8.4 (1) $\dfrac{1}{3^x+5} \leq \dfrac{1}{3^{x+1}-1} \quad / \ldots$

- $3^x+5 > 0 \quad \forall x \in \mathbb{R}$
- $3^{x+1} > 1 \Rightarrow x+1 > 0$

$x > -1$

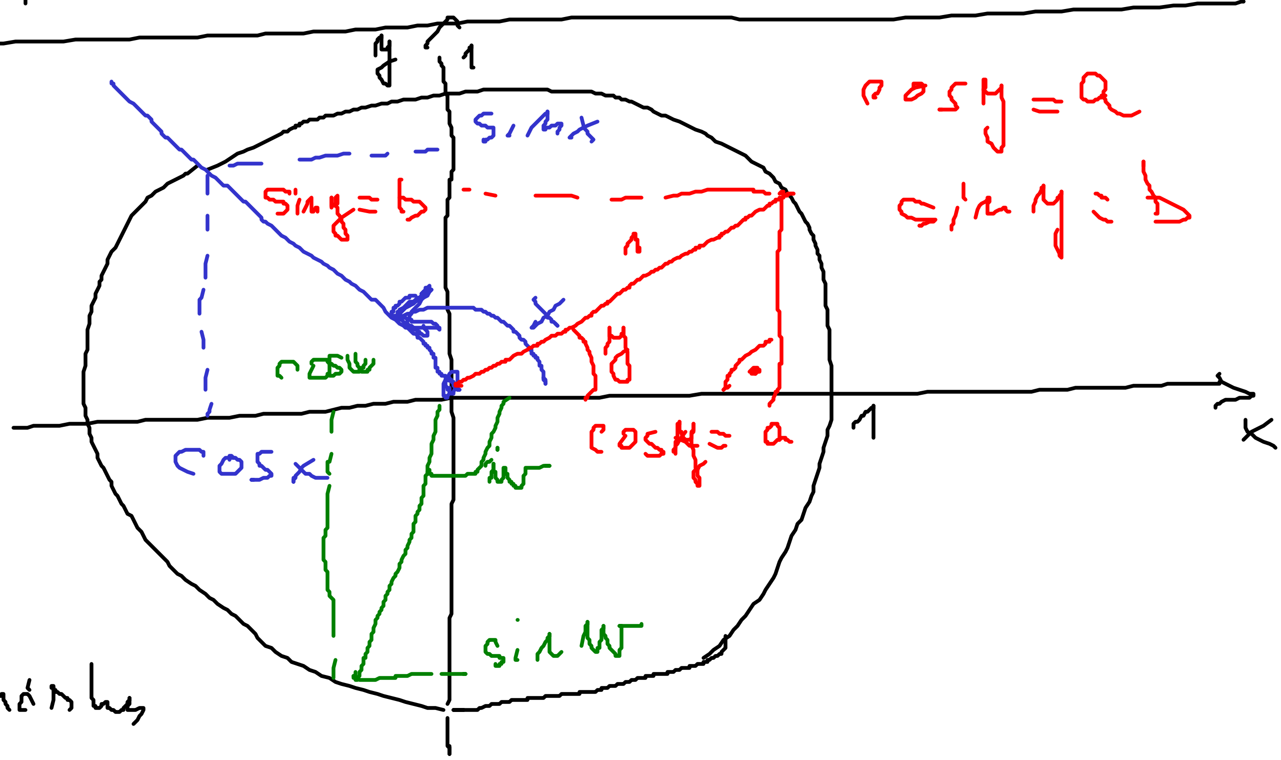
$3^{x+1} - 1 \leq 3^x + 5$

$3^x(3-1) \leq 6$

$3^x \leq 3 \Rightarrow x \leq 1$

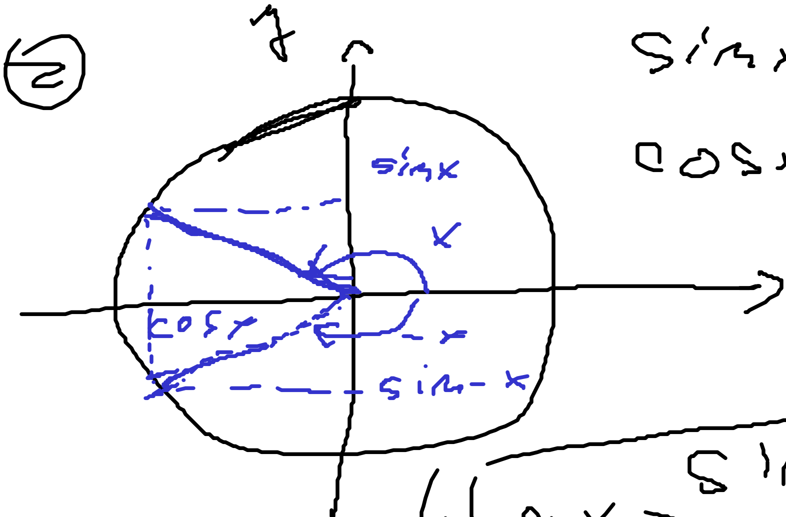
Závěr. $x \in (-1, 1]$

9.1



(1) z obrázku

(2)



$\sin x = -\sin(-x)$ z obrazku

$\cos x = \cos(-x)$

$$tg(-x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin x}{\cos x} = -tg x$$

$$\boxed{tg x = \frac{\sin x}{\cos x}}$$

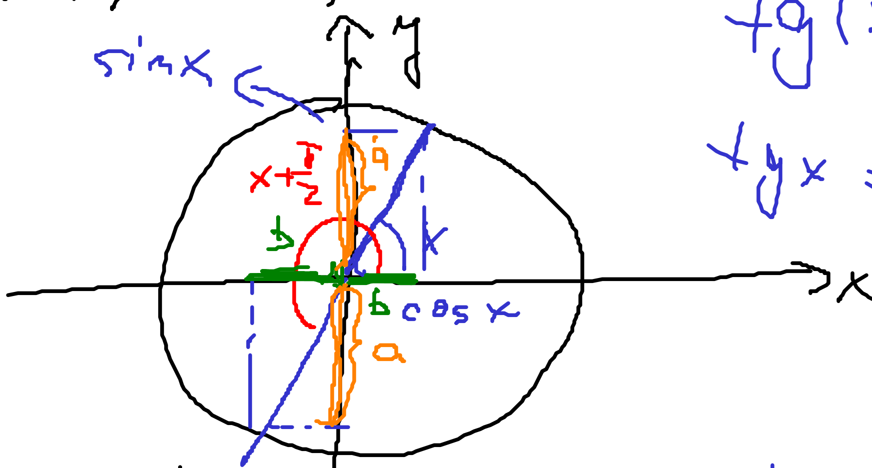
$x \neq \frac{\pi}{2} + k\pi,\ k \in Z$

(3) $\sin(x+2\pi) = \sin \pi$

$\cos(x+2\pi) = \cos \pi$

$tg(x+\pi) = \frac{a}{b}$

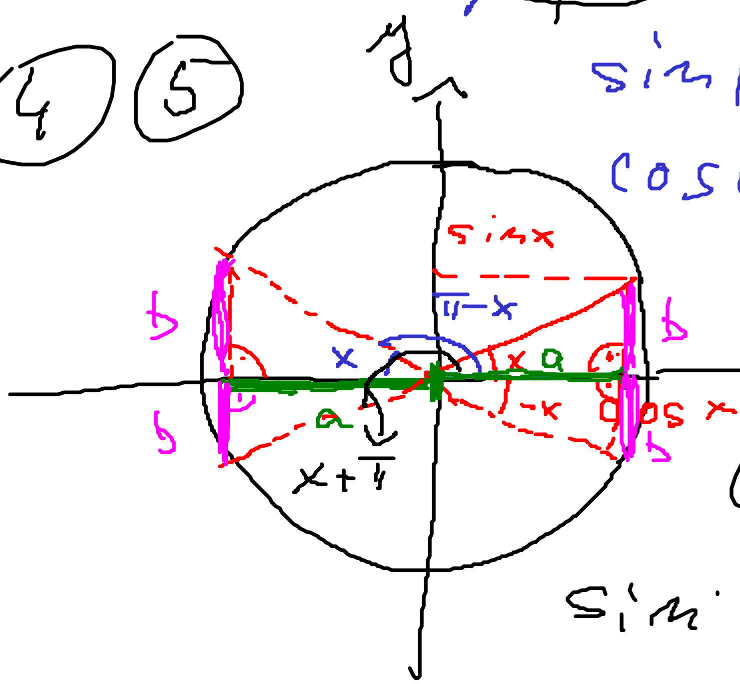
$tg x = \frac{a}{b}$



(4) (5)

$\sin(\pi - x) = b = \sin x = -\sin(x+\pi)$

$\cos(\pi - x) = -a = -\cos x =$
$= -\cos(x+\pi)$

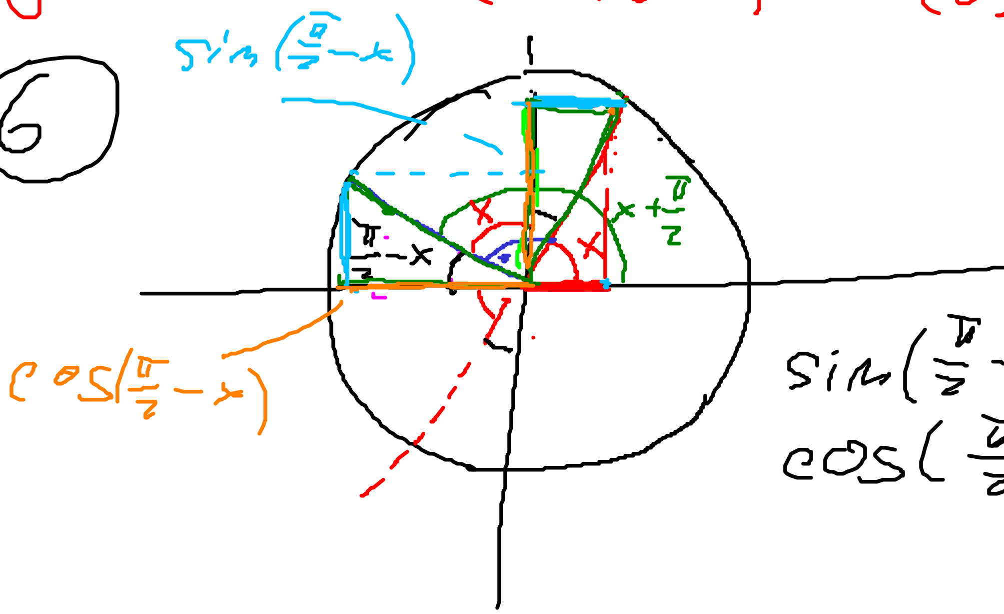


$\cos(2\pi - x) = \cos(-x) = \cos x$

$\sin(2\pi - x) = \sin(-x) = -\sin x$

$$\operatorname{tg}(\pi - x) = \frac{\sin(\pi - x)}{\cos(\pi - x)} = \frac{\sin x}{-\cos x} = -\operatorname{tg} x$$

6



$$\alpha = \pi - \left(\frac{\pi}{2} + x\right) = \frac{\pi}{2} - x$$

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

9.2 $\quad e^{ix} = \cos x + i \sin x \leftarrow$

$$\boxed{e^{z_1} \cdot e^{z_2} = e^{z_1 + z_2}, \quad \forall z_i \in \mathbb{C}}$$

$$e^{iy} = \cos y + i \sin y \leftarrow$$

vynásobit

$$e^{ix} \cdot e^{iy} = \underline{(\cos x \cos y - \sin x \sin y)} + i\,\underline{(\sin x \cos y + \cos x \sin y)}$$

$$\|$$

$$e^{i(x+y)} = \underline{\cos(x+y)} + i\,\underline{\sin(x+y)}$$

9.3 ① $\sin 2x = \sin(x+x) =$

$= \sin x \cos x + \cos x \sin x$

$= 2\sin x \cos x$

② $\sin x + \sin y = \ldots$

$$\left.\begin{array}{l} x = \alpha + \beta \\ y = \alpha - \beta \end{array}\right\} \left.\begin{array}{l} x+y = 2\alpha \\ x-y = 2\beta \end{array}\right\} \begin{array}{l} \alpha = \frac{x+y}{2} \\ \beta = \frac{x-y}{2} \end{array}$$

$\sin x = \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin y = \sin(\alpha-\beta) = \sin\alpha\cos(-\beta) + \cos\alpha\sin(-\beta)$

$\alpha + (-\beta) \quad = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$$\boxed{\begin{array}{l} \sin x + \sin y = 2\sin\alpha\cos\beta = \\ = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2} \end{array}}$$

$$\boxed{\begin{array}{l} \sin x - \sin y = 2\cos\alpha\sin\beta \\ = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2} \end{array}}$$

9.4 ① $\operatorname{tg}(x+y) =$

$$= \frac{\sin(x+y)}{\cos(x+y)} = \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y} \cdot \frac{\frac{1}{\cos x \cos y}}{\frac{1}{\cos x \cos y}}$$

$$= \frac{\operatorname{tg} x + \operatorname{tg} y}{1 - \operatorname{tg} x \operatorname{tg} y}$$

$x, y \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$

$x + y \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$

② - -

③ $\operatorname{tg} x \cdot \operatorname{tg}\left(\frac{\pi}{2} + x\right) = -1$

$$\cdot \operatorname{tg}\left(\frac{\pi}{2} + x\right) = \frac{\sin\left(\frac{\pi}{2} + x\right)}{\cos\left(\frac{\pi}{2} + x\right)} = \frac{\cos x}{-\sin x} = -\operatorname{cotg} x = -\frac{1}{\operatorname{tg} x}$$

NB: $\cos\left(x + \frac{\pi}{2}\right) = -\sin x$

· jiná úvaha – součtový vzorec pro tg:

$$\operatorname{tg}\frac{\pi}{2} = \frac{\sin\frac{\pi}{2}}{\cos\frac{\pi}{2}} = \frac{1}{0} = \infty$$

$$\operatorname{tg}\left(x + \frac{\pi}{2}\right) = \frac{\operatorname{tg} x + \operatorname{tg}\frac{\pi}{2}}{1 - \operatorname{tg} x \operatorname{tg}\frac{\pi}{2}} =$$

$$= \frac{\underbrace{\frac{\operatorname{tg} x}{\operatorname{tg}\frac{\pi}{2}}}_{\to 0} + 1}{\underbrace{\frac{1}{\operatorname{tg}\frac{\pi}{2}}}_{\to 0} - \operatorname{tg} x} = \frac{1}{-\operatorname{tg} x}$$

9.5 (1) $\sin x = \dfrac{2\,tg\frac{x}{2}}{1+tg^2\frac{x}{2}}$

$\frac{x}{2} \neq \frac{\pi}{4} + k\pi$

$x \neq \frac{\pi}{2} + 2k\pi$

$$\frac{2\,tg\frac{x}{2}}{1+tg^2\frac{x}{2}} = \frac{2\,\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{1+\left(\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\right)^2} \begin{array}{l} /\cos^2\frac{x}{2} \\ /\cos^2\frac{x}{2} \end{array} =$$

$$= \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}} = \sin\left(\frac{x}{2}+\frac{x}{2}\right) = \sin x$$

(3) $tg\, x = \dfrac{2\,tg\frac{x}{2}}{1-tg^2\frac{x}{2}}$

$\Downarrow$

$$tg\left(\frac{x}{2}+\frac{x}{2}\right) = \frac{tg\left(\frac{x}{2}\right)+tg\left(\frac{x}{2}\right)}{1-tg^2\left(\frac{x}{2}\right)}$$

9.6 (1) $\dfrac{\sin x+\cos x}{\cos^3 x} = 1+tg\,x+tg\,x^2+tg\,x^3$

$$1+tg\,x+tg^2x+tg^3x =$$

$$= \frac{\cos^3 x+\cos^2 x\sin x+\cos x\sin^2 x+\sin^3 x}{\cos^3 x}$$

$$= \frac{\cos x+\sin x}{\cos^3 x}$$

② $\dfrac{1+\sin 2x}{\cos 2x} = tg\left(\frac{\pi}{4} + x\right)$ $\qquad tg\frac{\pi}{4} = 1$

$$tg\left(x+\frac{\pi}{4}\right) = \frac{tg\,x + tg\frac{\pi}{4}}{1 - tg\,x\, tg\frac{\pi}{4}} = \frac{1 + \frac{\sin x}{\cos x}}{1 - \frac{\sin x}{\cos x}} \Big/ \cos x$$

$$= \frac{\cos x + \sin x}{\cos x - \sin x} \cdot \frac{\cos x + \sin x}{\cos x + \sin} =$$

$$= \frac{\sin^2 x + \cos^2 x + 2\sin x\cos x}{\cos^2 x - \sin^2 x} = \frac{1+\sin 2x}{\cos 2x}$$

③ $tg(3x) = tg(x+2x) =$

$$= \frac{tg\,x + tg\,2x}{1 - tg\,x\, tg\,2x} = \frac{tg\,x + \frac{2\,tg\,x}{(1-tg^2 x)}}{1 - tg\,x\,\frac{2\,tg\,x}{(1-tg^2x)}}$$

$$= \frac{tg\,x\,(1-tg^2 x) + 2\,tg\,x}{(1-tg^2x) - 2\,tg^2 x} =$$

$$\frac{tg\,3x}{tg\,x} = \frac{3 - tg^2 x}{1 - 3\,tg^2 x}$$

9.7 (1) $\cos 15^\circ$

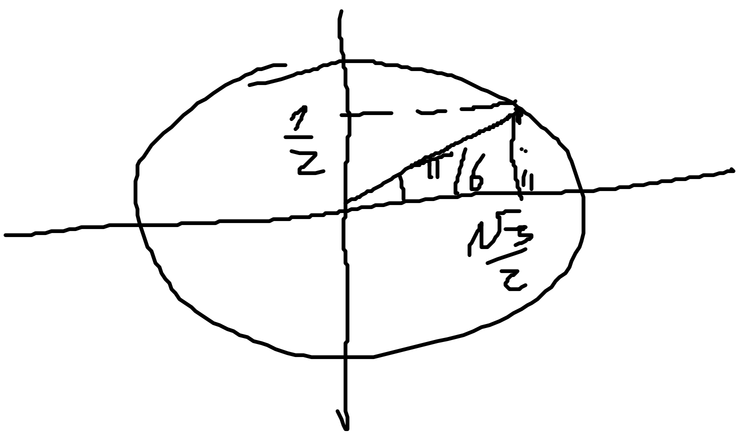
0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
0	30°	45°	60°	90°

$$15^\circ = \frac{\pi}{12}$$

$$\cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \underbrace{\cos\frac{\pi}{4}}_{\frac{\sqrt{2}}{2}}\ \underbrace{\cos\frac{\pi}{6}}_{\frac{\sqrt{3}}{2}} - \underbrace{\sin\frac{\pi}{4}}_{\frac{\sqrt{2}}{2}}\ \sin\left(-\frac{\pi}{6}\right)$$

$$\frac{\pi}{4} + \left(-\frac{\pi}{6}\right)$$

$$\sin\frac{\pi}{6} = \frac{1}{2}$$



$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\,\frac{1}{2}$$

$$= \frac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$$

(2) $\mathrm{tg}\, 75^\circ = \mathrm{tg}\,(30^\circ + 45^\circ)$

$$= \mathrm{tg}\left(\frac{\pi}{6} + \frac{\pi}{4}\right) = \frac{\mathrm{tg}\left(\frac{\pi}{6}\right) + 1}{1 - \mathrm{tg}\frac{\pi}{6}\cdot 1} =$$

$$= \frac{\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} + 1}{1 - \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}} = \frac{1 + \frac{\sqrt{3}}{2}}{1 - \frac{\sqrt{3}}{2}} =$$

$$= \frac{2 + \sqrt{3}}{2 - \sqrt{3}}$$