104 $X_1,\dots,X_n \sim Ex(\lambda)$ nez.

$f_{X_i}(x_i)=\begin{cases}\lambda e^{-\lambda x_i}\\ 0\end{cases}$ $\quad F_{X_i}(x_i)=\begin{cases}1-e^{-\lambda x_i} & x_i\geq 0\\ 0 & \text{inak}\end{cases}$

a) $Y=\max\{X_1,\dots,X_n\}$

$F_Y(y)=P(Y\leq y)=P(\max\{X_1,\dots,X_n\}\leq y)=P(X_1\leq y, X_2\leq y,\dots,X_n\leq y)\overset{\text{nez.}}{=}$
$=P(X_1\leq y)\cdot P(X_2\leq y)\cdot\ldots\cdot P(X_n\leq y)=F_{X_1}(y)\cdot\ldots\cdot F_{X_n}(y)=\underbrace{(1-e^{-\lambda y})\cdot\ldots\cdot(1-e^{-\lambda y})}_{n}=\underline{\underline{(1-e^{-\lambda y})^n}}$

$F_Y(y)=\begin{cases}0 & y<0\\ (1-e^{-\lambda y})^n & y\geq 0\end{cases}$

b) $Z=\min\{X_1,\dots,X_n\}$

$F_Z(z)=P(Z\leq z)=P(\min\{X_1,\dots,X_n\}\leq z)=1-P(\min\{X_1,\dots,X_n\}>z)=1-P(X_1>z, X_2>z,\dots X_n>z)$

z nez. platí: $P(X_1\leq z,\dots,X_n\leq z)=P(X_1\leq z)\cdot\ldots\cdot P(X_n\leq z)$ $\quad F(z,\dots,z)=F_{X_1}(z)\dots F_{X_n}(z)$

ak vymeníme ľubov. # javov za javy opačné tj $X_i>z$, stále javy nez. a platí!

$=1-P(X_1>z)\cdot P(X_2>z)\dots P(X_n>z)=1-(1-F_{X_1}(z))(1-F_{X_2}(z))\dots(1-F_{X_n}(z))$
$=1-(\cancel{1}-\cancel{1}+e^{-\lambda z})^n=1-e^{-\lambda n z}\quad\sim Ex(\lambda n)$

c) $P(Z>t)=1-P(Z\leq t)=1-F_Z(t)=1-(1-e^{-\lambda n\cdot t})=\underline{\underline{e^{-\lambda n}}}\quad t>0,\ 0$ inak

d) $P(Y\leq t)=F_Y(t)=\underline{\underline{(1-e^{-\lambda t})^n}}\quad t>0,\ 0$ inak

105 $X_1\sim Ex(\lambda_1)$, $X_2\sim Ex(\lambda_2)$ nez. $\quad f_{X_i}(x_i)=\begin{cases}\lambda_i e^{-\lambda_i x_i} & x_i\geq 0\\ 0 & \text{inak}\end{cases}\quad F_{X_i}(x_i)=\begin{cases}1-e^{-\lambda_i x_i} & x_i\geq 0\\ 0 & \text{inak}\end{cases}\quad t\geq 0$

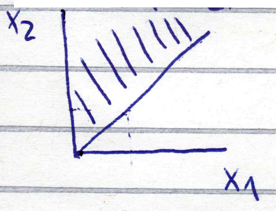
a) $P(X_1>t)=1-P(X_1\leq t)=1-F_{X_1}(t)=1-(1-e^{-\lambda_1 t})=\underline{\underline{e^{-\lambda_1 t}}}$

b) $P(X_1>t, X_2>t)=P(X_1>t)P(X_2>t)=(1-F_{X_1}(t))(1-F_{X_2}(t))=e^{-\lambda_1 t}\cdot e^{-\lambda_2 t}=e^{-(\lambda_1+\lambda_2)t}$

c) $P([X_1>t\wedge X_2\leq t]\vee(X_1\leq t\wedge X_2>t)])=P(X_1>t, X_2\leq t)+P(X_1\leq t, X_2>t)-$
$\underbrace{P(X_1\geq t, X_2\leq t, X_1\leq t, X_2>t)}_{\text{nemožný}}\overset{\text{nez.}}{=}P(X_1>t)\cdot P(X_2\leq t)+P(X_1\leq t)P(X_2>t)$
$=(1-F_{X_1}(t))F_{X_2}(t)+F_{X_1}(t)(1-F_{X_2}(t))=e^{-\lambda_1 t}(1-e^{-\lambda_2 t})+(1-e^{-\lambda_1 t})e^{-\lambda_2 t}$

d) $P(X_1>t\vee X_2>t)=1-P(X_1\leq t, X_2\leq t)=1-F_{X_1}(t)\cdot F_{X_2}(t)=1-(1-e^{-\lambda_1 t})(1-e^{-\lambda_2 t})$

e) $P(X_2>X_1)=\int_0^\infty\int_{x_1}^\infty f(x_1,x_2)\,dx_2\,dx_1\overset{\text{nez.}}{=}\int_0^\infty\int_{x_1}^\infty f(x_1)\cdot f(x_2)\,dx_2\,dx_1$



$=\int_0^\infty\int_{x_1}^\infty\lambda_1\lambda_2 e^{-\lambda_1 x_1}e^{-\lambda_2 x_2}\,dx_2\,dx_1=$

$=\int_0^\infty\lambda_1\lambda_2 e^{-\lambda_1 x_1}\left[e^{-\lambda_2 x_2}\frac{1}{-\lambda_2}\right]_{x_1}^\infty=\int_0^\infty\lambda_1 e^{-\lambda_1 x_1}(-0+e^{-\lambda_2 x_1})\,dx_1$

$=\lambda_1\int_0^\infty e^{-(\lambda_1+\lambda_2)x_1}dx_1=\lambda_1\cdot\left[e^{-(\lambda_1+\lambda_2)x_1}\frac{-1}{\lambda_1+\lambda_2}\right]_0^{\text{inf}}=\lambda_1\cdot\left(-0+\frac{1}{\lambda_1+\lambda_2}e^0\right)=\underline{\underline{\frac{\lambda_1}{\lambda_1+\lambda_2}}}$

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