**7.1** (1): $a>0$, $n\in\mathbb{Z}$

$n>0$: $a^n = \underbrace{a\cdot\ldots\cdot a}_{n}$

$n=0$: $1$

$n<0$: $a^n = \dfrac{1}{a^{-n}} = \left(\dfrac{1}{a}\right)^{\underset{>0}{-n}}$

(2) $a>1,\ a\in\mathbb{R},\ n<m$,

pak $\underline{a^n<a^m}$, $n,m\in\mathbb{Z}$

- $0\le n<m \Rightarrow$

$$a^m = a^n\cdot\underbrace{a^{m-n}}_{>1} > a^n$$

- $n<m\le 0$

$$a^m = \left(\frac{1}{a}\right)^{\underset{>0}{-m}} = \left(\frac{1}{a}\right)^{-m}\cdot\underbrace{\left(\frac{1}{a}\right)^{\underset{\in\mathbb{Z}_+}{-n+m}}}_{\in[0,1)} < \left(\frac{1}{a}\right)^{-n}$$

$$\begin{array}{ccc} \| & & \| \\ a^m & < & a^n \end{array}$$

- $n<0\le m$ - - -

(3) $a\in\mathbb{R}_+$, $x=\frac{p}{q}$, $p\in\mathbb{Z}$, $q\in\mathbb{N}$

Položimo $a^x = a^{\frac{p}{q}} = \underbrace{\sqrt[q]{a^p}}$

za značenje pojma
q-tá odmocnina

Je treba ukázať, že $x = \frac{p}{q} = \frac{kp}{kq}$, nezávisí na $k \in N$ ....

$$a^{\frac{kp}{kq}} = \sqrt[kq]{a^{kp}} = \sqrt[q]{\sqrt[k]{(a^p)^k}} = \sqrt[q]{a^p} = a^{\frac{p}{q}}$$

pro celočíselné kladné exponenty platí

- $(a^p)^k = a^{kp}$
- $\sqrt[kp]{b} = \sqrt[p]{\sqrt[k]{b}}$

$a, b \in \mathbb{R}_+$

4. $a > 0$, $x, y \in Q$

pak $\underline{a^{x+y} = a^x \cdot a^y}$ a $\underline{a^{xy} = (a^x)^y}$

- $x, y \in Z$ — kladné / kladné i záporné
- $x = \frac{p_1}{q}$, $y = \frac{p_2}{q}$

$$a^{\frac{p_1+p_2}{q}} = \sqrt[q]{a^{p_1+p_2}} = \sqrt[q]{a^{p_1} \cdot a^{p_2}} = \sqrt[q]{a^{p_1}} \cdot \sqrt[q]{a^{p_2}} = a^{\frac{p_1}{q}} \cdot a^{\frac{p_2}{q}}$$

měli bychom to zmyslet pořádně

- umocňování podobně

(5) $a \in \mathbb{R}_+$, $x \in \mathbb{R}$

$a^x := \sup\{a^y \mid y \in \mathbb{Q},\ y \leq x\}$

- $M \subseteq \mathbb{R}$, $c$ je horní závora $M$, jestliže $c \geq z$ $\forall z \in M$
- $\sup M := \min\{c \in \mathbb{R} \mid c \text{ horní závora } M\}$
- Př: $\sup (0,5] = 5$

  $\sup (0,5) = 5$

7.2: (1) Uvažme funkci $f(x)$, $D(f)$, $H(f)$ a předp., že $f$ je injektivní

Pak definuje funkci $g(x)$ t. ž. $g(x) = y \iff f(y) = x$

(zadáno jednoznačně pro injektivní $f(x)$)

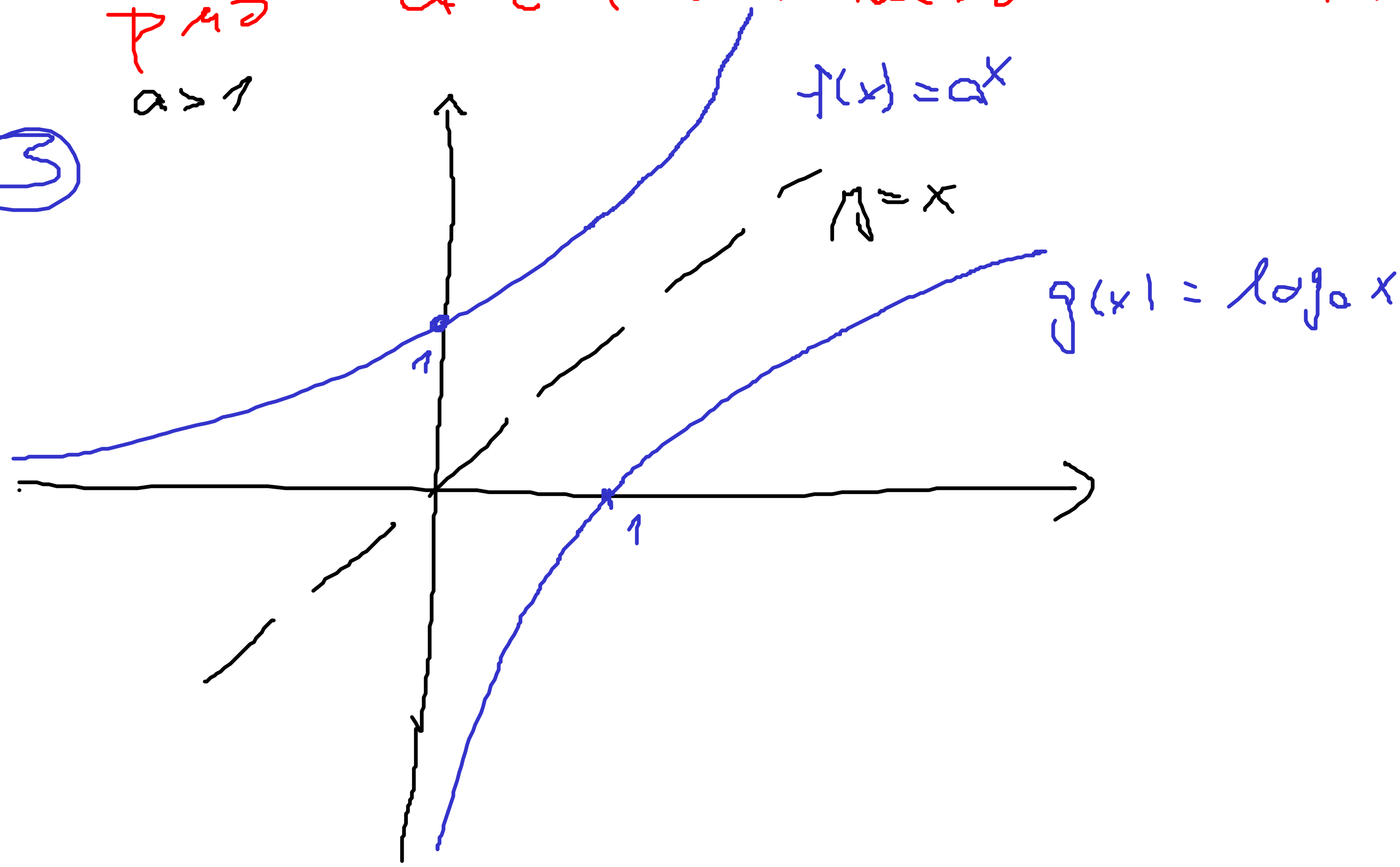
$D(g) = H(f)$

$H(g) = D(f)$

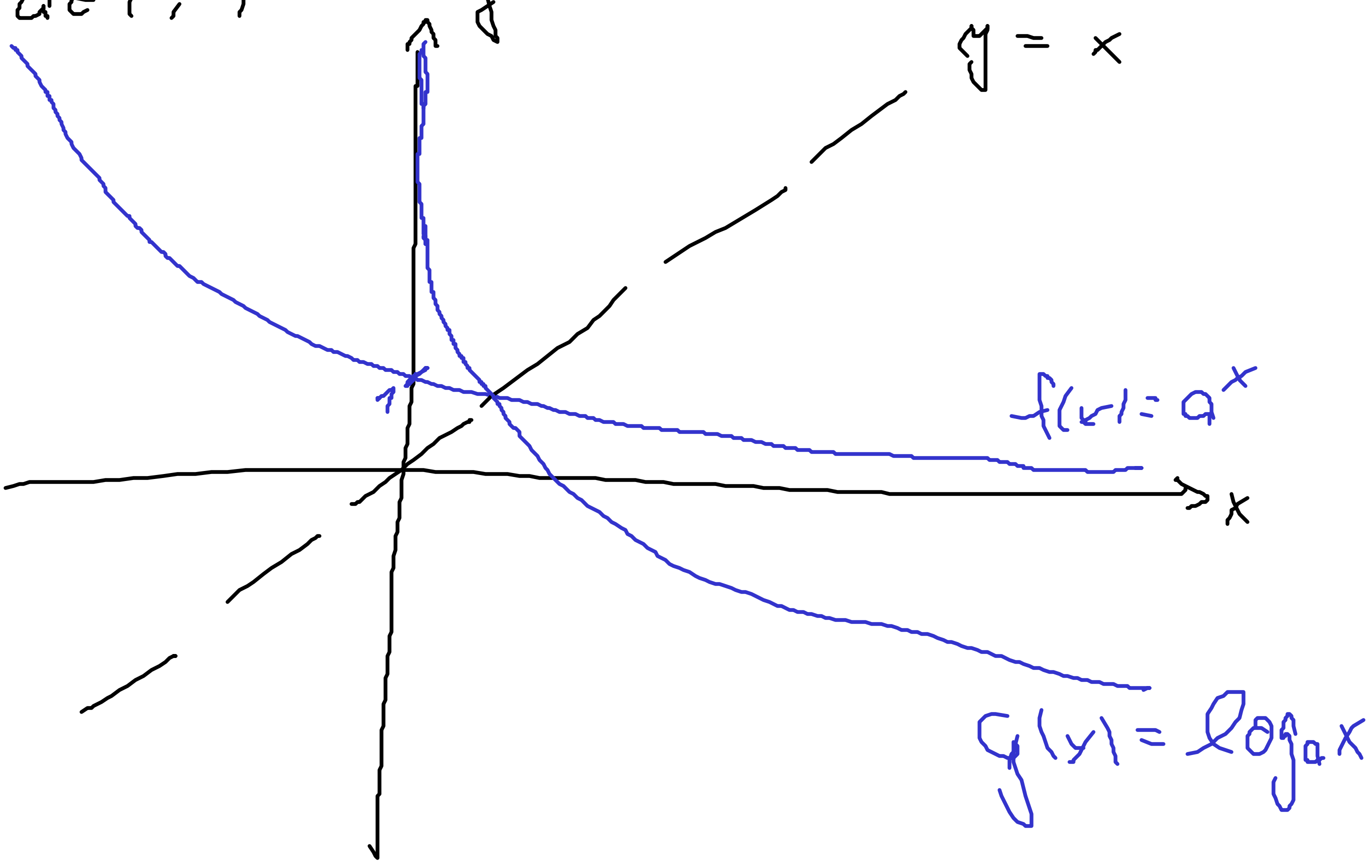
② Definujeme $g(x) = \log_a x$ jako inverzní funkci k $a^x = f(x)$ pro $a \in (1, \infty)$ nebo $a \in (0,1)$

$a > 1$

③




$a \in (0,1)$



7.3 (1) $\underline{\log_a xy = \log_a x + \log_a y}$

$a > 0$

$x, y > 0$

• $a^{x_1+y_1} = a^{x_1} \cdot a^{y_1} \quad / \log_a$

$x_1 + y_1 = \log_a \underbrace{(a^{x_1})}_{x} \cdot \underbrace{(a^{y_1})}_{y}$

$x = a^{x_1} / \log_a$

$\log_a x = x$

$\log_a x + \log_a y = \log_a xy$

(3) $\log_a x^y = y \cdot \log_a x$

• $(a^{x_1})^{y_1} = a^{x_1 y_1} \quad / \log_a$

$\log_a (a^{x_1})^{y_1} = x_1 y_1$

$\log_a x^{y_1} = y_1 \cdot \log_a x$

$x := a^{x_1}$

$\log_a x = x_1$

$y := y_1$

(4) $\log_a x = \dfrac{\log_b x}{\log_b a}$

• $a^{x_1} = b^{k x_1} \quad / \log_b$

$\log_b \underbrace{a^{x_1}}_{x} = k x_1$

$a = b^k$

$\log_b a = k$

$\overline{|a^{x_1} = x}, \; x_1 = \log_a x$

$$\log_b x = \log_b a \cdot \log_a x$$

$$\frac{\log_b x}{\log_b a} = \log_a x$$

7.4: (1) $$49^{1-\frac{1}{2}\log_7 25} =$$

$$= 49^{1-\log_7 (25)^{1/2}} = \frac{49}{49^{\log_7 5}} =$$

$$= \frac{49}{5^{\log_7 49}} = \frac{49}{5^2} = \frac{49}{25}$$

(2) $$\log\Big(\log \underbrace{\sqrt{\underbrace{\sqrt[5]{10}}_{10^{1/5}}}}_{10^{1/10}}\Big) = \log\left(\frac{1}{10}\log 10\right) =$$

$$= \log(10^{-1})$$

$$= -\log 10 = -1$$

(3) $$81^{\frac{1}{\log_5 3}} = 81^{\log_3 5} =$$

$$= 5^{\log_3 81} = 5^4 = 25^2$$

8.1: Pomocí a, b, c vyjádřete

(1) $x = \log_{100} 40$, $a = \log_2 5 \rightsquigarrow \frac{1}{a} = \log_5 2$

$$x = \log_{100}(5 \cdot 8) = \log_{100} 5 + \log_{100} 2^3 =$$

$$= \log_{100} 5 + 3 \cdot \log_{100} 2 =$$

$$= \frac{1}{\log_5 10^2} + 3 \frac{1}{\log_2 10^2} =$$

$$= \frac{1}{2(\log_5 2 + \log_5 5)} + 3 \frac{1}{2(\log_2 2 + \log_2 5)}$$

$$= \frac{1}{2(\frac{1}{a} + 1)} + \frac{3}{2(1+a)} =$$

$$= \frac{a}{2(a+1)} + \frac{3}{2(a+1)} = \frac{a+3}{2(a+1)}$$

(2) $x = \log_6 16$, $a = \log_{12} 27$

| | |
|---|---|
| $x = 4 \log_6 2$ | $a = 3 \log_{12} 3$ |
| $\frac{1}{x} = \frac{1}{4} \log_2 6$ | $\frac{1}{a} = \frac{1}{3} \log_3 12$ |
| $\frac{1}{x} = \frac{1}{4}(\log_2 2 + \log_2 3)$ | $\frac{1}{a} = \frac{1}{3}(\log_3 3 + \log_3 4)$ |

$$\frac{1}{x} = \frac{1}{4}(1+\log_2 3) \quad \Big| \quad \frac{1}{a} = \frac{1}{3}(1+2\log_3 2)$$

$$\hookrightarrow \frac{3}{a} - 1 = 2\log_3 2$$

$$\log_3 2 = \frac{1}{2}\,\frac{3-a}{a}$$

$$\Downarrow$$

$$\frac{1}{x} = \frac{1}{4}\left(1+\frac{2a}{3-a}\right)$$

$$= \frac{(3-a)+2a}{4(3-a)} = \frac{a+3}{4(3-a)}$$

$$\boxed{x = \frac{4(3-a)}{a+3}}$$

## 8.2. Řešte rovnici:

① $4^x + 2^{x+1} = 24$

$$4^x = (2^2)^x = 2^{2x} = (2^x)^2$$

• $(2^x)^2 + 2\cdot 2^x = 24$

$$y = \underbrace{2^x}_{>0} \rightsquigarrow y^2 + 2y - 24 = 0$$

$$(y+6)(y-4) = 0$$

$$\cancel{y = -6} \qquad y = 4$$

$$x = 2 \Leftarrow 2^x = 4$$

(2) $|x|^{x^2-2x} = 1$

- $x^2 - 2x = 0 \rightsquigarrow x \in \{\cancel{0}, 2\}$
- $|x| = 1 \rightsquigarrow x \in \{\pm 1\}$

Závěr $x \in \{\pm 1, 2\}$

(3) $6 \cdot 9^x - 13 \cdot 6^x + 6 \cdot 4^x = 0 \quad / \frac{1}{6^x}$

$$6 \cdot \left(\frac{3}{2}\right)^x - 13 + 6 \cdot \left(\frac{2}{3}\right)^x = 0$$

$a := \left(\frac{3}{2}\right)^x : \quad 6a - 13 + \frac{6}{a} = 0 \quad / \cdot a$

$$6a^2 - 13a + 6 = 0 \quad \ldots$$

(4) $\left(\frac{3}{5}\right)^x + \frac{7}{5} = 2^x$

- $x = 1$ je řešení: $\frac{3}{5} + \frac{7}{5} = \frac{10}{5} = 2$
- Jiná řešení neexistují

nebot' LS je klesající

PS rostoucí

8.3 (1)

$$\log 5 + \log(x+10) = \underbrace{1}_{\log 10} - \log(2x-1) + \log(21x-20)$$

$$\log\big(5(x+10)\big) = \log\left(\frac{10(21x-20)}{2x-1}\right)$$

$$5(x+10) = \frac{10(21x-20)}{2x-1}$$

$$(x+10)(2x-1) = 2(21x-20)$$

$$2x^2 + (20-1)x - 10 = 42x - 40$$

$$2x^2 - 23x + 30 = 0$$

$$x_{1,2} = \frac{23 \pm \sqrt{23^2 - 8\cdot 30}}{4} = \frac{23 \pm \sqrt{529-240}}{4} = \frac{23 \pm \sqrt{289}}{4} = \frac{23 \pm 17}{4} = \begin{cases} \frac{6}{4} = \frac{3}{2} \\ \frac{40}{4} = 10 \end{cases}$$

$$(20+3)^2 = 400 + 120 + 9$$

Záver: $x \in \left\{\frac{3}{2}, 10\right\}$