⑧ $X_1 \perp X_2$

$$f_{X_i}(x_i) = \begin{cases} 1 & x_i \in (0,1) \\ 0 & \text{inak} \end{cases} \qquad i=1,2$$

$$Y = X_1 + X_2 \qquad f_Y(y) = ?$$

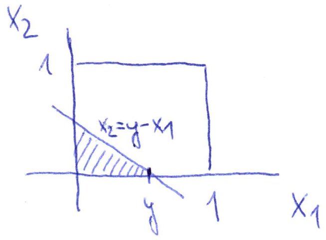
Rieš.: $F_Y(y) = P(Y \le y) = P(X_1 + X_2 \le y) = \int\limits_{h^{-1}(B_y)} f_{X_1,X_2}(x_1,x_2)\, dx_1\, dx_2$

$$f_{X_1,X_2}(x_1,x_2) \overset{\text{nez. } X_1 \text{ a } X_2}{=} f_{X_1}(x_1)\, f_{X_2}(x_2) = \begin{cases} 1 & x_1 \in (0,1) \wedge x_2 \in (0,1) \\ 0 & \text{inak} \end{cases}$$

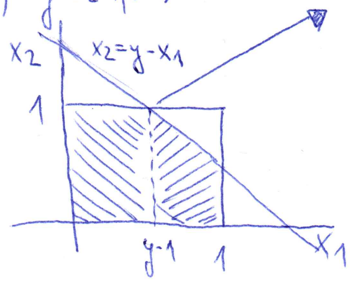
$Y = X_1 + X_2$, $x_1 \in (0,1)$, $x_2 \in (0,1)$ $\Rightarrow$ $y \in (0,2)$

$y = x_1 + x_2 \Rightarrow x_2 = y - x_1$

I.) $y \in (0,1)$



II.) $y \in [1,2)$



$y - x_1 = 1 \Rightarrow$

$x_1 = y - 1$

I.) $y \in (0,1)$

$$F_Y(y) = \int_0^y \int_0^{y-x_1} 1\, dx_2\, dx_1 = \int_0^y [x_2]_0^{y-x_1} dx_1 = \int_0^y (y - x_1)\, dx_1 = \left[y x_1 - \frac{x_1^2}{2} \right]_0^y$$

$$= \left(y \cdot y - \frac{y^2}{2} \right) = \frac{y^2}{2}$$

$$f_Y(y) = \frac{dF_Y(y)}{dy} = \frac{d\left(\frac{y^2}{2}\right)}{dy} = \frac{2 \cdot y}{2} = y$$

II.) $y \in [1,2)$

$$F_Y(y) = \iint 1\, dx_2\, dx_1 + \iint 1\, dx_2\, dx_1 = \int_0^{y-1}\int_0^1 1\, dx_2\, dx_1 + \int_{y-1}^1 \int_0^{y-x_1} 1\, dx_2\, dx_1 =$$

$$= \int_0^{y-1} [x_2]_0^1\, dx_1 + \int_{y-1}^1 [x_2]_0^{y-x_1}\, dx_1 = \int_0^{y-1} 1\, dx_1 + \int_{y-1}^1 (y - x_1)\, dx_1 = [x_1]_0^{y-1} + \left[y x_1 - \frac{x_1^2}{2} \right]_{y-1}^1 =$$

$$= (y-1) + \left[\left(y \cdot 1 - \frac{1^2}{2} \right) - \left(y(y-1) - \frac{(y-1)^2}{2} \right) \right] = y - 1 + \left[\left(y - \tfrac{1}{2} \right) - \left(y^2 - y - \frac{y^2 - 2y + 1}{2} \right) \right] =$$

$$= (y-1) + (y - 1/2) - \left(\frac{2y^2 - \cancel{2y} - y^2 + \cancel{2y} - 1}{2}\right) = 2y - 3/2 - \left(\frac{y^2-1}{2}\right) = \frac{-y^2}{2} + 2y - 1$$

$$f_Y(y) = \frac{dF_Y(y)}{dy} = \frac{d\left(\frac{-y^2}{2} + 2y - 1\right)}{dy} = \frac{-2y}{2} + 2 = 2 - y$$

<u>Záver:</u> $f_Y(y) = \begin{cases} y & y \in (0,1) \\ 2-y & y \in [1,2) \\ 0 & \text{inak} \end{cases}$