5.7 2. $27x^3+27x^2-4=0$

$x=\frac{p}{q},\ p\in\mathbb{Z},\ q\in\mathbb{N},\ (p;q)=1$

$p\,|\,4 \Rightarrow p\in\{\pm1,\pm2,\pm4\}$

$q\,|\,27 \Rightarrow q\in\{1,3,9,27\}$

	27	27	0	-4
$\frac{1}{3}$	27	36	12	0
$-\frac{2}{3}$	27	18	0	
$-\frac{2}{3}$	27	0		

$$27x^3+27x^2-4=\left(x-\frac{1}{3}\right)\left(27x^2+36x+12\right)$$

$$=\left(x-\frac{1}{3}\right)\left(x+\frac{2}{3}\right)\left(27x+18\right)=$$

$$=27\left(x-\frac{1}{3}\right)\left(x+\frac{2}{3}\right)^2$$

5.8

$$\begin{aligned} x^2 + ax + 8 &= 0 \\ x^2 + x + a &= 0 \end{aligned}$$

(≤ 1)

Určete $a \in \mathbb{R}$ t. ž. soustava má společný kořen.

$D = a^2 - 32 > 0$

$a \in (-\infty, -4\sqrt{2})$

$D = 1 - 4a > 0$

$a < \frac{1}{4}$

Vietovy vztahy:

$$ax^2 + bx + c = 0$$
$$x_1 + x_2 = -\frac{b}{a}$$
$$x_1 \cdot x_2 = \frac{c}{a}$$

$$x_1 + x_2 = -\frac{a}{1}$$
$$x_1 \cdot x_2 = 8$$

$$x_1 + x_2 = -1$$
$$x_1 x_2 = a$$

$$(a-1)x + 8 - a = 0$$

$$x = \frac{a-8}{a-1} \rightsquigarrow x^2 + x + a = 0$$

$$\left(\frac{a-8}{a-1}\right)^2 + \frac{a-8}{a-1} + a = 0 \quad / (a-1)^2$$

$$(a-8)^2 + (a-8)(a-1) + a(a-1)^2 = 0$$

$$(a^2 - 16a + 64) + (a^2 - 9a + 8) + (a^3 - 2a^2 + a) = 0$$

$$a^3 - 24a + 72 = 0$$

$$a = \frac{p}{q} \qquad p \mid 72, \quad q \mid 1$$

$$\Rightarrow \frac{p}{q} \in \{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \ldots\}$$

$$\begin{array}{c|cccc} & 1 & 0 & -24 & 72 \\ \hline -6 & 1 & -6 & 12 & 0 \\ \hline \end{array}$$

$$\boxed{a = -6}$$

$$a^3 - 24a + 72 =$$

$$= (a+6)\underbrace{(a^2 - 6a + 12)}$$

$$D = 36 - 4 \cdot 12 < 0$$

$$x^2 - 6x + 8 = (x-4)(x-2) \longrightarrow 4, 2$$

$$x^2 + x - 6 = (x+3)(x-2) \longrightarrow -3, 2$$

6.1 (1) $\sqrt{x+1} - 1 = \sqrt{x - \sqrt{x+8}} \quad / ()^2$

$$(x+1) - 2\sqrt{x+1} + 1 = x - \sqrt{x+8}$$

$$2 - 2\sqrt{x+1} = -\sqrt{x+8} \quad / ()^2$$

$$4 - 8\sqrt{x+1} + 4(x+1) = x + 8$$

$$3x = 8\sqrt{x+1} \quad / ()^2$$

$$9x^2 = 64(x+1)$$

$$9x^2 - 64x - 64 = 0$$

$$D = 64^2 + 4 \cdot 9 \cdot 64$$
$$= 64(64 + 4 \cdot 9)$$
$$= 64 \cdot 4 (16 + 9)$$
$$= 64 \cdot 4 \cdot 25$$

$$x_{1,2} = \frac{64 \pm 8 \cdot 2 \cdot 5}{18} = \frac{64 \pm 80}{18}$$

$$= \begin{cases} \frac{144}{18} = \frac{16}{2} = 8 \\ -\frac{24}{18} = -\frac{16}{18} = \cancel{-\frac{8}{9}} \end{cases}$$

ova bendi doći u nuj sledeć

$$\sqrt{x+1}-1=\sqrt{x-\sqrt{x+8}}$$

$$x=8 \leadsto \sqrt{9}-1=\sqrt{8-\sqrt{16}}$$

$$2=\sqrt{4} \quad \text{OK.}$$

$\underline{x=8}$ jediné riešenie

2. $\sqrt{3x+4}+\sqrt{x-4} = 2\sqrt{x} \quad / (\,)^2$

$(3x+4) + 2\sqrt{(3x+4)(x-4)} + (x-4) = 4x$

$2\sqrt{(3x+4)(x-4)} = 0$ $\quad \cancel{-\frac{4}{3}}$ $\quad 4$

$x=4 \rightarrow \sqrt{16} + 0 = 2\sqrt{4}$

$4 \qquad 4 = 4$

jediné řešení

6.2 (1) $3 > x + 3\sqrt{1-x^2}$

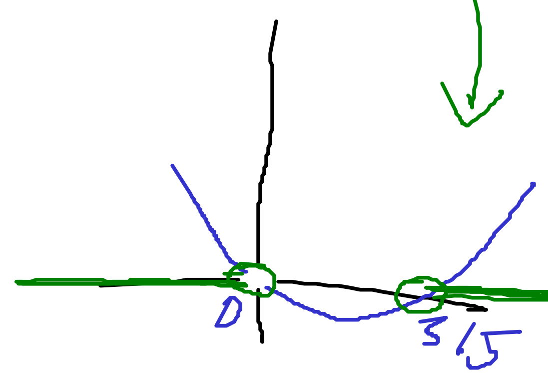
$\rightarrow \underbrace{3-x}_{\geq 0} > 3\underbrace{\sqrt{1-x^2}}_{\geq 0} \quad / (\,)^2 \quad \rightarrow x \in [-1, 1]$

$9 - 6x + x^2 > 9(1-x^2)$

$10x^2 - 6x > 0$

$5x^2 - 3x > 0$

$5x\left(x - \frac{3}{5}\right) > 0$



$= 0$ ряд $\quad x_1 = 0$

$x_2 = \frac{3}{5}$

Zaměm :

$x \in [-1, 0) \cup (\frac{3}{5}, 1]$

② $\sqrt{x+3} - \sqrt{x-1} > \sqrt{2x-1}$ $\quad x \geq 1$

$\hookrightarrow x \geq -3$ $\quad x \geq 1$ $\quad x \geq \frac{1}{2}$

$\sqrt{x+3} > \sqrt{x-1} + \sqrt{2x-1} \quad /()^2$

$x+3 > (x-1) + 2\sqrt{(x-1)(2x-1)} + (2x-1)$

$\underbrace{-2x+5}_{\geq 0} > \underbrace{2\sqrt{(x-1)(2x-1)}}_{\geq 0} \quad /()^2$

$x \leq \frac{5}{2}$ $\quad x \in [1, \frac{5}{2}]$

$4x^2 - 20x + 25 > 4(2x^2 - x - 2x + 1)$

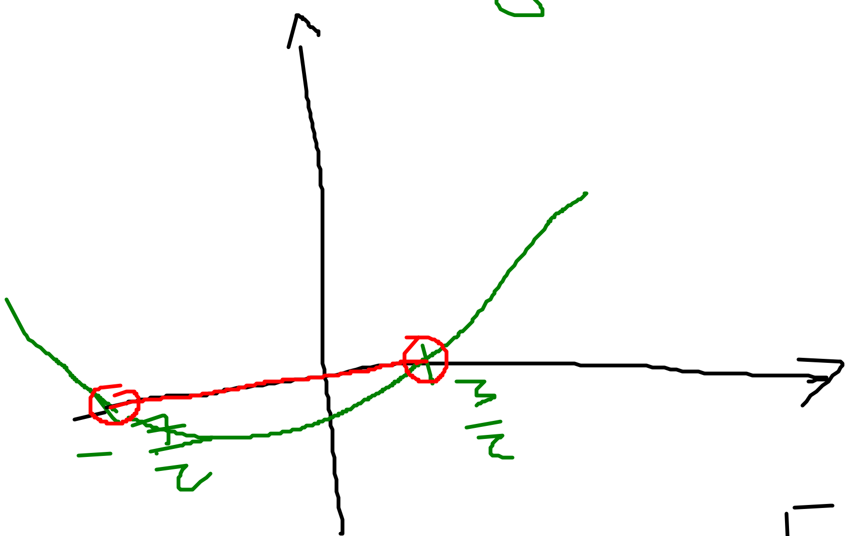
$0 > \underbrace{4x^2 + 8x - 21}_{=0}$

$D = 64 + 4 \cdot 4 \cdot 21 =$

$= 16(4+21)$

$= 16 \cdot 25$

$$x_{1,2} = \frac{-8 \pm 4\cdot 5}{8} = -1 \pm \frac{5}{2} = \begin{cases} \frac{3}{2} \\ -\frac{7}{2} \end{cases}$$



$x \in [1; \frac{5}{2}]$

Závěr: $x \in [1, \frac{5}{2})$

Vietovy vzorce

$ax^2 + bx + c = 0$ s kořeny x_1, x_2

$$x_1 + x_2 = -\frac{b}{a}$$

$$x_1 x_2 = \frac{c}{a}$$

Dk: x_1, x_2 kořeny

$$ax^2 + bx + c = a(x - x_1)(x - x_2) =$$

$$= a\left[x^2 - (x_1 + x_2)x + x_1 x_2\right]$$

$$= ax^2 - a(x_1 + x_2)x + a x_1 x_2$$

$b = -a(x_1 + x_2)$

$c = a x_1 x_2$

6.3 $3x^2+8x+4=0$ má kořeny x_1, x_2

Určete čísla

(1) $x_1^2+x_2^2=(x_1+x_2)^2-2x_1x_2=$

$$=\left(-\frac{8}{3}\right)^2-2\cdot\frac{4}{3}=$$

$$=\frac{64}{9}-\frac{8}{3}=\frac{64-24}{9}=\frac{40}{9}$$

(2) $x_1^3+x_2^3=(x_1+x_2)^3-3x_1x_2^2-3x_1^2x_2$

$$=(x_1+x_2)^3-3x_1x_2(x_1+x_2)$$

$$(x_1+x_2)^3=x_1^3+3x_1x_2^2+3x_1^2x_2+x_2^3$$

$$=\left(-\frac{8}{3}\right)^3-3\frac{4}{3}\left(-\frac{8}{3}\right)=$$

$$=-\frac{8}{3}\left[\left(\frac{8}{3}\right)^2-4\right]=-\frac{8}{3}\,\frac{64-36}{9}=$$

$$=-\frac{8}{3}\cdot\frac{28}{9}$$

(3) $\frac{1}{x_1}+\frac{1}{x_2}=\frac{x_2+x_1}{x_1 x_2}=\frac{-\frac{8}{3}}{\frac{4}{3}}=-2$

(4) $x_1-x_2=\sqrt{(x_1+x_2)^2-4x_1x_2}=$

$$(x_1-x_2)^2=(x_1+x_2)^2-4x_1x_2$$

$=\sqrt{\left(-\frac{8}{3}\right)^2-4\cdot\frac{4}{3}}=\sqrt{\frac{64}{9}-\frac{16}{3}}$

$=4\sqrt{\frac{4}{9}-\frac{3}{9}}=4\sqrt{\frac{1}{9}}=\frac{4}{3}$

Pořádně: $x_1-x_2=\pm\frac{4}{3}$

(5) $x_1^2x_2+x_1x_2^2=x_1x_2(x_1+x_2)$

$=\frac{4}{3}\left(-\frac{8}{3}\right)=-\frac{32}{9}$

(6) $x_1^2-x_2^2=(x_1+x_2)(x_1-x_2)$

$$= \left(-\frac{8}{3}\right)\left(+\frac{4}{3}\right) = +\frac{32}{9}$$

Vietovy vzťahy pre

$$ax^3 + bx^2 + cx + d = 0$$

x_1, x_2, x_3 korene

$$a(x-x_1)(x-x_2)(x-x_3) = 0$$

$$a\left(x^2 - (x_1+x_2)x + x_1x_2\right)(x-x_3) = 0$$

$$a\left[x^3 - (x_1+x_2+x_3)x^2 + (x_1x_2 + x_1x_3 + x_2x_3)x - x_1x_2x_3\right] = 0$$

$$x_1 + x_2 + x_3 = -\frac{b}{a}$$

$$x_1x_2 + x_1x_3 + x_2x_3 = \frac{c}{a}$$

$$x_1x_2x_3 = -\frac{d}{a}$$