5.1 $f(x)=|\overbrace{\underbrace{2x-3}_{\le 0}}^{\ge 0}|-|\overbrace{\underbrace{x+2}_{\ge 0}}^{\ge 0}|+|\overbrace{\underbrace{10-3x}_{\ge 0}}^{\ge 0}|-1$

na intervalu $[-5,5]$

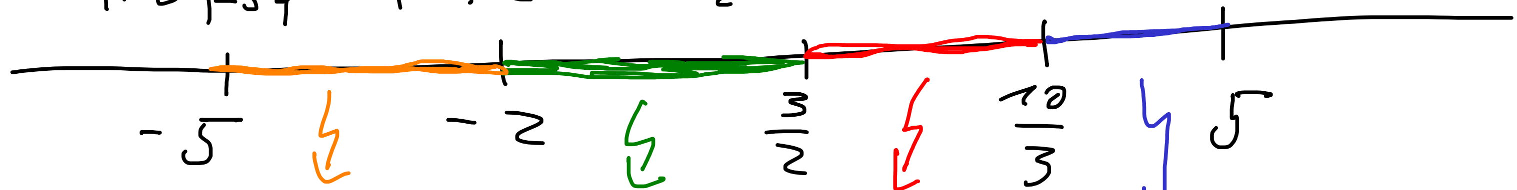
- obor hodnot
- max. intervaly monotónnosti
- vyriešte nerovnicu $f(x)<2$
- graf

$-\frac{20}{3}+4=\frac{-20+12}{3}=\frac{-8}{3}$

body $\frac{3}{2}$, -2, $\frac{10}{3}$

$f\left(\frac{10}{3}\right)=\frac{-8}{3}$ $\quad f(5)=4$

$f(-5)=34$ $\quad f(-2)=22$ $\quad f\left(\frac{3}{2}\right)=1$



$f(x)=-4x+14$ $\quad f(x)=-6x+10$ $\quad f(x)=-2x+4$ $\quad f(x)=4x-16$

$x\in\left[\frac{10}{3},5\right] \Rightarrow f(x)=(2x-3)-(x+2)-(10-3x)-1$

$=4x-16$

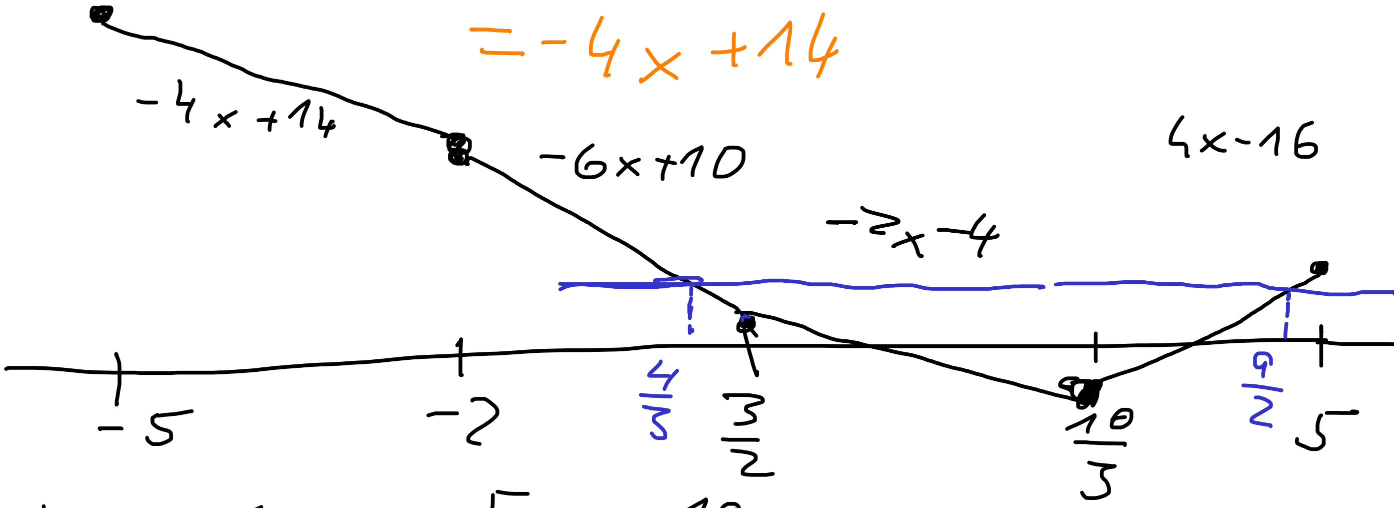
$x\in\left[\frac{3}{2},\frac{10}{3}\right] \Rightarrow f(x)=(2x-3)-(x+2)+(10-3x)-1$

$=-2x+4$

$x\in\left[-2,\frac{3}{2}\right] \Rightarrow f(x)=-(2x-3)-(x+2)+(10-3x)-1$

$=-6x+10$

$x\in[-5,-2] \Rightarrow f(x)=-(2x-3)+(x+2)+(10-3x)-1$



- $f(x)$ klesá na $[-5, \frac{10}{3}]$

  rastie na $[\frac{10}{3}, 5]$

- $H(f) = [-\frac{8}{3} ; 34]$

- nerovnica $f(x) < 2$

$-6x + 10 = 2$

$6x = 8$

$x = \frac{4}{3}$

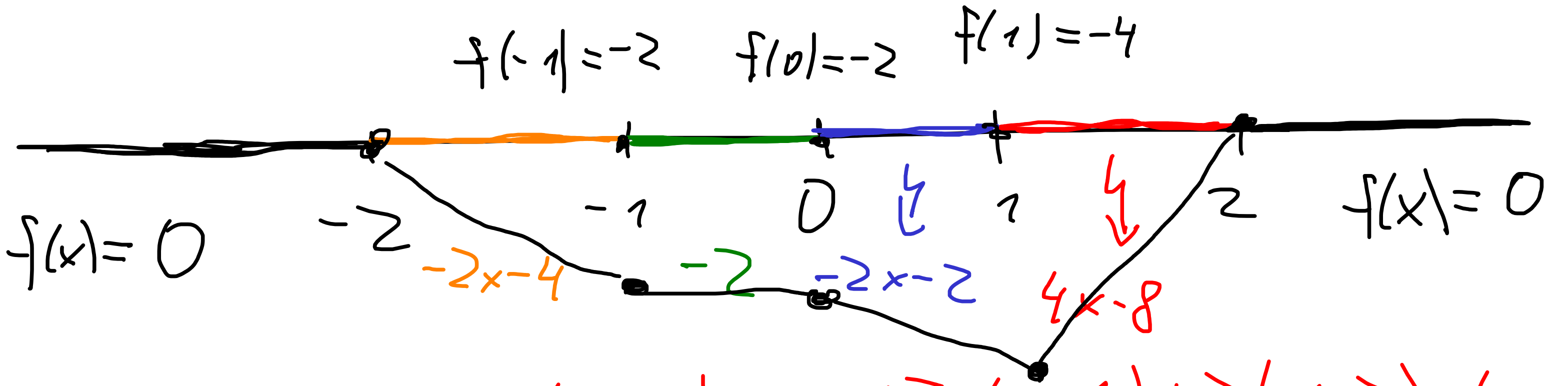
$4x - 16 = 2$

$4x = 18$

$x = \frac{9}{2}$

$x \in (\frac{4}{3}, \frac{9}{2})$

5. 2(a) $f(x)=|x+1|-|x|+3|x-1|-2|x-2|-|x+2|=0$



$x\in[1,2] \Rightarrow f(x) = (x+1) - x + 3(x-1) + 2(x-2) - (x+2)$
$= 4x-8$

$x\in[0,1] \Rightarrow f(x) = (x+1) - x - 3(x-1) + 2(x-2) - (x+2)$
$= -2x-2$

$x\in[-1,0] \Rightarrow f(x) = (x+1) + x - 3(x-1) + 2(x-2) - (x+2)$
$= -2$

$x\in[-2,-1] \Rightarrow f(x) = -(x+1) + x - 3(x-1) + 2(x-2) - (x+2)$
$= -2x-4$

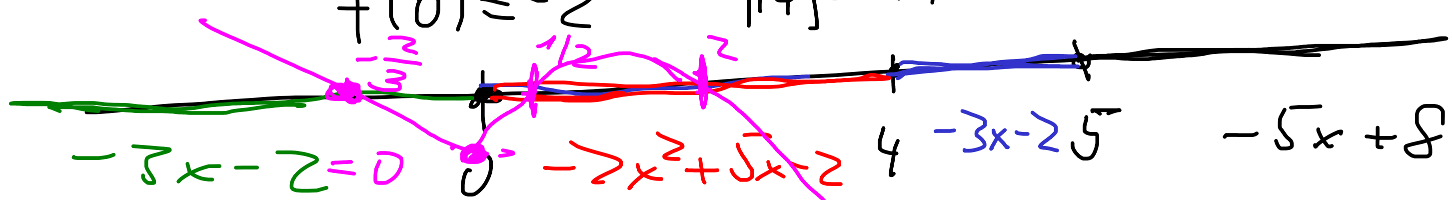
Zeilen: $x\in(-\infty,-2]\cup[2,\infty)$

(b) $\dfrac{|x^2-4x|+3}{x^2+|x-5|} = 1$ $\qquad x^2+|x-5|>0$

$$|x^2-4x|+3 = x^2+|x-5|$$

$$f(x) = |x(x-4)| - x^2 - |x-5| + 3 = 0$$

$f(0)=-2 \qquad f(4)=-14 \qquad f(5)=-17$



$$x \in [5,\infty) \Rightarrow f(x) = (x^2-4x) - x^2 - x + 5 + 3 = -5x+8$$

$$x \in [4,5] \Rightarrow f(x) = (x^2-4x) - x^2 + (x-5) + 3 = -3x-2$$

$$x \in [0,4] \Rightarrow f(x) = -(x^2-4x) - x^2 + (x-5) + 3 = -2x^2+5x-2$$

$$x \in (-\infty,0] \Rightarrow f(x) = (x^2-4x) - x^2 + (x-5) + 3 = -3x-2$$

Dále řešíme $-2x^2+5x-2=0$

$$2x^2-5x+2=0$$

$$x_{1,2} = \frac{5 \pm \sqrt{25-16}}{4} = \frac{5 \pm 3}{4} = \begin{cases} 2 \\ \frac{1}{2} \end{cases}$$

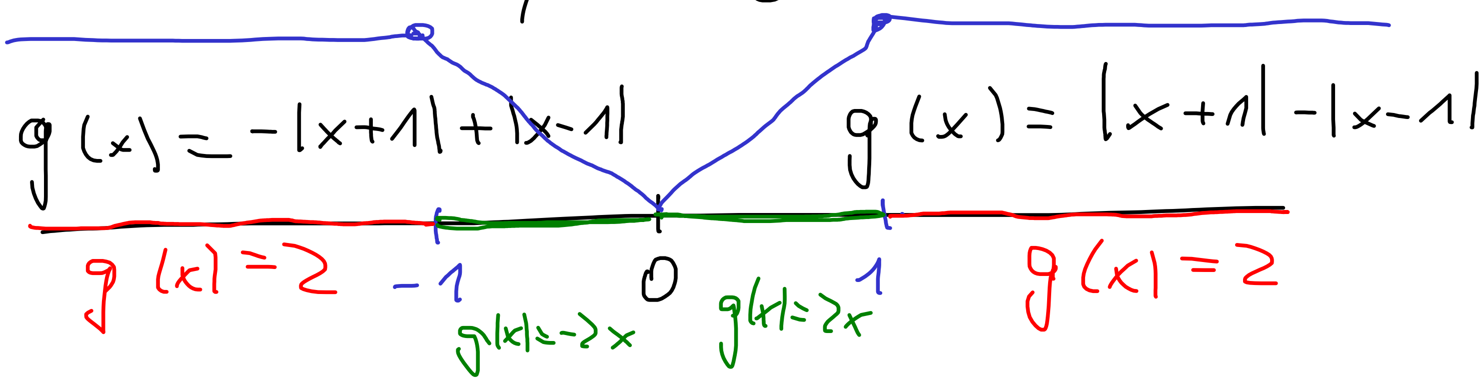
Řešení jsou: $-\frac{2}{3}, \frac{1}{2}, 2$

5.3 • $f(x) = ||x+1|+|x-1|| = |x+1|+|x-1|$ DÚ

• $g(x) = \underbrace{||x+1|-|x-1||}_{=0 \text{ pro}}$ $\quad |x+1| = |x-1| \; /()^2$

$x^2+2x+1 = x^2-2x+1$

pro $x=0$ $\qquad x = 0$



g(x) sudá funkce

$g(-x) = ||-x+1|-|-x-1|| = ||x-1|-|x+1||$

$= ||x+1|-|x-1||$

5.5 (a) $x(x-1)(x+\frac{1}{2}) \quad /\cdot 2$

$$f(x) = x(x-1)(2x+1)$$

(b) $f(x) = (x+1)(x^2+1)$

(c) $f(x) = (x-1)^3$

(d) $f(x) = (x+1)(x^2-2)$

5.6 Hledání racionálních kořenů polynomů s celočíselnými koeficienty $a_i \in \mathbb{Z}$

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$$

$\frac{p}{q}$ je kořen $\quad (p,q) = 1$

$p \in \mathbb{Z} \quad p, q$ nesoudělná

$q \in \mathbb{N}$

$$f\left(\frac{p}{q}\right) = a_n\left(\frac{p}{q}\right)^n + a_{n-1}\left(\frac{p}{q}\right)^{n-1} + \ldots + a_1\frac{p}{q} + a_0 = 0 \quad /\cdot q^n$$

$$\underbrace{\underbrace{a_n p^n + a_{n-1}p^{n-1}q + \ldots + a_1 p q^{n-1}}_{p|} + a_0 q^n}_{\text{}} = 0$$

$\Rightarrow p | a_0 q^n \Rightarrow p | a_0$

$q | a_n p^n \Leftarrow \quad \underbrace{a_{n-1}p^{n-1}q + \ldots + a_0 q^n}_{q|}$

$\Downarrow q | a_n$

5.7 (a) $f(x) = 2x^3 + x^2 - 4x - 3 = 0$

$\frac{p}{q}$ rac. kořen $\Rightarrow p \mid 3 \Rightarrow p \in \{\pm 1, \pm 3\}$

$(p, q) = 1$ $\quad q \mid 2 \Rightarrow q \in \{1, 2\}$

$p \in \mathbb{Z}, q \in \mathbb{N}$

$$\frac{p}{q} \in \{\underbrace{\pm 1, \pm 3, \pm \tfrac{1}{2}, \pm \tfrac{3}{2}}_{8 \text{ kandidátů na rac. kořeny}}\}$$

Hornerovo schéma

	2	1	-4	-3	
1	2	3	-1	-4 $\neq 0$	není kořen
3	2	7	17	$\neq 0$	není kořen
-1	2	-1	-3	0	$f(x) = (x+1)(2x^2 - x - 3)$
-1	2	-3	0		$f(x) = (x+1)^2(2x-3)$

Kořeny jsou: $-1, \frac{3}{2}$

↳ dvojnásobný

(b) $f(x) = 27x^3 + 27x^2 - 4$

$\frac{p}{q}$ rac. kořen $\Rightarrow p \mid -4 \Rightarrow p \in \{\pm 1, \pm 2, \pm 4\}$

$q \mid 27 \Rightarrow q \in \{1, 3, 9, 27\}$

$$\frac{p}{q} \in \left\{ \pm 1, \pm 2, \pm 4, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{1}{9}, \pm \frac{2}{9}, \pm \frac{4}{9}, \pm \frac{1}{27}, \pm \frac{2}{27}, \pm \frac{4}{27} \right\}$$

$\Rightarrow$ 24 kandidátov na rac. korene

$x \geq 1 \Rightarrow f(x) > 0$

1, 2, 4 nejsou koreny