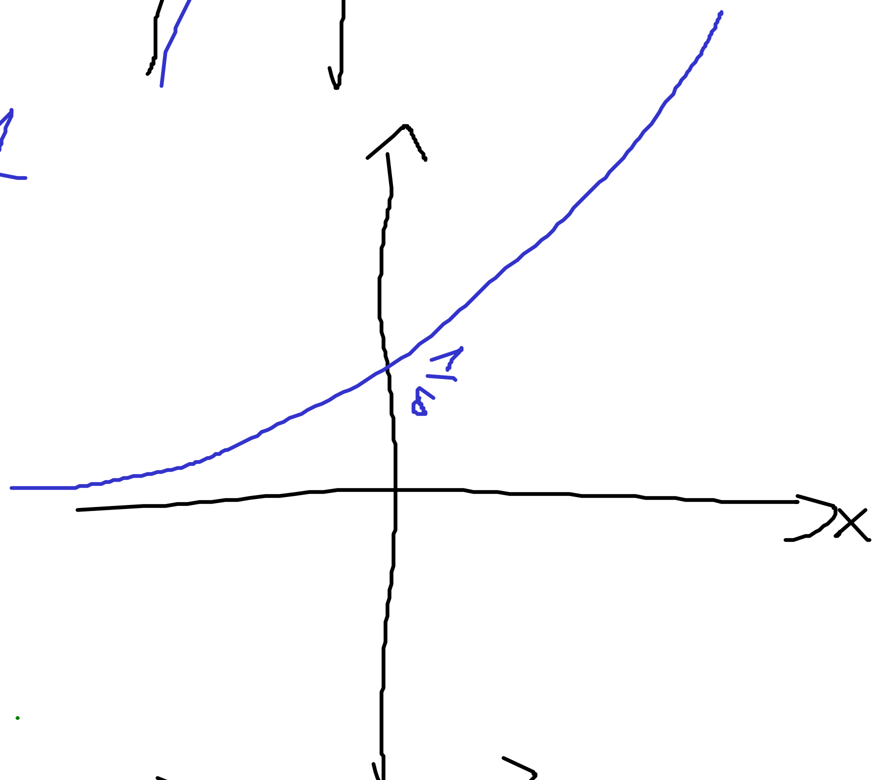
$D(f)=(-2,\infty)$

$H(f)=\mathbb{R}$

rostoucí, inj, surj



• $f(x) = 2^{x-3}$



$f(0) = 2^{-3} = \frac{1}{8}$

$D(f) = \mathbb{R}$

$H(f) = \mathbb{R}_+$

rost, inj,

není surj.

• $f(x) = (x-1)^2 + (x+2)^2$

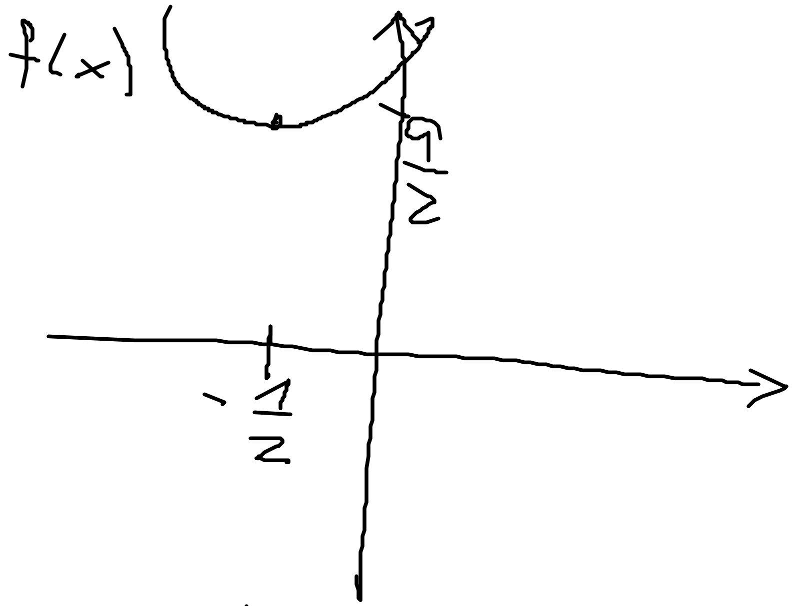
$f(x) = x^2 - 2x + 1 + x^2 + 4x + 4$

$= 2x^2 + 2x + 5$

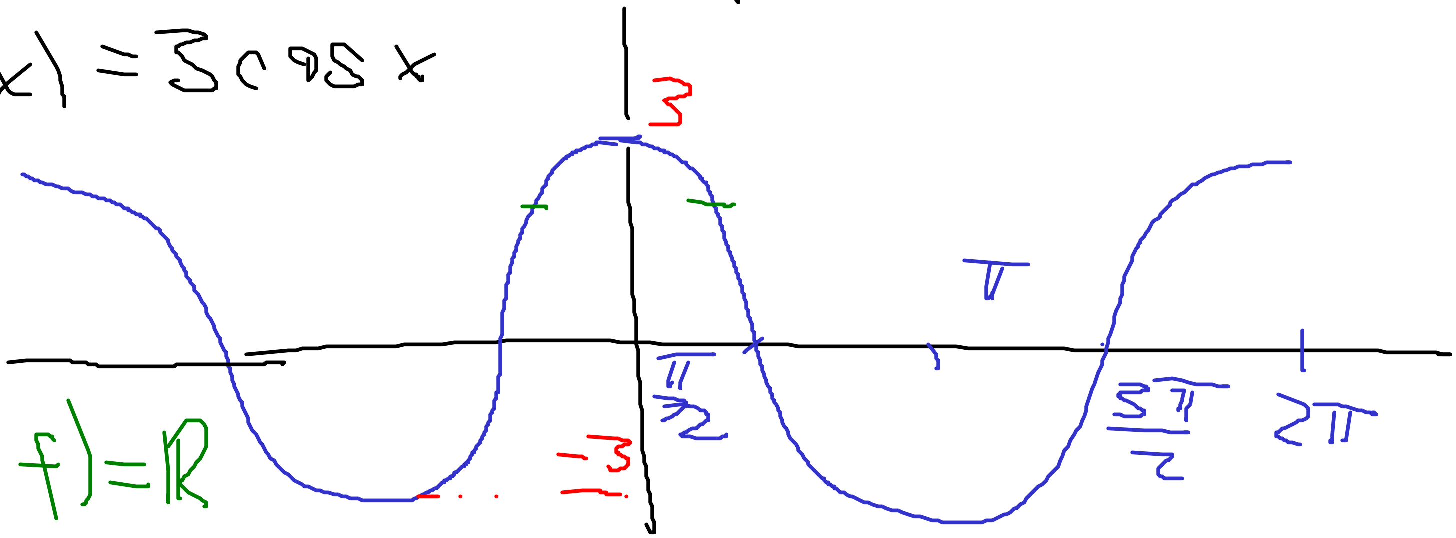
$= 2\left(x^2 + x + \frac{5}{2}\right)$

$= 2\left(\left(x + \frac{1}{2}\right)^2 - \frac{1}{4} + \frac{5}{2}\right)$

$= 2\left(x + \frac{1}{2}\right)^2 + \frac{9}{2}$



$f(x) = 3\cos x$



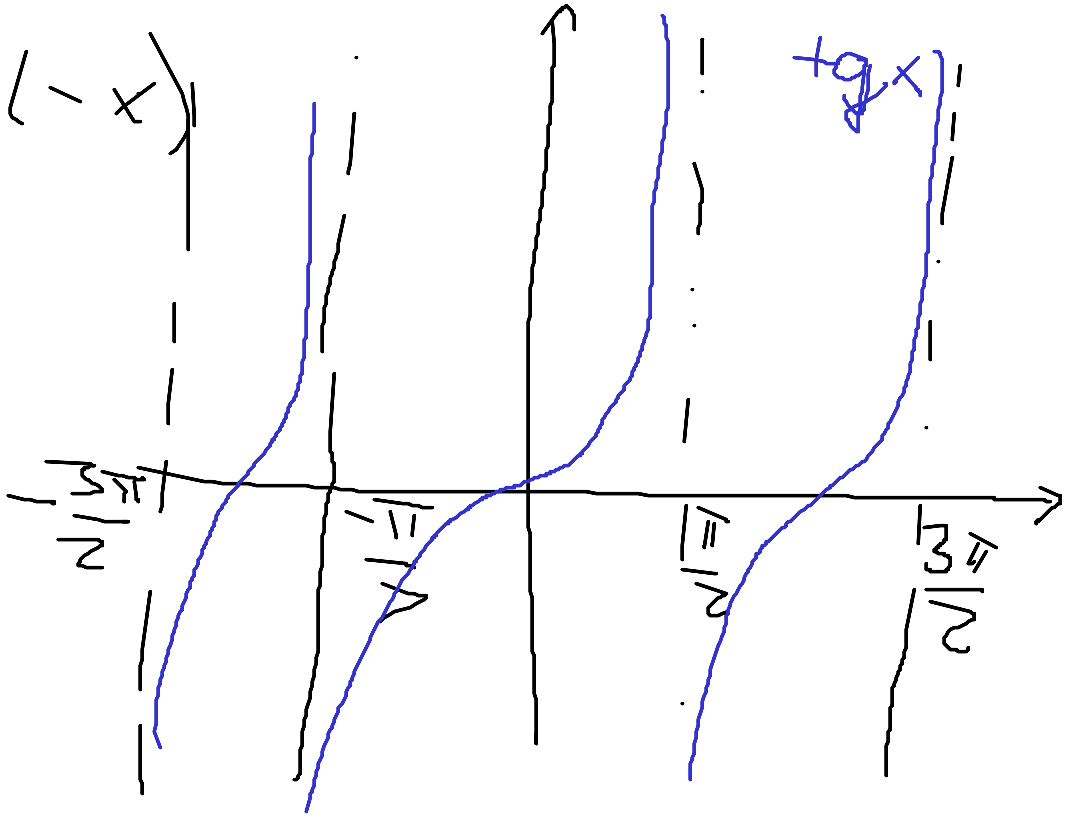
$D(f) = \mathbb{R}$

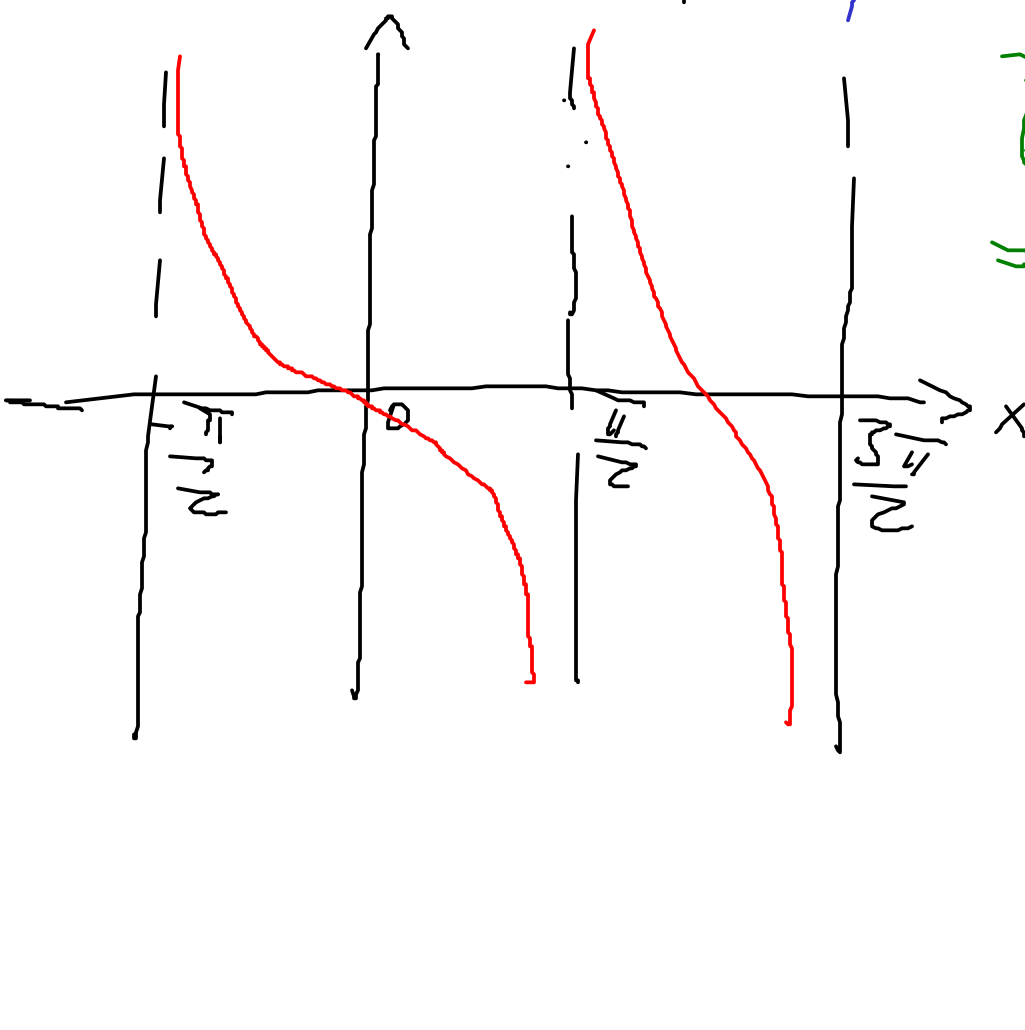
$H(f) = \langle -3, 3 \rangle$

není inj ani surj.

na int $[0, \pi]$ klesá

$f(x) = tg(-x)$





$D(f) =$

$= \mathbb{R} \setminus \{\frac{k\pi}{2} \mid k \text{ lich}\}$

$H(f) = \mathbb{R}$

3.2 $f(x) = \dfrac{1}{\log_{10}(x^2-1)-1}$

$D(f)$: $x^2-1>0$

$\log_{10}(x^2-1) \neq 1 \; (10^1)$

$x \in (-\infty,-1) \cup (1,\infty)$ $\wedge$ $x \neq \pm\sqrt{11}$

$x^2-1 \neq 10$

$x^2 \neq 11$

$x \neq \pm\sqrt{11}$

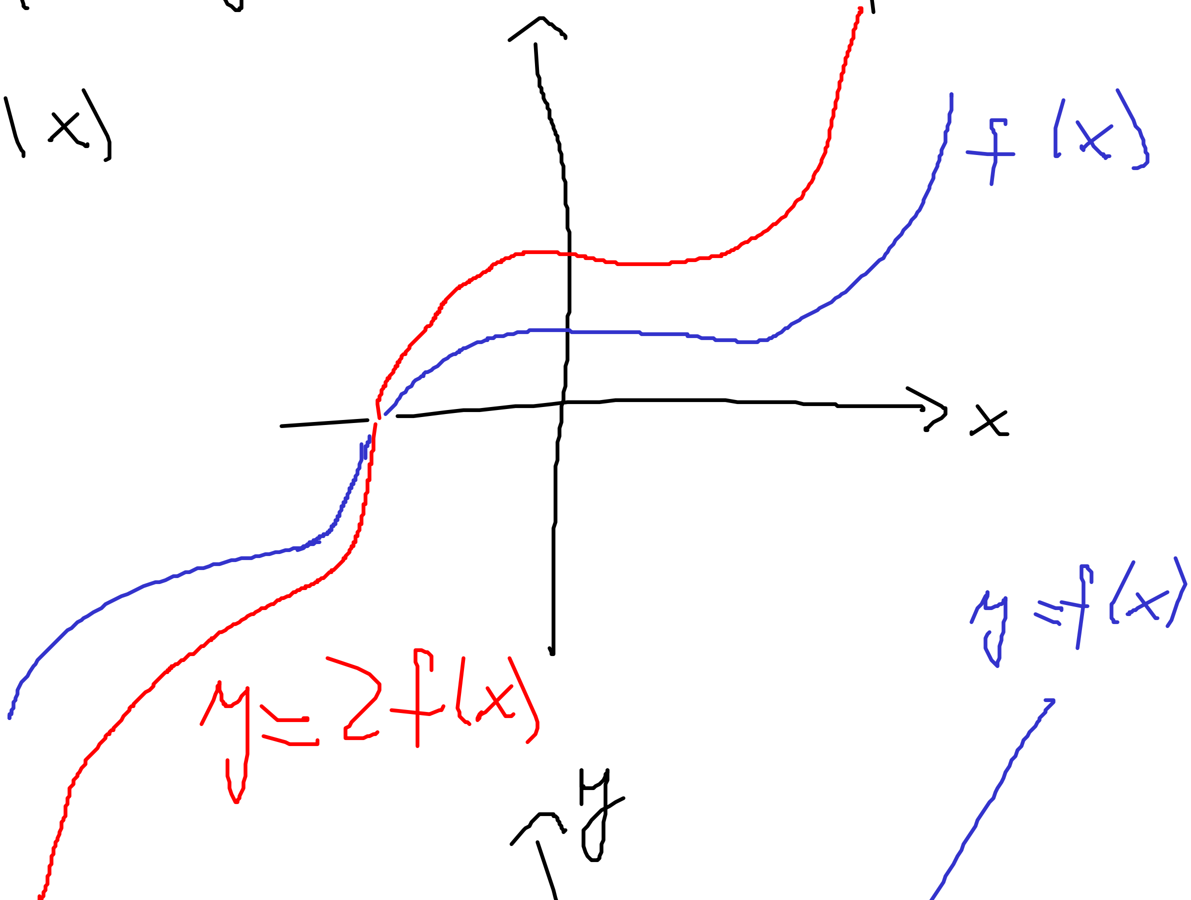
Graf jmenovatel

$\log_{10}(x^2-1)-1 = \underbrace{\log_{10}\dfrac{x^2-1}{10}}$

$H(f) = \mathbb{R} \setminus \{0\}$

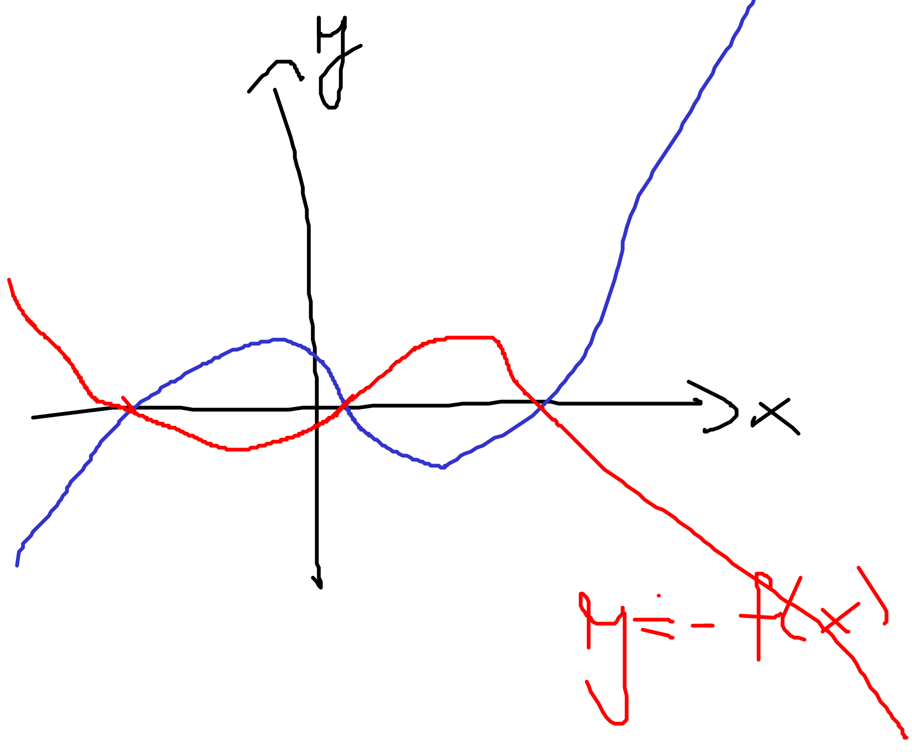
3.3 Jak se mění graf funkce $f(x)$, přejdeme-li k funkci

• $y = 2f(x)$



• $y = -f(x)$

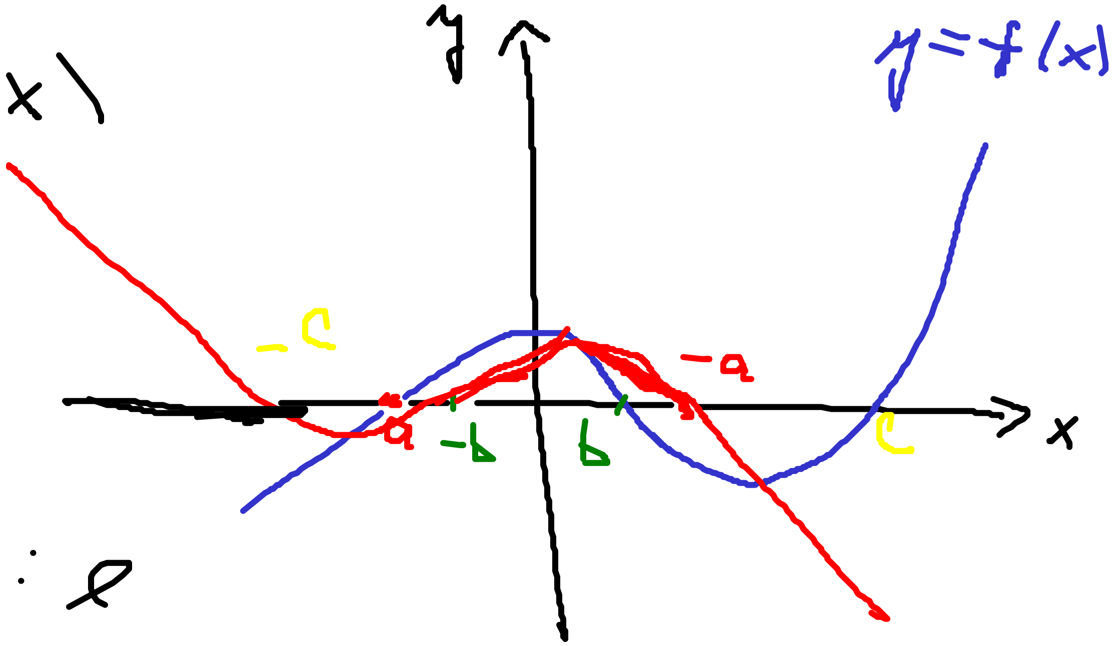
symetrie podle osy x



• $y = f(-x)$

↓

symetrie podle osy y

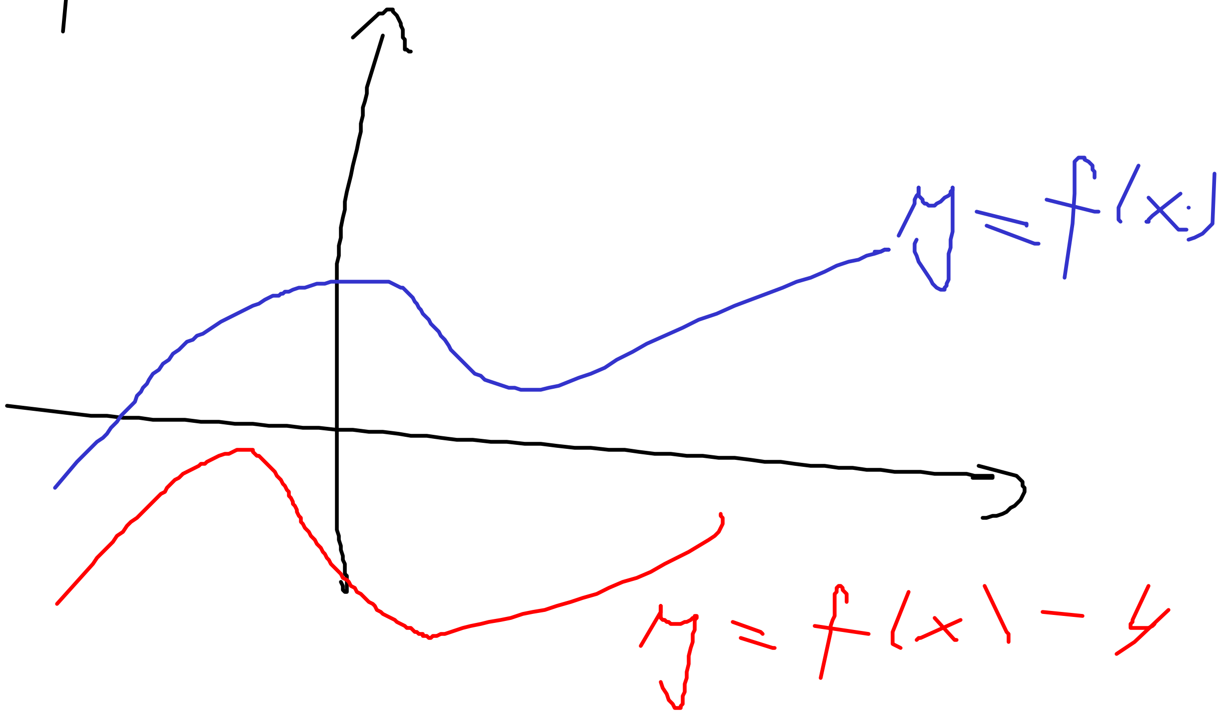


• $y = f(x+3)$

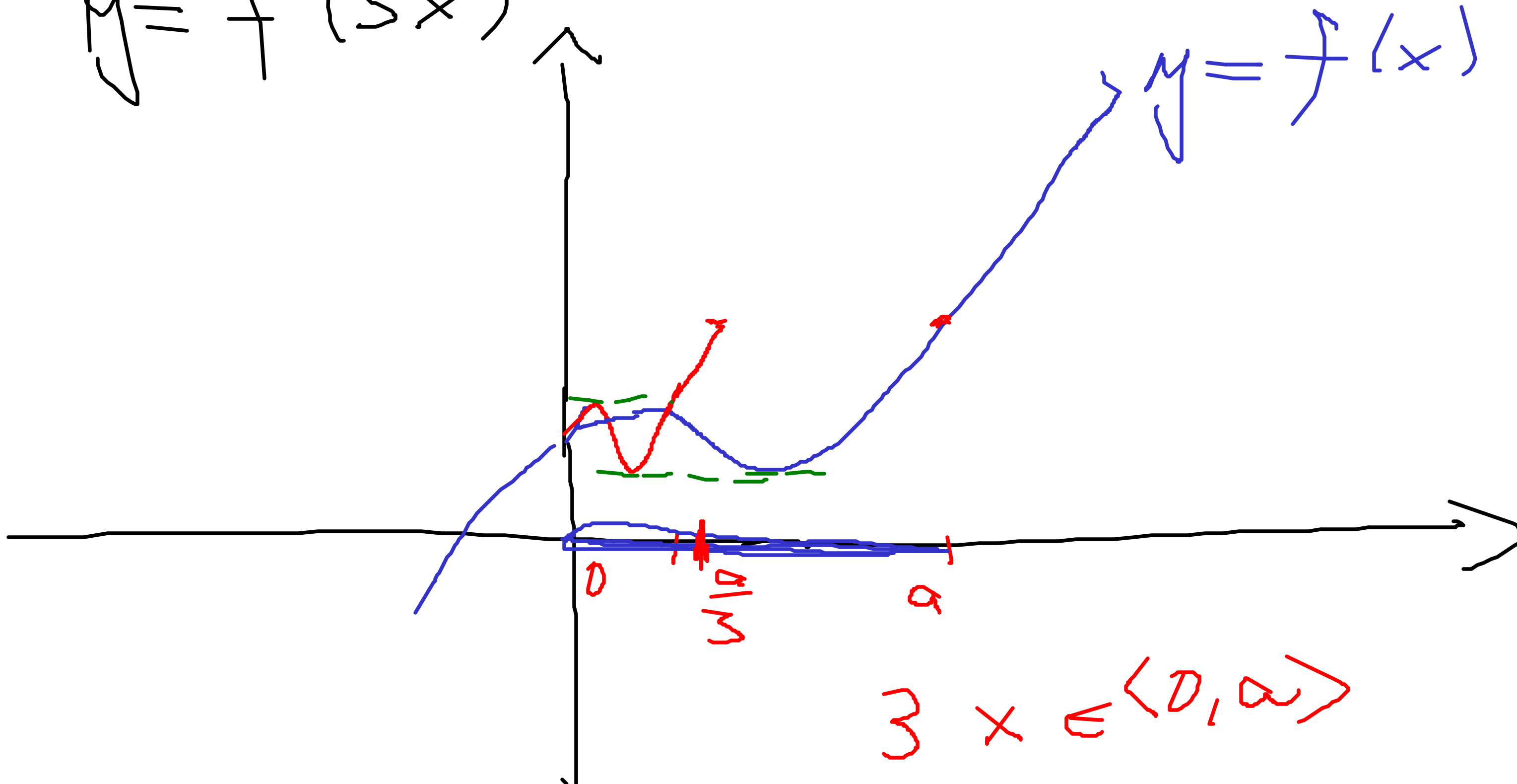
$$\left.\begin{array}{l} f(a) = 0 \\ f(x+3) = 0 \end{array}\right\} \quad \begin{array}{l} x+3 = a \\ x = a-3 \end{array}$$

Posun původní funkce doleva.

- $y = f(x) - 4$ posun "dolů"

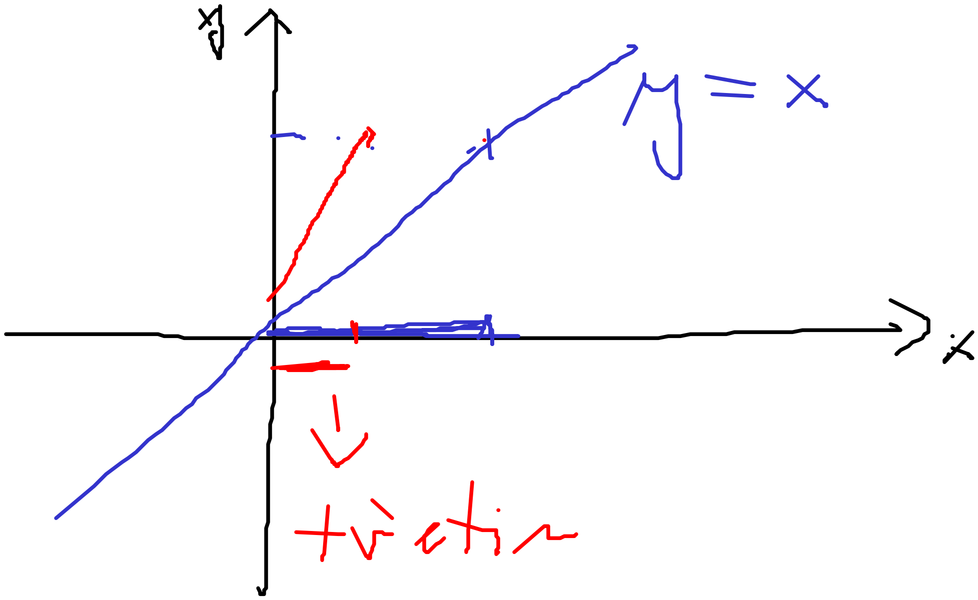


- $y = f(3x)$



$3x \in \langle 0, a \rangle$

$x \in \langle 0, \frac{a}{3} \rangle$



$y = 3x$

3.4 $g(x) = \frac{3}{x+5} + 2$

složeni funkcija

$x \mapsto \frac{1}{x}$

$x \mapsto x + k_1$

$x \mapsto k_2 x$

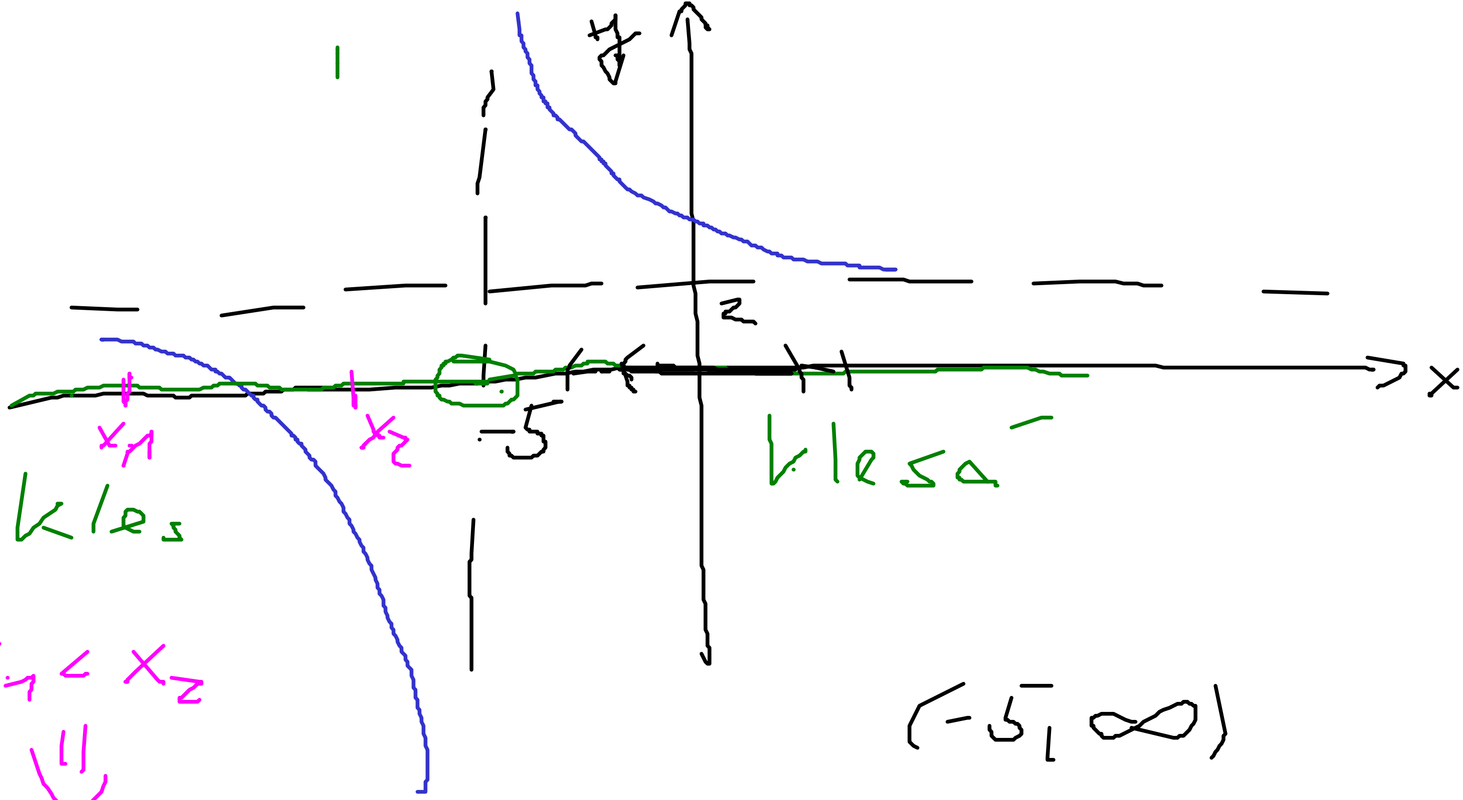
$f_1(x) = x + 5$

$f_2(x) = \frac{1}{x}$

$f_3(x) = 3x$

$f_4(x) = x + 2$

$h(x) = (f_4 \circ f_3 \circ f_2 \circ f_1)(x)$



$x_1 < x_2$

$\Downarrow$

$f(x_1) > f(x_2)$

$(-5, \infty)$

$(-\infty, 5]$

3.5 Def: Fce $f(x)$ je <u>rostoucí</u> na intervalu $I \subseteq D(f)$, jestliže: $\forall x_1, x_2 \in I$ t.z. $x_1 < x_2$ platí $f(x_1) < f(x_2)$

<u>Def</u>, I <u>je maximální interval</u>, kde je fce $f(x)$ rostoucí, jestliže f je rostoucí na I a dále pro $\forall$ interval J t.z. $I \subsetneq J \subseteq D(f)$ platí, že $f(x)$ není rostoucí na J.

$f \circ g$

<u>Věta</u>: Nechť g je fce rostoucí na intervalu I a f rostoucí na intervalu J a dále $\{g(x) \mid x \in I\} \subseteq J$. Pak $f \circ g$ je rostoucí na I

Dk: $f \circ g$ má být rostoucí

$x_1, x_2 \in I$ t. ž. $x_1 < x_2$

Pak $\underbrace{g(x_1)}_{\in J} < \underbrace{g(x_2)}_{\in J}$ neboť g rostoucí na I

$\underbrace{\qquad\qquad\qquad\qquad}$

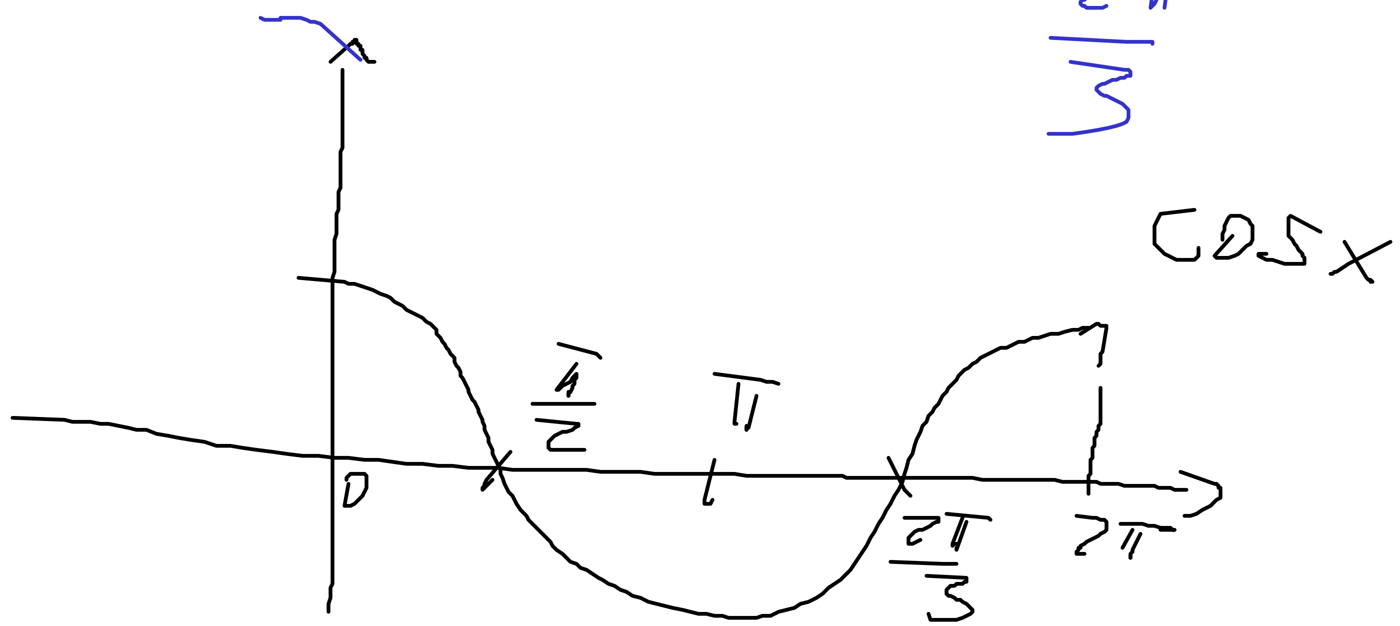
$f(g(x_1)) < f(g(x_2))$ neboť f rostoucí na J

$\square$

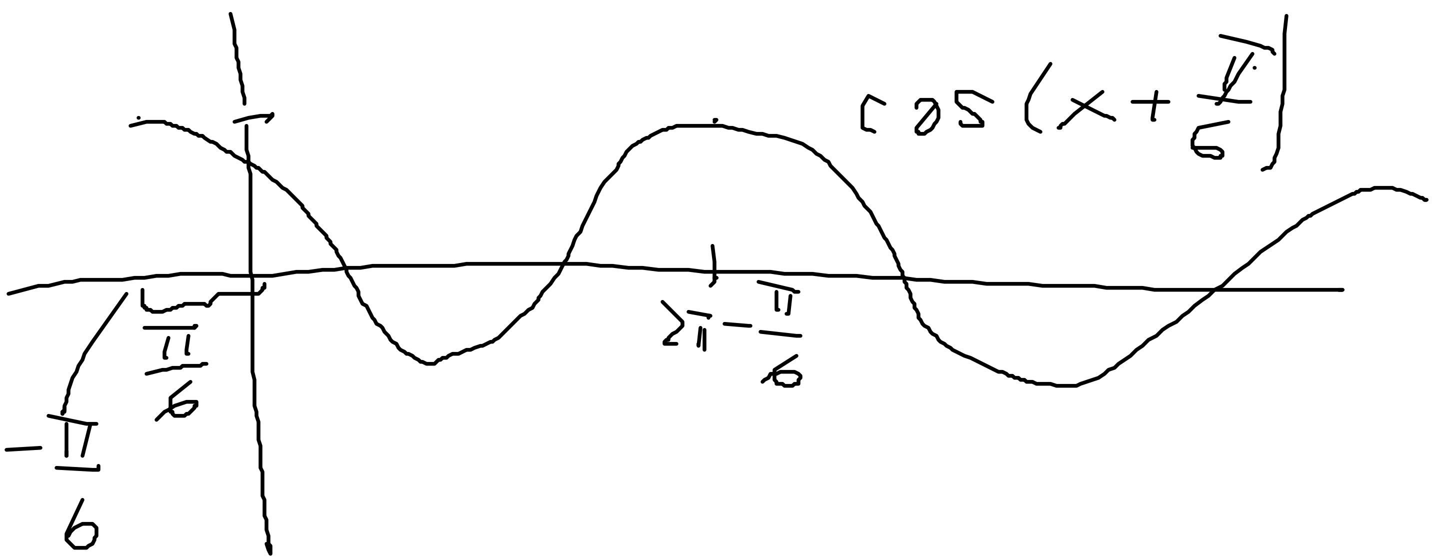
3.6 $f(x) = 2\cos\left(3x + \frac{\pi}{2}\right) - 1$

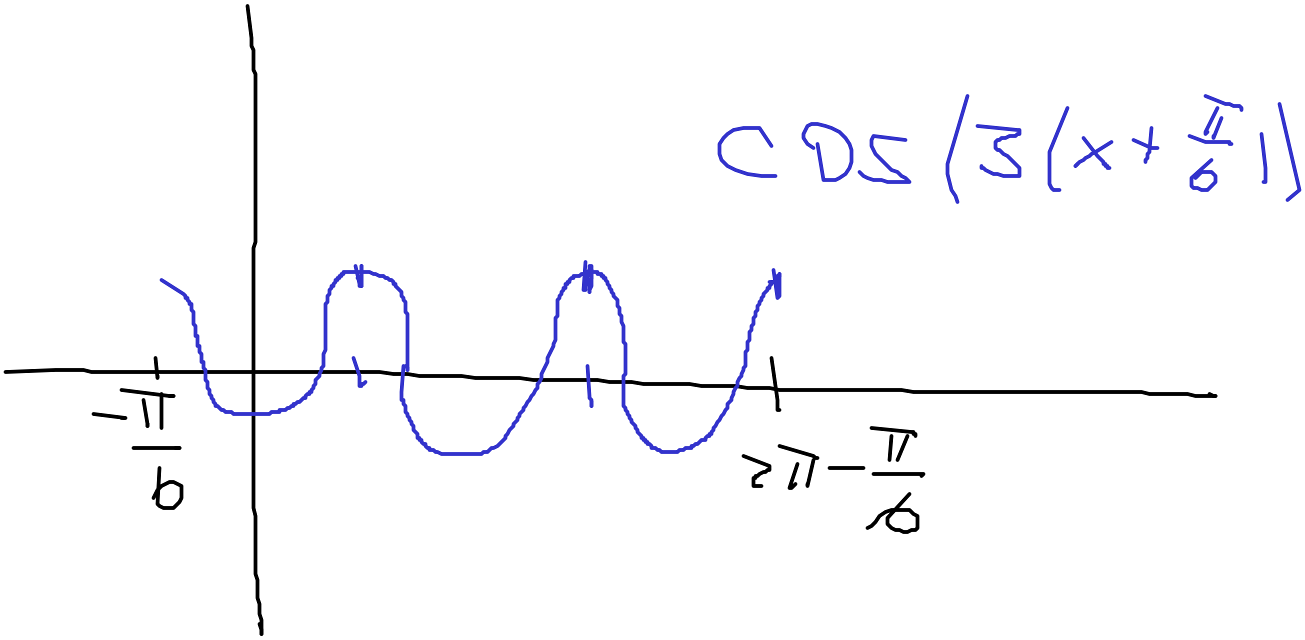
$\cos(x)$ perioda 2π

$\Rightarrow \cos\left(3x + \frac{\pi}{2}\right)$ má periodu $\frac{2\pi}{3}$



$\cos\left(3\left(x + \frac{\pi}{6}\right)\right)$





$\cos(3(x+\frac{\pi}{6}))$
$-\frac{\pi}{6}$
$2\pi-\frac{\pi}{6}$