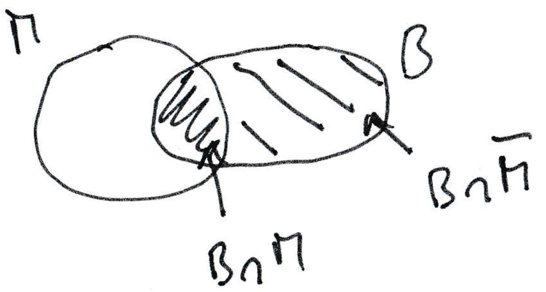
$(\Omega, \mathcal{A}, P) \xrightarrow[X,\ F]{} (\mathbb{R}, \mathcal{B}, \mu_F)$ $\qquad F(x) = P(X \leq x)$

D.V.4. 1) $p(x) = P(X = x) \geq 0$

$$\sum_{x \in M} p(x) = \sum_{x \in M} P(X = x) = P\left(\bigcup_{x \in M} \{X = x\}\right) = P(X \in M) = 1$$

2) $B \in \mathcal{B}$ $\quad P(X \in B) = P(\{X \in M \cap B\} \cup \{X \in \bar{M} \cap B\}) =$



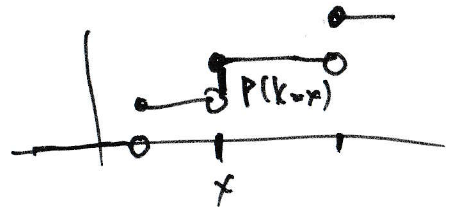
$$= \underline{P(X \in M \cap B)} + \underbrace{P(X \in \bar{M} \cap B)}_{0}$$

$$= P\left(\bigcup_{x \in M \cap B} \{X = x\}\right) = \sum_{x \in M \cap B} P(X = x) = \sum_{x \in M \cap B} p(x)$$

3) $F(x) = P(X \leq x) = \underline{P(X \in B_x)}$, kde $B_x = (-\infty; x\rangle$

$$= \sum_{t \in M;\ t \leq x} p(t)$$

4) $p(x) = P(X = x)$



$$\begin{array}{cccccc} 1 & 2 & 3 & & & n \\ \bullet & \bullet & \bullet & \cdots\cdots & & \bullet \\ \vee & \times & \vee & \vee & & \times \\ \theta & (1-\theta) & \theta & \theta & & (1-\theta) \end{array}$$

$$P(X=k) = \binom{n}{k} \theta^k (1-\theta)^{n-k}$$

X … počet bílých koulí z n vybraných

$n = N$

$n - (N-K)$ černých

$$\left.\begin{array}{l} x \geq 0 \\ x \geq n-(N-k) \end{array}\right.$$

$\Rightarrow$

$$x \geq \max\{0; n-N+k\}$$

$$\left.\begin{array}{l} x \leq n \\ x \leq K \end{array}\right\} \Rightarrow x \leq \min\{n, k\}$$

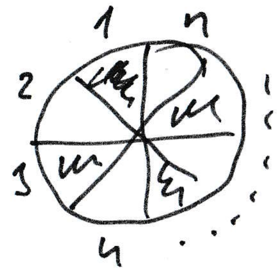
$$p(x) = P(X=x) = \frac{\binom{K}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$$

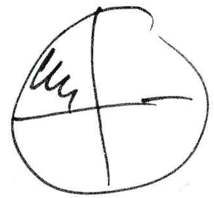
Poisson

λ .... ~~po~~ očekávaný počet událostí za 1 h

x .... počet událostí za 1 h

X_n .... počet událostí za n n-tin hodiny





$\lambda = 2$

$$X_n \sim Bi(n, \theta_n) \qquad \theta_n = \frac{\lambda}{n}$$

$$\theta_4 = \frac{2}{4}$$

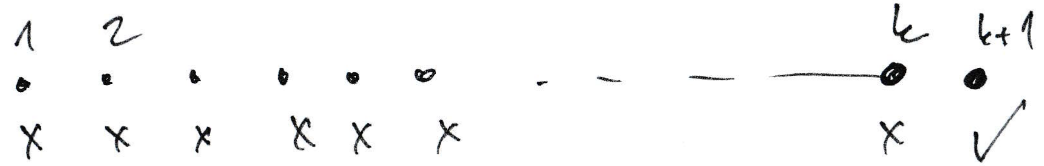
$$P(X_n = x) = \binom{n}{x}\left(\frac{\lambda}{n}\right)^x \left(1-\frac{\lambda}{n}\right)^{n-x} \xrightarrow[n\to\infty]{} ? \quad (P_0(\lambda))$$

$$\lim_{n\to\infty} P(X_n = x) = \lim_{n\to\infty} \frac{n!}{x!(n-x)!} \cdot \left(\frac{\lambda}{n}\right)^x \left(1-\frac{\lambda}{n}\right)^{n-x} = \lim_{n\to\infty} \frac{\lambda^x}{x!} \cdot \frac{n \cdot (n-1) \cdot \ldots \cdot (n-x+1)}{n \cdot n \cdot \ldots \cdot n} \cdot \left(1-\frac{\lambda}{n}\right)^{n-x}$$

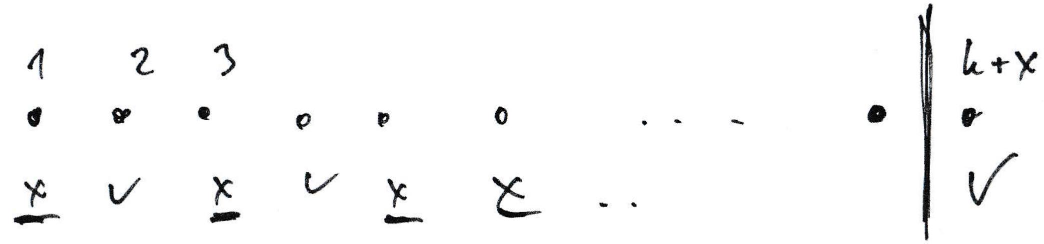
$$1 \quad \left(1-\frac{1}{n}\right) \cdots 1-\frac{x-1}{n}$$

$$= \frac{\lambda^x}{x!} \lim_{n\to\infty} 1 \cdot 1 \cdot \ldots \cdot 1 \cdot \left(1-\frac{\lambda}{n}\right)^n \cdot \left(1-\frac{\lambda}{n}\right)^{-x}$$

$$= \frac{\lambda^x}{x!} \lim_{n\to\infty} \left(1-\frac{\lambda}{n}\right)^n \cdot 1 = \underline{\underline{\frac{\lambda^x}{x!} e^{-\lambda}}}$$



$$p(k) = P(X=k) = (1-\theta)^k \cdot \theta$$



$$p(x) = P(X=x) = \binom{k+x-1}{k-1}(1-\theta)^x \theta^k$$