1) $\displaystyle\int_{-\infty}^{\infty} f(x)\,dx = P(X \in \mathbb{R}) = 1$

2) $\displaystyle F(x) = P(X \leq x) = P(X \in \underbrace{(-\infty, x\rangle}_{B}) = \int_B f(t)\,dt = \int_{-\infty}^{x} f(t)\,dt$

3) $\displaystyle F(x) = \int_{-\infty}^{x} f(t)\,dt$

4) $B \in \mathbb{B}$ $\displaystyle P(X \in B) = \int_B f(x)\,dx = \int_B g(x)\,dx \rightarrow \int_B \big(f(x) - g(x)\big)dx = 0$

$f(x), g(x)$

$\Downarrow$

$f(x) = g(x)$ s.v.

5) předch. V.

6) $\displaystyle P(a < X \leq b) = F(b) - F(a) = \int_a^b f(t)\,dt$

$\displaystyle P(X = x) = F(x) - \lim_{y \to x^-} F(y) = F(x) - F(x) = 0$

7) $\displaystyle f(x) = F'(x) = \lim_{h \to 0} \frac{F(x+\frac{h}{2}) - F(x-\frac{h}{2})}{h} = \lim_{h \to 0} \frac{P(x-\frac{h}{2} < X \leq x+\frac{h}{2})}{h} - \lim_{h \to 0} \frac{o(h)}{h}$

A .... náh. jev $\qquad$ X.... čas, než nastane jev A

$Q(t)$ .... pst, že A <u>nenastane</u> v průběhu čas. int. délky t

$\underline{Q(0) = 1}$, $Q(t)$ klesající

t_1, t_2 délky 2 navaz. intervalů

$$\overbrace{\quad}^{t_1}\overbrace{\quad}^{t_2}$$

$$Q(t_1+t_2) = Q(t_1)\cdot Q(t_2) \Leftrightarrow Q(t+\Delta t) = Q(t)\cdot Q(\Delta t)$$

$$\ln Q(t+\Delta t) = \ln Q(t) + \ln Q(\Delta t)$$

$$(\ln Q(t))' = \lim_{\Delta t \to 0^+} \frac{\ln Q(t+\Delta t) - \ln Q(t)}{\Delta t} = \lim_{\Delta t \to 0^+} \frac{\ln Q(\Delta t)}{\Delta t} = -\lambda, \quad \lambda > 0$$

nezávisí na t!

$$\frac{d \ln Q(t)}{dt} = -\lambda \;; \quad Q(0) = 1$$

$$\int d \ln Q(t) = -\lambda \int dt$$

$$\ln Q(t) = -\lambda t + C$$

$$Q(t) = e^{-\lambda t} \cdot e^C$$

$$\boxed{Q(t) = e^{-\lambda t}}$$

$$F(t) = P(X \leq t) = 1 - Q(t) = \underline{\underline{1 - e^{-\lambda t}}} \;; \quad t \geq 0$$

$$f(t) = F'(t) = \underline{\underline{\lambda e^{-\lambda t}}}$$